Chapters 6, 7, & 8 HW Cover Page:

Problems Done:

6.1: 5, 8, 9, 13, 16, 21, 24, 28, 29, 32 (ALL)

7.1: 6, 10, 15, 18, 25, 30, 35, 40, 45, 51 (ALL)

8.1: 2, 4, 5, 8, 10, 12, 16, 19, 23, 24 (ALL)

8.2: 6, 11, 15, 20, 25, 31, 37, 43, 51 (ALL)

8.3: 5, 10, 11, 15, 20, 26, 32, 38, 42 (ALL)

Problems Not Done: NONE

Ch. 6 Sect. 1 Prob. 5

Let $C = \{n \in \mathbb{Z} \mid n = 6r - 5$ for some integer $r\}$ and

$D = \{m \in \mathbb{Z} \mid m = 3s + 1$ for some integer $s\}$.

Prove or Disprove.

a) $C \subseteq D$ (For $C \subseteq D$, every element in C would need to be in D)

(For $C \not\subseteq D$, one element in C wouldn't be in D)

$n = 6r - 5$

$= 6r - 6 + 1 = 3(2r - 2) + 1$

$\rightarrow$ Let $s = 2r - 2$ where s is an integer.

So, $n = 3s + 1$ for some integer s.

Hence, every element in C is in D and $C \subseteq D$.

b) $D \subseteq C$

$\rightarrow$ $D \not\subseteq C$ because there are elements in D that aren't in C. For example, 4 is in D because $4 = 3 \cdot 1 + 1$, but 4 isn't in C since $4 = 6r - 5$ would imply that $r = 9/6$ which isn't an integer.

Hence, $D \not\subseteq C$.

Ch. 6 Sect. 1 Prob. 8

Write in words how to read each. Then write the shorthand notn.

a) $\{x \in U \mid x \in A \text{ and } x \in B\}$

"The set of all x in U such that x is in A and x is in B." $\rightarrow A \cap B$

b) $\{x \in U \mid x \in A \text{ or } x \in B\}$

"The set of all x in U such that x is in A or x is in B." $\rightarrow A \cup B$

c) $\{x \in U \mid x \in A \text{ and } x \notin B\}$

"The set of all x in U such that x is in A and x is not in B." $\rightarrow A - B$

d) $\{x \in U \mid x \notin A\}$

"The set of all x in U such that x is not in A."

$\rightarrow A^c$.

Ch. 6 Sect. 1 Prob. 9

Complete the following w/o using the symbols

$\cup$, $\cap$, or $-$.

a) $x \notin A \cup B$ iff ...

$x \notin A$ and $x \notin B$.

b) $x \notin A \cap B$ iff ...

$x \notin A$ or $x \notin B$

c) $x \notin A - B$ iff ...

$x \notin A$ or $x \in B$

Ch. 6 Sect. 1 Prob. 13

Indicate which are true and which are false.

a) $\mathbb{Z}^+ \subseteq \mathbb{Q}$ TRUE, any integer can be written as a rational over 1. (ex. $4 = 4/1$ (rational)).

b) $\mathbb{R}^- \subseteq \mathbb{Q}$ False, since the set of rational numbers include irrationals which aren't in $\mathbb{Q}$.

c) $\mathbb{Q} \subseteq \mathbb{Z}$ False, not all rationals are integers.

d) $\mathbb{Z}^- \cup \mathbb{Z}^+ = \mathbb{Z}$ False, we're missing the 0 vector.

e) $\mathbb{Z}^- \cap \mathbb{Z}^+ = \emptyset$ True, no elements in common.

f) $\mathbb{Q} \cap \mathbb{R} = \mathbb{Q}$ True, since $\mathbb{Q} \subseteq \mathbb{R}$ and all rationals are reals and all reals are rationals.

g) $\mathbb{Q} \cup \mathbb{Z} = \mathbb{Q}$ True, all integers are rationals.

h) $\mathbb{Z}^+ \cap \mathbb{R} = \mathbb{Z}^+$ True, since $\mathbb{Z}^+ \subseteq \mathbb{R}$ so any element in $\mathbb{Z}^+$ is also in $\mathbb{R}$.

i) $\mathbb{Z} \cup \mathbb{Q} = \mathbb{Z}$ False, because not all rationals are integers.

Ch. 6 Sect. 1 Prob. 16

Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, and $C = \{b, c, e\}$.

a) Find $A \cup (B \cap C)$, $(A \cup B) \cap C$ and $(A \cup B) \cap (A \cup C)$, which of these are equal?

① $A \cup (B \cap C) = \{a, b, c\} \cup \{b, c\}$
$= \{a, b, c\}$

② $(A \cup B) \cap C = \{a, b, c, d\} \cap \{b, c, e\}$
$= \{b, c\}$

③ $(A \cup B) \cap (A \cup C) = \{a, b, c, d\} \cap \{a, b, c, e\}$
$= \{a, b, c\}$

$\rightarrow A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

b) Find $A \cap (B \cup C)$, $(A \cap B) \cup C$, and $(A \cap B) \cup (A \cap C)$.

① $A \cap (B \cup C) = \{a, b, c\} \cap \{b, c, d, e\}$
$= \{b, c\}$

② $(A \cap B) \cup C = \{b, c\} \cup \{b, c, e\}$
$= \{b, c, e\}$

③ $(A \cap B) \cup (A \cap C) = \{b, c\} \cup \{b, c\}$
$= \{b, c\}$

$\rightarrow A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

c) Find $(A - B) - C$ and $A - (B - C)$. Are these equal?

