Problem 2: Selection and Bubble sorts.

Part 1: Mathematical Analysis of Selection Sort:

```
indx, min;
for( pos = 0, pos < n-1, pos++ ){
    min = a[pos]; indx = pos;
    for( i = pos+1; i < n; i++){
        if ( a[i] < min){
            min = a[i];
            indx = i;
        }
    a[indx] = a[pos];
    a[pos] = min;
}
```

O → represents operations, which equate to clock cycles.

O_p → operations in the 1st loop.

O_i → operations in the 2nd loop.

PO_s → operations for the swap (only happens sometimes.)

— That is why there is a probability P.

Then:

$$\sum_{pos=0}^{n-2} O_p + \sum_{i=pos+1}^{n-1} (O_i + PO_s).$$

$*\ \sum_{i=m}^{n} 1 = n-m+1.$ So, $\sum_{i=pos+1}^{n-1} = (n-1)-(pos+1)+1$

$= n-pos-1.$

Let $O_{is} = O_i + PO_s$. Then,

$$\sum_{pos=0}^{n-2} O_p + (n-pos-1)\,O_{is}$$

$$= \sum_{pos=0}^{n-2} O_p + (n-1)O_{is} - pos \cdot O_{is}$$

$*\ \sum_{i=0}^{n} i = \frac{n \cdot (n+1)}{2}.$ So, $\sum_{pos=0}^{n-2} O_{is} \cdot pos = O_{is} \cdot \frac{(n-2)(n-1)}{2}.$

Then:

$$(n-1)(O_p + (n-1)O_{is}) + \frac{(n-2)(n-1)}{2} O_{is}$$

$$= (n-1)^2 O_{is} + (n-1)O_p + \frac{(n-2)(n-1)}{2} O_{is}.$$

$$= (3O_{is})n^2 + (2O_p - 7O_{is})n + (4O_{is} - 2O_p)$$

$$= c''n^2 + c'n + c,$$

where $c'' = 3O_{is}$

$c' = 2O_p - 7O_{is}$

$c = 4O_{is} - 2O_p.$

* $f(n)$ = 2nd order polynomial.

∴ Selection Sort = $O(n^2)$ for all $n > 0$

where $c'' > 3O_{is}$.

Part 2: Mathematical Analysis of Bubble Sort:

```
swap;
do {
    swap= false;
    for (i=0; i<n-1; i++){
        if (a[i] > a[i+1]) {
            temp= a[i];
            a[i] = a[i+1];
            a[i+1] = temp;
            swap= true;
        }
    }
}while(swap);
```

O_{dw} – operations in our do-while loop.

O_i – operations in our for loop.

PO_s – swap operations with a probability P.

Then:

$$\sum Odw + \sum_{i=0}^{n-2} (Oi + pOs).$$

* $\sum_{i=m}^{n} 1 = n-m+1$. So, $\sum_{i=0}^{n-2} = n-2-0+1 = n-1$.

Let $Ois = Oi + pOs$.

$\Rightarrow \sum Odw + (n-1) Ois$.

$\uparrow$

This outer loop runs at most $n-1$ times. Suppose it runs $n-1$ times, then:

$$(n-1)[\,Odw + (n-1)\,Ois\,]$$

$$= (n-1)^2\,Ois + (n-1)\,Odw.$$

$$= (Ois)\,n^2 + (-2Ois + Odw)\,n + (Ois - Odw).$$

$$= c''\,n^2 + c'n + c,$$

where $c'' = Ois$

$c' = -2Ois + Odw$

$c = Ois - Odw$.

* $f(n) =$ 2nd order polynomial.

$\therefore$ Bubble sort $= O(n^2)$ for all $n > 0$

where $c'' > Ois$.

Selection and bubble sorts are both $O(n^2)$, but the selection sort is more efficient.