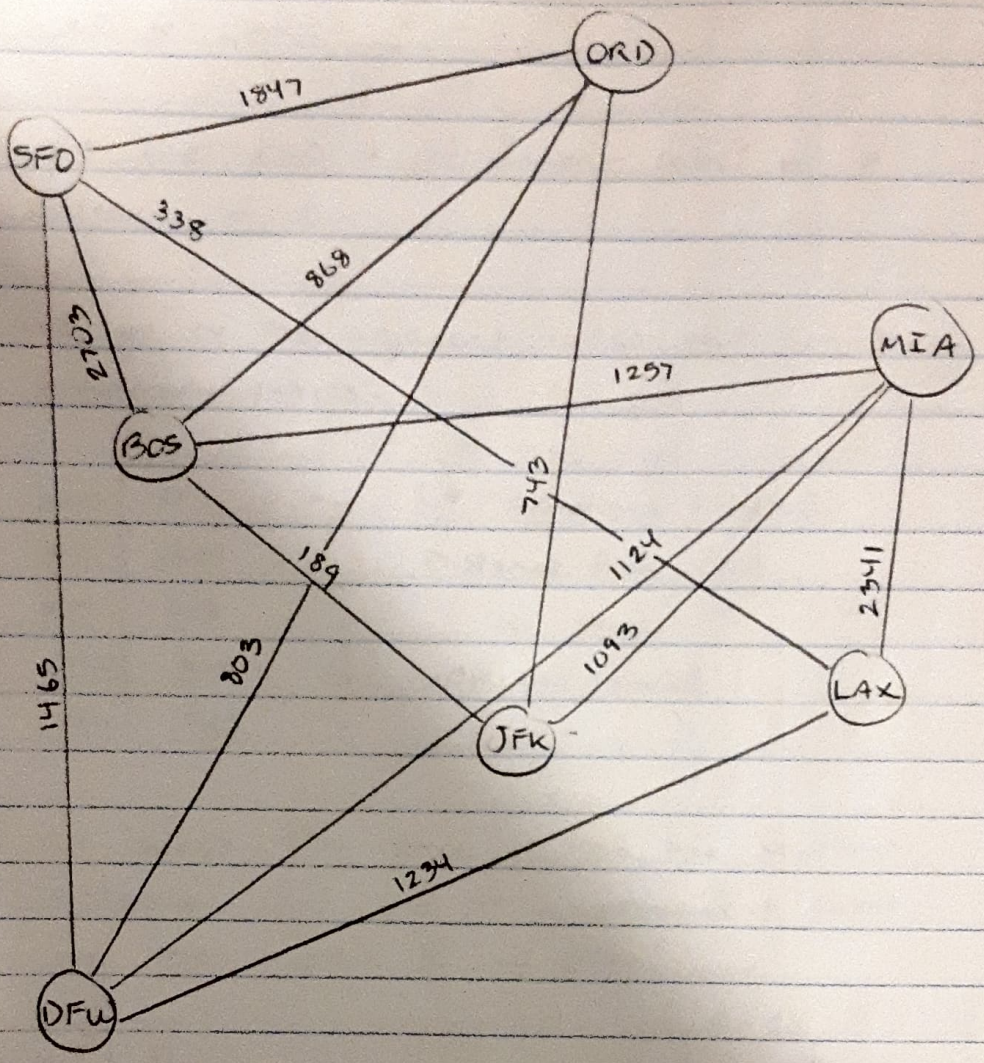
Problem 6: Graph Algorithms.




The edge-vertex structure provided for problem 6 translates to the graph above. This graph will be referenced for the problems to follow.

A) Find the shortest distance from ORD to LAX.

① we give ORD a permanent label w/ a distance of 0.

② we go to the adjacent nodes and give them temporary labels.

| | | |
|---|---|---|
| * | ORD | 0 |
| | SFO | 1847 |
| | BOS | 868 |
| | DFW | 803 |
| | JFK | 743 |

(* - Permanent Labels)
Distance from ORD to those adjacent nodes.

③ we select JFK since it has the shortest distance and give it a permanent label, we then repeat ② w/ JFK.

| | | |
|---|---|---|
| * | ORD | 0 |
| * | JFK | 743 |
| | BOS | 868 |
| | MIA | 1836 |

Distance from ORD to these nodes.

* 743 + 184 = 932, but since 932 > 868, we leave the temp. label for BOS alone.

④ we select the next shortest distance that has no permanent label. This will be DFW. we give it a permanent label and repeat ② w/ it.

| | |
|---|---|
| * ORD | 0 |
| * JFK | 743 |
| * DFW | 803 |
| SFO | 1847. |
| MIA | 1836 |
| LAX | 2037 |

* The new sum for SFO isn't smaller, so we leave its label alone.
* The same goes for MIA.