① $(A - B) - C = \{a\} - \{b, c, e\}$
$= \{a\}$

② $A - (B - C) = \{a, b, c\} - \{d\}$
$= \{a, b, c\}$

→ They are NOT equal.

Ch. 6 Sect. 1 Prob. 21

Let $C_i = \{i, -i\}$ for all nonnegative integers i.

a) $\bigcup_{i=0}^{4} C_i = C_0 \cup C_1 \cup C_2 \cup C_3 \cup C_4$

$= \{0,0\} \cup \{1,-1\} \cup \cdots \cup \{4,-4\}$

$= \{-4,-3,-2,-1,0,1,2,3,4\}$

b) $\bigcap_{i=0}^{4} C_i = C_0 \cap C_1 \cap C_2 \cap C_3 \cap C_4$

$= \{0,0\} \cap \cdots \cap \{4,-4\}$

$= \phi$, since no elements are common.

c) Are $C_0, C_1, C_2, \ldots$ mutually disjoint?

Yes, because they have no elements in common between them.

d) $\bigcup_{i=0}^{n} C_i = C_0 \cup C_1 \cup \cdots \cup C_n$

$= \{-n, -(n-1), \ldots, 0, \ldots, n-1, n\}$

e) $\bigcap_{i=0}^{n} C_i = \phi$ because they're mutually disjoint.

f) $\bigcup_{i=0}^{\infty} C_i = C_0 \cup C_1 \cup \cdots$

$= \{-\infty, \ldots, -1, 0, 1, \ldots, \infty\}$

$= Z$ (set of all integers)

g) $\bigcap_{i=0}^{\infty} C_i = \phi$ because they're mutually disjoint.

Ch. 6 Sect 1 Prob. 24

Let $W_i = \{x \in \mathbb{R} \mid x > i\} = (i, \infty)$ for all nonnegative integers i.

a) $\bigcup_{i=0}^{4} W_i = W_0 \cup W_1 \cup W_2 \cup W_3 \cup W_4$

$= (0,\infty) \cup (1,\infty) \cup (2,\infty) \cup (3,\infty) \cup (4,\infty)$

$= \{0, 1, 2, \ldots, \infty\} = (0,\infty)$

b) $\bigcap_{i=0}^{4} W_i = W_0 \cap \ldots \cap W_4$

$= (0,\infty) \cap (1,\infty) \cap (2,\infty) \cap (3,\infty) \cap (4,\infty)$

$= (4,\infty)$

c) Are $W_0, W_1, W_2, \ldots$ mutually disjoint?

No, because they have elements in common.

d) $\bigcup_{i=0}^{n} W_i = W_0 \cup W_1 \cup \ldots \cup W_n$

$= (0,\infty)$.

e) $\bigcap_{i=0}^{n} W_i = W_0 \cap W_1 \cap \ldots \cap W_n$

$= (n,\infty)$

f) $\bigcup_{i=0}^{\infty} W_i = W_0 \cup W_1 \cup \ldots$

$= (0,\infty)$

g) $\bigcap_{i=0}^{\infty} W_i = W_0 \cap W_1 \cap \ldots$

$= \emptyset$, because it goes out to infinity and there is never a final W_i that we can use to determine our left bound. $(\infty,\infty) = \emptyset$

Ch. 6 Sect 1 Prob. 28

Let E be the set of all even integers and O be the set of all odd integers. Is $\{E, O\}$ a partition of $\mathbb{Z}$?

Yes, because all integers are either even or odd. For ex, 0 is even because $0 = 2 \cdot 0$. We also know that integers can't be both even and odd.

* A finite collection of sets $\{A_1, A_2, \ldots, A_n\}$ is a partition of a set A iff.

① A is the union of all the A_i

② The sets are mutually disjoint.

- We know that integers can't be both even and odd, so E and O are mutually disjoint.
- We know that $\mathbb{Z} = E \cup O$ because integers are either even or odd.

Ch. 6 Sect. 1 Probs. 29

Let R be the set of all real numbers. Is $\{R^+, R^-, \{0\}\}$ a partition of R?

① Need to be mutually disjoint.

② R needs to be the union of all our sets.

∘ $R = R^+ \cup R^- \cup \{0\}$ → True, so R is the set of all positive reals, negative reals, and includes the element 0.
Hence, it is the union of all 3.

∘ Moreover, $R^+ \cap R^- = \phi$, $R^+ \cap 0 = \phi$, $R^- \cap 0 = \phi$, so the sets are all mutually disjoint.
(They share no common elements)

Therefore, $\{R^+, R^-, \{0\}\}$ is a partition of R.

Ch. 6 Sect 1 Prob. 32

a) Suppose $A = \{1\}$ and $B = \{u, v\}$. Find $\mathcal{P}(A \times B)$

$A \times B = \{(1,u), (1,v)\}$

$\mathcal{P}(A \times B)$ = set of all subsets of $A \times B$.

