Quiz 8 : 9-25-2022

Aamir K.

① Given $P(A) = 0.2$ and $P(B) = 0.3$. If A and B are independent events, what is $P(A \cap B)$? what is $P(A \cup B)$?

$$P(A \cap B) = P(A) \cdot P(B) = 0.2 * 0.3 = 0.06$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.2 + 0.3 - 0.06$$

$$= 0.44.$$

② If a 7-sided die $\{0,1,2,3,4,5,6\}$ is biased w/ $P(1) = P(6) = 3/35$ and $P(2) = P(3) = P(4) = P(5) = 6/35$, what is the expected value?

$$1 \cdot P(1) + 2 \cdot P(2) + 3 \cdot P(3) + 4 \cdot P(4) + 5 \cdot P(5) + 6 \cdot P(6)$$

$$= 3/35 + 12/35 + 18/35 + 24/35 + 30/35 + 18/35$$

$$= 105/35 = 3.$$

Expected Value $= 3.$

③ Given $(u^2 - v^2)^{10}$, what is the coefficient of $u^{14} v^6$?

$$(a+b)^n = \sum_{j=0}^{n} \binom{n}{j} a^j b^{n-j} \quad (a = u^2 \text{ and } b = -v^2)$$

Then, $? \, u^{14} v^6 = \binom{10}{7} {u^2}^7 {v^2}^3 = \binom{10}{7} u^{14} v^6$

$$\binom{10}{7} = \frac{10!}{7!\,3!} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}\,3!} = \frac{720}{6} = 120$$

So, the coefficient of $u^{14} v^6$ is -120.

It's -120 because $-v^2$ is raised to an odd power.