⑤ we select the next node w/ the shortest distance. That will be BOS. we give it a permanent label and repeat ② w/ it.

| | |
|---|---|
| * ORD | 0 |
| * JFK | 743 |
| * DFW | 803 |
| * BOS | 868 |
| SFO | 1847 |
| MIA | 1836 |

* we don't update the distances for SFO and MIA as the new sums are not smaller.

⑥ we select the next node w/ the shortest distance. That will be MIA. we give it a permanent label and repeat ② w/ it.

| | |
|---|---|
| * ORD | 0 |
| * JFK | 743 |
| * DFW | 803 |
| * BOS | 868 |
| * MIA | 1836 |
| LAX | 2037 |

* we don't update the distance for LAX as the new sum is not smaller.

⑦ we select SFO as our next node. we permanently label it and repeat ② w/ it.

| | |
|---|---|
| * ORD | 0 |
| * JFK | 743 |
| * DFW | 803 |
| * BOS | 868 |
| * MIA | 1836 |
| * SFO | 1847 |
| LAX | 2037 |

* we don't update the distance for LAX as the new sum is not smaller.

⑧ Lastly, we select LAX and give it a permanent label. Since our ending vertex has a permanent label, we're done.

⑨ We need to back track from LAX to ORD. From LAX, we go to DFW. From DFW, we go to ORD.

Shortest path: ORD → DFW → LAX.

DONE.

B) Find the shortest distance from JFK to SFO.

① we give JFK a permanent label w/ a distance of 0.

② we go to the adjacent nodes and give them temporary labels.

| | |
|---|---|
| * JFK | 0 |
| ORD | 743 |
| BOS | 189 |
| MIA | 1093 |

} shortest distances will be stored.

③ we select the node w/ the shortest distance. That will be BOS. we give it a permanent label and repeat ② w/ it.

| | |
|---|---|
| * JFK | 0 |
| * BOS | 189 |
| SFO | 2892 |
| ORD | 743 |
| MIA | 1093 |

* we don't update the labels of ORD and MIA as the new sums are not smaller.

④ we select the next node w/ the shortest distance. That will be ORD. we give it a permanent label and repeat ② w/ it.

| | |
|---|---|
| * JFK | 0 |
| * BOS | 189 |
| * ORD | 743 |
| SFO | 2590 |
| DFW | 1946 |

* we updated the distance for SFO because the new sum was smaller.

⑤ we select MIA as our next node and give it a permanent label. we also repeat ② w/ it.

| | |
|---|---|
| * JFK | 0 |
| * BOS | 189 |
| * ORD | 743 |
| * MIA | 1093 |
| LAX | 3434 |
| DFW | 1946 |

* we don't update the distance for DFW as the new sum is not smaller.

⑥ we select DFW as our next node and give it a permanent label. we also repeat ② w/ it.

| | |
|---|---|
| * JFK | 0 |
| * BOS | 189 |
| * ORD | 743 |
| * MIA | 1093 |
| * DFW | 1946 |
| LAX | 2780 |
| SFO | 2590 |

* we update the distance for LAX. we don't update the distance for SFO.

⑦ we select SFO as our next node and give it a permanent label. since our ending vertex has a permanent label, we're done.

⑧ we can now backtrack from SFO to JFK. From SFO, we go to ORD. From ORD, we go to JFK.

Shortest path: JFK → ORD → SFO.

DONE.

c) Find the minimum spanning tree.

① Sorting the edges.

| | | |
|---|---|---|
| JFK | → BOS | 189 |
| SFO | → LAX | 338 |
| ORD | → JFK | 743 |
| DFW | → ORD | 803 |
| ORD | → BOS | 868 |
| MIA | → JFK | 1093 |
| DFW | → MIA | 1124 |
| LAX | → DFW | 1234 |
| MIA | → BOS | 1257 |
| SFO | → DFW | 1465 |
| SFO | → ORD | 1847 |
| LAX | → MIA | 2341 |
| SFO | → BOS | 2703 |

② Selecting the shortest edge.

1. JFK → BOS 189

③ Selecting the next shortest edge that doesn't form a cycle.

1. JFK → BOS 189
2. SFO → LAX 338

④ Repeating ③ until all vertices are connected. (until we have a tree → N nodes, N-1 edges)

1. JFK → BOS 189
2. SFO → LAX 338
3. ORD → JFK 743

Then,

1. JFK → BOS 189
2. SFO → LAX 338
3. ORD → JFK 743
4. DFW → ORD 803

Then,

1. JFK → BOS 189
2. SFO → LAX 338
3. ORD → JFK 743
4. DFW → ORD 803
5. MIA → JFK 1093

Then,

1. JFK → BOS 189
2. SFO → LAX 338
3. ORD → JFK 743
4. DFW → ORD 803
5. MIA → JFK 1093
6. LAX → DFW 1234.

we have 7 vertices and 6 edges.
we're done, and the total cost of our MST = 4400.