$= \{\emptyset, \{(1,u)\}, \{(1,v)\}, \{(1,u),(1,v)\}\}$.

b) Suppose $X = \{a, b\}$ and $Y = \{x, y\}$. Find $\mathcal{P}(X \times Y)$

$X \times Y = \{(a,x), (a,y), (b,x), (b,y)\}$

$\mathcal{P}(X \times Y) = \{\emptyset, \{(a,x)\}, \{(a,y)\}, \{(b,x)\}, \{(b,y)\},$
$\{(a,x),(a,y)\}, \{(a,x),(b,x)\}, \{(a,x),(b,y)\},$
$\{(a,y),(b,x)\}, \{(a,y),(b,y)\}, \{(b,x),(b,y)\},$
$\{(a,x),(a,y),(b,x)\}, \{(a,x),(a,y),(b,y)\},$
$\{(a,x),(b,x),(b,y)\}, \{(a,y),(b,x),(b,y)\},$
$\{(a,x),(a,y),(b,x),(b,y)\}\}$

Ch. 7 Sect. 1 Prob. 6

Find functions defined on the set of nonnegative integers that define the sequences whose 1st 6 terms are given.

a) $1, -1/3, 1/5, -1/7, 1/9, -1/11$

b) $0, -2, 4, -6, 8, -10$

For a) we have:

- we should start from 0, since we're dealing w/ the set of nonnegative integers $\{0, 1, 2, 3, \dots\}$.

$1, -1/3, 1/5, -1/7, 1/9, -1/11$

Terms 0 1 2 3 4 5

$\downarrow \quad \downarrow \quad \downarrow$

$\frac{1}{2\cdot 0+1} \quad \frac{1}{2\cdot 2+1} \quad \frac{1}{2\cdot 4+1}$

* odd integers: $2k+1$ for some k. (This is how we get $2n+1$)

- we also need an alternator.

$\rightarrow$ $f: \mathbb{Z}^{\text{nonneg}} \to \mathbb{R} = \dfrac{(-1)^n}{2n+1}, \ n \in \mathbb{Z}^{\text{nonneg}}$.

For b) we have:

$0, -2, 4, -6, 8, -10$

Terms 0 1 2 3 4 5

$\rightarrow$ $f: \mathbb{Z}^{\text{nonneg}} \to \mathbb{R} = (-1)^n \cdot 2n, \ n \in \mathbb{Z}^{\text{nonneg}}$.

Ch. 7 Sect. 1 Prob. 10

Let D be the set of all finite subsets of positive integers. Define a function $T: Z^+ \to D$ as follows: For each positive integer n, $T(n)$ = the set of positive divisors of n.

Find the following:

a) $T(1)$ c) $T(17)$ e) $T(18)$

b) $T(15)$ d) $T(5)$ f) $T(21)$

a) $T(1) = \{1\}$

b) $T(15) = \{1, 3, 5, 15\}$

c) $T(17) = \{1, 17\}$

d) $T(5) = \{1, 5\}$

e) $T(18) = \{1, 2, 3, 6, 9, 18\}$

f) $T(21) = \{1, 3, 7, 21\}$.

Ch. 7 Sect. 1 Prob. 15

Let F and G be functions from the set of all real numbers to itself. Define the product functions $F \cdot G : R \to R$ and $G \cdot F : R \to R$ as follows: For all $x \in R$,

$$(F \cdot G)(x) = F(x) \cdot G(x)$$

$$(G \cdot F)(x) = G(x) \cdot F(x)$$

Does $F \cdot G = G \cdot F$?

$$\begin{aligned} (F \cdot G)(x) &= F(x) \cdot G(x) && \text{(By def. of } F \cdot G \text{ .)} \\ &= G(x) \cdot F(x) && \text{(By commutative law)} \\ &= (G \cdot F)(x) && \text{(By def. of } G \cdot F \text{ .)} \end{aligned}$$

$\therefore$ $F \cdot G = G \cdot F$ For all real numbers.

Ch. 7 Sect. 1 Prob. 18

Find exact values for each:

a) $\log_3 81 \rightarrow 3^x = 81, \quad x = 4$

b) $\log_2 1024 \rightarrow 2^x = 1024, \quad x = 10$

c) $\log_3 (1/27) \rightarrow 3^x = 1/27, \quad x = -3$

d) $\log_2 1 \rightarrow 2^x = 1, \quad x = 0$

e) $\log_{10} (1/10) \rightarrow 10^x = 1/10, \quad x = -1$

f) $\log_3 3 \rightarrow 3^x = 3, \quad x = 1$

g) $\log_2 (2^k) \rightarrow 2^x = 2^k, \quad x = k.$

Ch. 7 Sect. 1 Prob. 23

Let $A = \{2, 3, 5\}$ and $B = \{x, y\}$. Let p_1 and p_2 be the projections of $A \times B$ onto the 1st and 2nd coordinates. That is, for each pair $(a, b) \in A \times B$, $p_1(a, b) = a$ and $p_2(a, b) = b$.

a) Find $p_1(2, y)$ and $p_1(5, x)$. What is the range of p_1?

b) Find $p_2(2, y)$ and $p_2(5, x)$. What is the range of p_2?

a):

$p_1(2, y) = 2$ and $p_1(5, x) = 5$

Range of p_1 is A because p_1 returns the 1st coordinate which can be any element of A.

b):

$p_2(2, y) = y$ and $p_2(5, x) = x$

Range of p_2 is B because p_2 returns the 2nd coordinate which can be any element of B.

Ch. 7 Sect. 1 Prob. 30

Draw arrow diagrams for the Boolean functions defined by the following input/output tables.

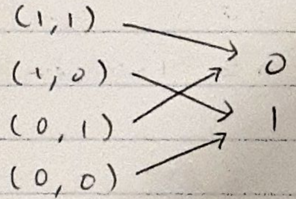
a.

