Quiz 4a : (3-30-2022) Aamir Khan

① List the steps of a proof.

1. Restate the theorem.
2. Mark the start of the proof w/ "Proof".
3. Introduce and explain all variables you use.
4. Use complete sentences.
5. Provide all reasons for your statements/ assertions.
6. Display all equations and inequalities.

② List common mistakes.

1. Arguing from examples
2. Jumping to conclusions
3. Circular reasoning
4. Confusing variables
5. Confusing "any" with "some".

③ Is $0.491491491... = 0.\overline{491} \in \mathbb{Q}$? Derive.

Let $x = 0.\overline{491}$. If we take $1000x$, we get: $491.\overline{491}$.

Then, $1000x - x = 491.\overline{491} - 0.\overline{491} = 491$

$999x = 491$

$x = \frac{491}{999} \in \mathbb{Q}$ since 491 and 999 are integers and $999 \neq 0$.

(4) Prove the following: $r, s \in \mathbb{Q} \rightarrow (\frac{3}{4}r + \frac{2}{3}s) \in \mathbb{Q}$. Derive.

By definition of rational, $r = \frac{a}{b}$ and $s = \frac{c}{d}$

for some integers $a, b, c,$ and $d$ where $b \neq 0$ and $d \neq 0$.

Then $\frac{3}{4}r + \frac{2}{3}s = \frac{3}{4}\left(\frac{a}{b}\right) + \frac{2}{3}\left(\frac{c}{d}\right) = \frac{3a}{4b} + \frac{2c}{3d}$

$\frac{3a}{4b} + \frac{2c}{3d} = \frac{9ad}{12bd} + \frac{8bc}{12bd} = \frac{9ad + 8bc}{12bd}$.

Let $p = 9ad + 8bc$ and $q = 12bd$. Then $p$ and $q$ are integers because sums and products of integers are integers and $q \neq 0$ by the zero product property.

So, $\frac{3}{4}r + \frac{2}{3}s = \frac{p}{q}$ where $p$ and $q$ are integers and $q \neq 0$.

$\therefore$ by definition of rational, $\frac{3}{4}r + \frac{2}{3}s \in \mathbb{Q}$

if $r$ and $s$ are rational numbers.

⑤ Prove the following: $r, s \in \mathbb{Z}^+$, $r \% 2 = 1$, $s \% 2 = 0) \rightarrow (r * s) \% 2 = 0$

Proof:

Suppose $r$ and $s$ are positive integers such that $r \% 2 = 1$ and $s \% 2 = 0$.

By definition of quotient-remainder ~~divisibility~~, $r \% 2 = 1$ means $r = 2k + 1$ for some integer $k$.

Then $r$ is odd by definition of odd.

Similarly, $s \% 2 = 0$ means that $s = 2p$ for some integer $p$.

Then $s$ is even by definition of even.

$$
\begin{aligned}
\text{Then } (r * s) \% 2 &= ((2k+1) * (2p)) \% 2 \\
&= (4kp + 2p) \% 2 \\
&= (2 \cdot (2kp + p)) \% 2
\end{aligned}
$$

Let $z = 2kp + p$. Then $z$ is an integer since the product and sums of integers are integers.

So, $(r * s) \% 2 = 2z \% 2$ where $z$ is an integer and $r * s = 2z$ is even by definition of even.

Since $r * s$ is even, it equals twice some integer with no remainder as we've shown above.

$(r * s = 2z + \underline{0} \rightarrow$ remainder of $0)$

$\therefore (r * s) \% 2 = 0$ if $r \% 2 = 1$ and $s \% 2 = 0$.