P	Q	R
1	1	0
1	0	1
0	1	0
0	0	1

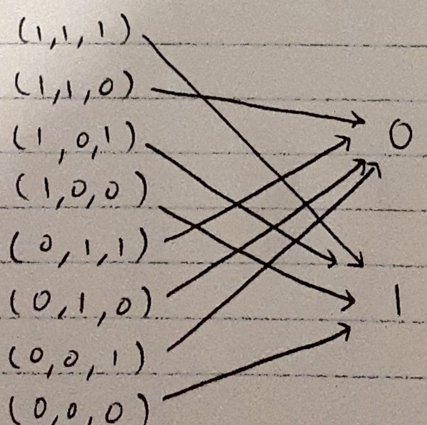
b.

P	Q	R	S
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1

a:



b:



Ch. 7 Sect. 1 Prob. 35

Let $J_5 = \{0,1,2,3,4\}$. Then $J_5 - \{0\} = \{1,2,3,4\}$. Student A tries to define a function $R: J_5 - \{0\} \rightarrow J_5 - \{0\}$ as follows:

For each $x \in J_5 - \{0\}$,

$R(x)$ is the number y so that $(xy) \bmod 5 = 1$.

Student B claims that R is not well-defined. Who is right?

* For a function to be well-defined, it must satisfy the 2 properties for being a function:

① Every x has a y.

② No x can have different y's.

Student B is correct because we see that for $R(3)$, it can be 2 since $(3 \cdot 2) \bmod 5 = 1$, but it can also be 7 since $(3 \cdot 7) \bmod 5 = 1$. Here, 3 maps to both 2 and to 7, so this function is not well-defined.

Ch. 7 Sect. 1 Prob. 40

Let X and Y be sets, let A and B be any subsets of X, and let F be a function from X to Y. Fill in the blanks in the following proof that $F(A) \cup F(B) \subseteq F(A \cup B)$.

Proof: Let y be any element $F(A) \cup F(B)$. By definition of union, $y \in F(A)$ or $y \in F(B)$.

Case 1, $y \in F(A)$: In this case, by def. of $F(A)$, $y = F(x)$ for some $x \in A$. Since $A \subseteq A \cup B$, it follows from the def. of union that $x \in B$. Hence, $y = F(x)$ for some $x \in A \cup B$, and thus, by def. of $F(A \cup B)$, $y \in F(A \cup B)$.

Case 2, $y \in F(B)$: In this case, by def. of $F(B)$, $y = F(x)$ for some $x \in B$. Since $B \subseteq A \cup B$, it follows from the def. of union that $x \in A$. Hence, $y = F(x)$ for some $x \in A \cup B$, and thus, by def. of $F(A \cup B)$, $y \in F(A \cup B)$.

Therefore, regardless of whether $y \in F(A)$ or $y \in F(B)$, we have that $y \in F(A \cup B)$.

Ch. 7 Sect. 1 Prob. 45

Let X and Y be sets, and let C and D be any subsets of Y. Determine if true or false.

For all subsets C and D of Y, if $C \subseteq D$, then $F^{-1}(C) \subseteq F^{-1}(D)$.

TRUE.

Proof: Let F be a function from a set X to a set Y, and suppose $C \subseteq Y$, $D \subseteq Y$, and $C \subseteq D$. Now, suppose $x \in F^{-1}(C)$, then $F(x) \in C$. Since $C \subseteq D$, $F(x) \subseteq D$. By definition of inverse image, $x \in F^{-1}(D)$. So, $F^{-1}(C) \subseteq F^{-1}(D)$.

Ch. 7 Sect. 1 Prob. 51

For each integer $n \geq 1$, $\phi(n)$ is the number of positive integers less than or equal to n that have no common factors with n except ± 1. For example, $\phi(10) = 4$ because there are 4 positive integers ≤ 10 that have no common factors with 10 except ± 1; namely, 1, 3, 7, 9.

(Basically, $\phi(n)$ = the # of integers that are coprime w/ n)

Find the following:

a) $\phi(15) = 8$ because 1, 2, 4, 7, 8, 11, 13, and 14 are coprime w/ 15.

b) $\phi(2) = 1$ because 1 is the only integer ≤ 2 that is coprime w/ 2.

c) $\phi(5) = 4$ because we have 1, 2, 3, 4.

d) $\phi(12) = 4$ because we have 1, 5, 7, 11.

e) $\phi(11) = 10$ because we have 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

Notice that 11 is prime, which is why 1-10 is valid.

f) $\phi(1) = 1$ because 1 is the only integer ≤ 1 that is coprime w/ 1.

Ch. 8 Sect. 1 Prob. 2

Prove that for all integers m and n, $m-n$ is even iff both m and n are even or both are odd.

Proof: Let m and n be integers.

① Case 1, both are even: Then, $m = 2k$ and $n = 2r$ for some integers k and r. $m-n = 2k-2r = 2(k-r)$. Let $z = k-r$, then z is an integer as the difference of integers is an integer. Hence $m-n = 2z$ for some z, and so $m-n$ is even by definition of even.

② Case 2, both are odd: Then $m = 2k+1$ and $n = 2r+1$ for some integers k and r. $m-n = (2k+1)-(2r+1) = 2k-2r = 2(k-r)$. Let $z = k-r$, and so z is an integer. Hence, $m-n = 2z$ for some z, and so $m-n$ is even by definition of even.

$\therefore$ $m-n$ is even iff both are even or both are odd.

Ch. 8 Sect. 1 Prob. 4

Define a relation P on $\mathbb{Z}$ as follows: for all $m, n \in \mathbb{Z}$,

$m \, P \, n \Leftrightarrow m$ and n have a common prime factor.

a. Is $15 \, P \, 25$?

Yes, because 5 is a prime factor of both.

b. Is $22 \, P \, 27$?

No, they have no common prime factors.

c. Is $0 \, P \, 5$?

Yes, because both can be divided by 5.

d. Is $8 \, P \, 8$?

Yes, because both are divisible by 2.

Ch. 8 Sect. 1 Prob. 5

Let $X = \{a, b, c\}$. Define a relation R on $\mathcal{P}(X)$ as follows:

For all $A, B \in \mathcal{P}(X)$, $A \, R \, B \Leftrightarrow$ A has the same # of elements as B.

a. Is $\{a, b\} \, R \, \{b, c\}$?

Yes, they both have 2 elements

b. Is $\{a\} \, R \, \{a, b\}$?

No, they don't have the same # of elements.

c. Is $\{c\} \, R \, \{b\}$?

Yes, both have 1 element.

Ch. 8 Sect. 1 Prob. 8

Let A be the set of all strings of a's and b's of length 4.
Define a relation R on A as follows: For all $s, t \in A$,

$s \, R \, t \Leftrightarrow s$ has the same first 2 characters as t.

a. Is abaa R abba?

Yes, the first 2 characters in both are ab.

b. Is aabb R bbaa?

No, aa $\neq$ bb.

c. Is aaaa R aaab?

Yes, aa = aa,

d. Is baaa R abaa?

No, ba $\neq$ ab

Ch. 8 Sect. 1 Prob. 10

Let $A = \{3, 4, 5\}$ and $B = \{4, 5, 6\}$ and let R be the "less than" relation. That is, for all $(x, y) \in A \times B$,

$$x \, R \, y \Leftrightarrow x < y.$$

State which ordered pairs are in R and R^{-1}.

In R:

$\{(3,4), (3,5), (3,6), (4,5), (4,6), (5,6)\}$

In R^{-1}:

$\{(4,3), (5,3), (6,3), (5,4), (6,4), (6,5)\}$

Ch. 8 Sect. 1 Prob. 12

a. Suppose a function $F: X \to Y$ is 1-to-1, but not onto.
Is F^{-1} a function?

F^{-1} is not a function because if F is not onto, then not all $y \in Y$ map back to an $x \in X$. This would mean that not all elements in the domain of F^{-1} have an element in the codomain, so it would fail the requirement that all elements in the domain need to map to an element in the codomain.

b. Suppose a function $F: X \to Y$ is onto but not 1-to-1.
Is F^{-1} a function?

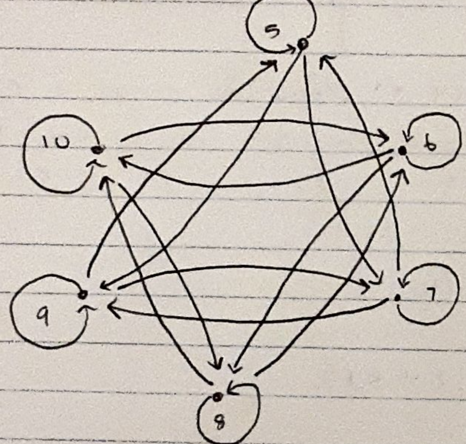
F^{-1} is not a function because if F is not 1-to-1, then many $x \in X$ can have the same $y \in Y$. This would mean that for F^{-1}, a $y \in Y$ will have different x's, so it would fail the property that the same element in the domain can't point to different elements in the codomain.

Ch. 8 Sech 1 Prob. 16

Draw a directed graph.

Let $A = \{5,6,7,8,9,10\}$ and define a relation S on A as follows: For all $x, y \in A$, $x S y \Leftrightarrow 2 \mid (x-y)$.

$S = \{(5,5),(5,7),(5,9),(6,6),(6,8),(6,10),$
$(7,5),(7,7),(7,9),(8,6),(8,8),(8,10),$
$(9,5),(9,7),(9,9),(10,6),(10,8),(10,10)\}$



Ch. 8 Sect. 1 Prob. 19

$R \cup S = \{(x,y) \in A \times B \mid (x,y) \in R \text{ or } (x,y) \in S\}$

$R \cap S = \{(x,y) \in A \times B \mid (x,y) \in R \text{ and } (x,y) \in S\}$

Let $A = \{2,4\}$ and $B = \{6,8,10\}$ and define relations R and S from A to B as follows: For all $(x,y) \in A \times B$,

$x \, R \, y \Leftrightarrow x \mid y$ and

$x \, S \, y \Leftrightarrow y - 4 = x$.

which ordered pairs are in $A \times B$, R, S, $R \cup S$, and $R \cap S$.

$A \times B = \{(2,6),(2,8),(2,10),(4,6),(4,8),(4,10)\}$

$R = \{(2,6),(2,8),(2,10),(4,8)\}$

$S = \{(2,6),(4,8)\}$

$R \cup S = \{(2,6),(2,8),(2,10),(4,8)\}$

$R \cap S = \{(2,6),(4,8)\}$

Ch. 8 Sech 1 prob. 23

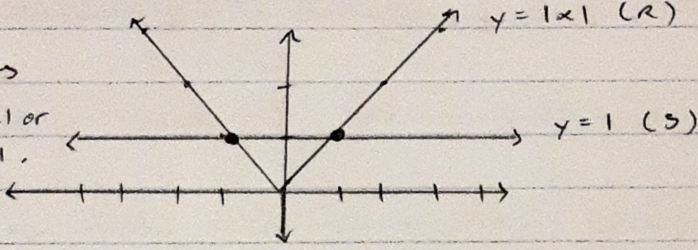
Define relations R and S on $\mathbb{R}$ as follows:

$R = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = |x|\}$ and

$S = \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = 1\}$.

Graph R, S, $R \cup S$, and $R \cap S$.

$R \cup S$ = points on either $y=|x|$ or $y=1$.



$R \cap S$ = points on both $y=|x|$ and $y=1$.

This would be the intersections.

$\{(1,1), (-1,1)\}$.

Ch. 8 Sect. 1 Prob. 24

In Ex 8.1.7 the result of the query SELECT Patient_ID #, Name FROM S WHERE Primary_Diagnosis = X is the projection onto the first 2 coordinates of the intersection of the set $A_1 \times A_2 \times A_3 \times \{X\}$ with the database.

a. Find the result of the query where X = pneumonia.

b. Find the result of the query where X = appendicitis.

(we want the patient ID # and the name which are our first 2 coordinates)

For a. we have:

{(574329, Tak Kurosawa),
(011985, John Schmidt)}

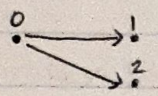
For b. we have:

{(466581, Mary Lazars),
(778400, Jamal Baskers)}

Ch. 8 Sect. 2 Prob. 6

$A = \{0, 1, 2, 3\}$ and $R_6 = \{(0,1), (0,2)\}$.

a. Draw the directed graph.



b. Reflexive?

No, because $(0,0), (1,1), (2,2)$, and $(3,3)$ aren't in R_6.

(R is reflexive iff for all $x \in A$, $(x,x) \in R$)

c. Symmetric?

No, because $(1,0)$ and $(2,0)$ aren't in R_6.

(R is symmetric iff for all $x, y \in A$, if $(x,y) \in R$ then $(y,x) \in R$)

d. Transitive?

No, because we need atleast 3 terms and it would need to satisfy: for all $x, y, z \in A$, if $(x,y) \in R$ and $(y,z) \in R$, then $(x,z) \in R$.

Ch. 8. Sect 2 Prob. 11

Determine whether reflexive, symmetric, transitive, or none.

D is the relation defined on $\mathbb{R}$ as follows: For all $x, y \in \mathbb{R}$,

$x \, D \, y \Leftrightarrow xy \geq 0$.

① For all real numbers x, $x \, D \, x \Leftrightarrow x^2 \geq 0$,

which is true since x^2 will never be < 0.

Hence, D is reflexive.

② For all $x, y \in \mathbb{R}$; $x \, D \, y \Leftrightarrow xy \geq 0$

$\Leftrightarrow yx \geq 0$

$y \, D \, x \Leftrightarrow yx \geq 0$.

This is true since products are commutative,

so D is symmetric.

③ D is not transitive. For example,

Let $x = 2$, $y = 0$, $z = -3$.

Then, $x \, D \, y \Leftrightarrow xy \geq 0 \quad = \quad 0 \geq 0$ True

$y \, D \, z \Leftrightarrow yz \geq 0 \quad = \quad 0 \geq 0$ True

$x \, D \, z \Leftrightarrow xz \geq 0 \quad = \quad -6 \geq 0$ False

Ch. 8 Sect. 2 Prob. 15

Same instructions as #11 on the previous page.

D is the "divides" relation on Z^+: For all positive integers m and n, $m \, D \, n \Leftrightarrow m \mid n$.

① D is reflexive because for all $x \in Z^+$, $x \, D \, x \Leftrightarrow x \mid x \rightarrow$ True since $x = x \cdot 1$.

② D is not symmetric. For example, let $x = 2$, $y = 4$. Then, $x \, D \, y \Leftrightarrow x \mid y$ and $2 \mid 4$ is true since $4 = 2 \cdot 2$. But $4 \nmid 2$ since $2 \neq 4k$ for some $k \in Z^+$.

③ D is transitive because for all $x, y, z \in Z^+$, if $x \mid y$ and $y \mid z$, then $x \mid z$ by the transitivity of divisibility.

Ch 8 Sect. 2 Prob. 20

Same instructions as # 11 (2 pages back).

Let $X = \{a, b, c\}$ and $\mathcal{P}(X)$ be the power set of X. A relation E is defined on $\mathcal{P}(X)$ as follows: for all $A, B \in \mathcal{P}(X)$,

$A \mathrel{E} B \Leftrightarrow$ the # of elements in A equals the # of elements in B.

① E is reflexive because for all $A \in \mathcal{P}(X)$, $A \mathrel{E} A \Leftrightarrow$ # of elements in A = # of elements in A which is true.

② E is symmetric because for all $A, B \in \mathcal{P}(X)$, if $A \mathrel{E} B$, then $B \mathrel{E} A$ since the # of elements in both are equal.

③ E is transitive because for all $A, B, C \in \mathcal{P}(X)$, if $A \mathrel{E} B$ and $B \mathrel{E} C$, then $A \mathrel{E} C$ because they all have the same # of elements.

Ch. 8 Sect. 2 Prob. 25

Same instructions as # 11 (3 pages back).

Let A be the set of all strings of a's and b's of length 4. Define a relation R on A as follows:

For all $s, t \in A$, $s R t \Leftrightarrow s$ has the same first 2 characters as t.

① R is reflexive because for all $s \in A$,

$s R s \Leftrightarrow$ they have the same 1st 2 chars

which is true since they're the same string.

② R is reflexive because for all $s, t \in A$, if

$s R t$, then $t R s$ since they have the

same 1st 2 characters.

③ R is transitive because for all $s, t, r \in A$,

if $s R t$ and $t R r$, then $s R r$ since

the 1st 2 chars are the same in all strings.

Ch. 8 Sect. 2 Prob. 31

Same instructions as #11 (4 pages back).

Let A be the set of people living in the world today. A relation R is defined on A as follows: For all $p, q \in A$, $pRq \Leftrightarrow p$ lives within 100 miles of q.

① R is reflexive since pRp is true for all $p \in A$ since anyone lives within 100 miles of themself.

② R is symmetric because for all $p, q \in A$, if p lives within 100 miles of q, then that means q lives within 100 miles of p.

③ R is not transitive. For example, say p lives within 100 miles of q and q lives within 100 miles of r (not within the same 100 miles between p and q), then the distance between p and r isn't going to be within 100 miles (within 200 miles instead).

Ch. 8 Sect. 2 Prob. 37

Assume that R and S are relations on a set A.

Prove or Disprove.

If R and S are reflexive, is $R \cap S$ reflexive?

Yes, $R \cap S$ is reflexive.

Since R and S are reflexive, for $x \in A$,
$(x,x) \in R$ and $(x,x) \in S$. It is in
both R and S, so $(x,x) \in R \cap S$.
Thus, $R \cap S$ is reflexive.

Ch. 8 Sect. 2 Prob. 43

Determine whether irreflexive, asymmetric, intransitive, or none of these.

* A relation on a set A is:

① irreflexive iff for all $x \in A$, $x \not R x$

② asymmetric iff for all $x, y \in A$, if $x R y$ then $y \not R x$

③ intransitive iff for all $x, y, z \in A$, if $x R y$ and $y R z$, then $x \not R z$.

$A = \{0, 1, 2, 3\}$

$R = \{(0,0), (0,1), (0,3), (1,1), (1,0), (2,3), (3,3)\}$

- R is not irreflexive because $(0,0) \in R$, which means that $x R x$ when $x = 0$.
- R is not asymmetric because $(0,1)$ and $(1,0) \in R$.
- R is not intransitive because $(0,1) \in R$, $(1,0) \in R$, and $(0,0) \in R$.

Ch. 8 Sect. 2 Prob. 51

R is a relation defined on $A = \{0, 1, 2, 3\}$.

Let $R = \{(0,1), (0,2), (1,1), (1,3), (2,2), (3,0)\}$. Find R^t, the transitive closure of R.

R^t is transitive and $R \subseteq R^t$.

$R^t = \{(0,1), (1,1), (1,3), (0,3), (0,2), (2,2),$
$(3,0), (3,1), (3,2), (3,3), (0,0), (1,0),$
$(1,2)\}$

Ch. 8 Sect. 3 Prob. 5

The relation R is an equivalence relation on the set A.

Find the distinct equivalence classes of R.

$A = \{1, 2, 3, 4, \dots, 20\}$. R is defined on A as follows:

For all $x, y \in A$, $x R y \Leftrightarrow 4 \mid (x - y)$.

* $[a] = \{x \in A \mid x R a\}$ (Equivalence class of a)

$[1] = \{x \in A \mid x R 1\} = \{1, 5, 9, 13, 17\}$

$[2] = \{x \in A \mid x R 2\} = \{2, 6, 10, 14, 18\}$

$[3] = \{x \in A \mid x R 3\} = \{3, 7, 11, 15, 19\}$

$[4] = \{x \in A \mid x R 4\} = \{4, 8, 12, 16, 20\}$

→ All other equivalence classes are 1 of the above,

so these are our distinct equivalence classes.

Ch. 8 Sect 3 Prob. 10

Same instructions as # 9 on previous page.

$A = \{-5,-4,-3,-2,-1,0,1,2,3,4,5\}$. R is defined on

A as follows: For all $m, n \in A$,

$m \, R \, n \Leftrightarrow 3 \mid (m^2 - n^2)$.

$[-5] = \{m \in A \mid m \, R \, -5\} = \{m \in A \mid 3 \mid (m^2 - 25)\}$

$= \{-5,-4,-2,-1,1,2,4,5\}$.

$[-4] = \{m \in A \mid m \, R \, -4\} = \{m \in A \mid 3 \mid (m^2 - 16)\}$

$= \{-5,-4,-2,-1,1,2,4,5\}$.

$[-3] = \{m \in A \mid m \, R \, -3\} = \{m \in A \mid 3 \mid (m^2 - 9)\}$

$= \{-3,0,3\}$

⋮

Our distinct classes are:

$\{-5,-4,-2,-1,1,2,4,5\}$

and

$\{-3,0,3\}$

(All others are 1 of the above.)

Ch. 8 Sect. 3 Prob. 11

Same instructions as # 9 (2 pages back).

$A = \{-4,-3,-2,-1,0,1,2,3,4\}$. R is defined on A as follows: For all $(m,n) \in A$,

$m \, R \, n \Leftrightarrow 4 \mid (m^2 - n^2)$.

$[0] = \{m \in A \mid m \, R \, 0\} = \{m \in A \mid 4 \mid m^2\}$
$= \{-4,-2,0,2,4\}$

$[1] = \{m \in A \mid m \, R \, 1\} = \{m \in A \mid 4 \mid (m^2 - 1)\}$
$= \{-3,-1,1,3\}$

All other classes are 1 of the above.

Ch. 8 Sect. 3 Prob. 15

Determine which of the following congruence relations are true or false.

a. $17 \equiv 2 \pmod{5}$

b. $4 \equiv -5 \pmod{7}$

Definition:

* $m \equiv n \pmod{d}$ iff $d \mid (m-n)$.

c. $-2 \equiv -8 \pmod{3}$

d. $-6 \equiv 22 \pmod{2}$

a. True, since $5 \mid (17-2) = 5 \mid 15 \rightarrow$ True.

b. False, since $7 \mid (4-(-5)) = 7 \mid 9 \rightarrow$ False

c. True, since $3 \mid (-2-(-8)) = 3 \mid 6 \rightarrow$ True

d. True, since $2 \mid (-6-22) = 2 \mid -28 \rightarrow$ True

Ch. 8 Sect. 3 prob. 20

Prove that the relation is an equivalence relation, and describe the distinct equivalence classes of it.

E is the relation defined on $\mathbb{Z}$ as follows:

For all $m, n \in \mathbb{Z}$, $m E n \Leftrightarrow 2 \mid (m-n)$.

① E is reflexive because for all $m \in \mathbb{Z}$,

$m E m \Leftrightarrow 2 \mid (m-m) = 2 \mid 0$, True.

② E is symmetric because for all $m, n \in \mathbb{Z}$,

$m E n \Leftrightarrow 2 \mid (m-n) = 2 \mid (-(n-m)) \Leftrightarrow n E m$, True.

③ E is transitive because for all $m, n, s \in \mathbb{Z}$,

if $m E n$ and $n E s$, then

$2 \mid (m-n)$ and $2 \mid (n-s)$ and

$2 \mid ((m-n) + (n-s))$

$= 2 \mid (m-s) \Leftrightarrow m E s$, true.

→ so, E is an equivalence relation.

→ Our 2 distinct classes are:

$\{x \in \mathbb{Z} \mid x = 2k$ for some $k\}$

and $\{x \in \mathbb{Z} \mid x = 2k+1$ for some $k\}$

The difference of 2 odd or even numbers is divisible by 2, which is why these are our 2 classes.

Ch. 8 Sect. 3 Prob. 26

Same instructions as # 20 on previous page.

D is the relation defined on Z as follows:

For all $m, n \in \mathbb{Z}$, $m \; D \; n \Leftrightarrow 3 \mid (m^2 - n^2)$.

① D is reflexive because for all $m \in \mathbb{Z}$,

$m \; D \; m \Leftrightarrow 3 \mid (m^2 - m^2) = 3 \mid 0$, True.

② D is symmetric because for all $m, n \in \mathbb{Z}$,

$m \; D \; n \Leftrightarrow 3 \mid (m^2 - n^2) = 3 \mid (-(n^2 - m^2)) \Leftrightarrow n \; D \; m$.

③ D is transitive because for all $m, n, s \in \mathbb{Z}$,

$m \; D \; n \Leftrightarrow 3 \mid (m^2 - n^2)$ and $n \; D \; s \Leftrightarrow 3 \mid (n^2 - s^2)$.

Then, $3 \mid ((m^2 - n^2) + (n^2 - s^2))$

$= 3 \mid (m^2 - s^2) \Leftrightarrow m \; D \; s$.

Hence, D is an equivalence relation.

→ Our equivalence classes are: For all $m \in \mathbb{Z}$,

$[m] = \{ x \in \mathbb{Z} \mid x \; D \; m \} = \{ x \in \mathbb{Z} \mid x = 3r - m \text{ for some } r \}$

$= \{ x \in \mathbb{Z} \mid x = 3r + m \text{ for some } r \}$

* Realize that $3 \mid (m^2 - n^2) = 3 \mid ((m+n)(m-n))$

which implies that $3 \mid (m+n)$ and

$3 \mid (m-n)$.

→ Then, $m + n = 3r$
$m - n = 3r$ for some r.

$(m = 3r \pm n)$

ch. 8 Sect. 3 Prob. 32

Let A be the set of all straight lines in the Cartesian plane.
Define a relation $\parallel$ on A as follows:
For all l_1 and $l_2 \in A$, $l_1 \parallel l_2 \Leftrightarrow l_1$ is parallel to l_2.

Then, $\parallel$ is an equivalence relation on A. Describe the equivalence classes of this relation.

We have 1 equivalence class for each real number θ such that $0 \leq \theta < \pi$. In each class, one line passes through the origin, making an angle θ with respect to the positive horizontal axis.

Ch. 8 Sect. 3 Prob. 38

Let R be an equivalence relation on a set A. Prove.

For all a, b, and c in A, if $b\,R\,c$ and $c \in [a]$, then $b \in [a]$.

Proof: Suppose R is an equivalence relation on a set A and a, b, and c are elements in A such that $b\,R\,c$ and $c \in [a]$.

Since $c \in [a]$, $c\,R\,a$. Then,
$b\,R\,c$ and $c\,R\,a$ imply that
$b\,R\,a$ as R is transitive.
Hence, $b \in [a]$ by definition of class.

Ch. 8 Sect. 3 Prob. 42

Let A be the set of all ordered pairs of integers for which the 2nd element of the pair is nonzero.

$$A = \mathbb{Z} \times (\mathbb{Z} - \{0\}).$$

Let R be the relation on A as follows:

For all $(a,b), (c,d) \in A$,

$(a,b) \, R \, (c,d) \Leftrightarrow ad = bc$.

① R is reflexive because for all $(a,b) \in A$,

$(a,b) \, R \, (a,b) \Leftrightarrow ab = ba$, True.

② R is symmetric because for all $(a,b), (c,d) \in A$,

$(a,b) \, R \, (c,d) \Leftrightarrow ad = bc = da = bc \Leftrightarrow (c,d) \, R \, (a,b)$.

③ List 4 distinct elements in $[(1,3)]$:

$(-1,-3), (4,12), (-4,-12), (5,15)$

④ List 4 distinct elements in $[(2,5)]$

$(-2,-5), (4,10), (-4,10), (6,15)$