2/13/22
Aamir
Khan

Chapter 1 HW cover page

Problems solved:

Section 1: 1, 4, 7, 10

Section 2: 1, 4, 7, 10

Section 3: 1, 4, 7, 10, 13, 16, 19

Problems Not solved:

Section 4: * My textbook edition only had until section 3, so I couldn't solve any section 4 problems *

Chapter 1 Section 1 Problem 1

Instructions: In each of the 1-6, fill in the blanks using a variable or variables to rewrite the given statement.

1. Is there a real number whose square is -1?

a. Is there a real number x such that $x^2 = -1$?

b. Does there exist a real number x such that $x^2 = -1$?

Chapter 1 Section 1 Problem 4

Instructions: In each of the 1-6, fill in the blanks using a variable or variables to rewrite the given statement.

4. Given any real number, there is a real number that is greater.

a. Given any real number r, there is a real number s such that s is greater than r.

b. For any real number r, there is a real number s such that $s > r$.

Chapter 1 Section 1 Problem 7

Instructions: Rewrite the following statements less formally, without using variables. Determine, as best as you can, whether the statements are true or false.

a. There are real numbers u and v with the property that $u+v < u-v$.

→ There are real numbers whose sum is less than their difference. TRUE

b. There is a real number x such that $x^2 < x$.

→ There is a real number whose square is less than itself. TRUE

c. For all positive integers n, $n^2 \geq n$.

→ For all positive integers, their square is greater than or equal to the integer. TRUE

d. For all real numbers a and b, $|a+b| \leq |a|+|b|$

→ The absolute value of the sum of two real numbers is less than or equal to the sum of their absolute values. TRUE

Chapter 1 Section 1 Problem 10

Instructions: In each of 8-13, fill in the blanks to rewrite the given statement.

10. Every nonzero real number has a reciprocal.

a. All nonzero real numbers have a reciprocal.

b. For all nonzero real numbers r, there is a reciprocal for r.

c. For all nonzero real numbers r, there is a real number s such that s is a reciprocal for r.

Chapter 1 Section 2 Problem 1

1. Which of the following sets are equal?

$A = \{a, b, c, d\}$ $B = \{d, e, a, c\}$

$C = \{d, b, a, c\}$ $D = \{a, a, d, e, c, e\}$

Sets A and C are equal. $A = C$

Sets B and D are equal. $B = D$.

Chapter 1 Section 2 Problem 4

4a. Is $2 \in \{2\}$? YES

4b. How many elements are in the set $\{2,2,2,2\}$?
There is only 1 element in the set. 2

4c. How many elements are in the set $\{0,\{0\}\}$?
There are 2 elements in the set. 0 & $\{0\}$

4d. Is $\{0\} \in \{\{0\},\{1\}\}$? YES

4e. Is $0 \in \{\{0\},\{1\}\}$? NO

Chapter 1 Section 2 Problem 7

7. Use the set-roster notation to indicate the elements in each of the following sets.

a. $S = \{n \in Z \mid n = (-1)^k$, for some integer $k\}$

Answer: $\{1, -1\}$

b. $T = \{m \in Z \mid m = 1 + (-1)^i$, for some integer $i\}$

Answer: $\{0, 2\}$

c. $U = \{r \in Z \mid 2 \leq r \leq -2\}$

Answer: $\emptyset$, there is no such r that $2 \leq r \leq -2$.

d. $V = \{s \in Z \mid s > 2$ or $s < 3\}$

Answer: $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ or Z.

e. $W = \{t \in Z \mid 1 < t < -3\}$

Answer: $\emptyset$, no such elements.

f. $X = \{u \in Z \mid u \leq 4$ or $u \geq 1\}$

Answer: $\{\ldots, -2, -1, 0, 1, 2, 3, 4, \ldots\}$ or Z.

Chapter 1 Section 2 Problem 10

10a. Is $((-2)^2, -2^2) = (-2^2, (-2)^2)$?
NO, $((-2)^2, -2^2) = (4, -4)$ while
$(-2^2, (-2)^2) = (-4, 4)$. $(4, -4) \neq (-4, 4)$.

10b. Is $(5, -5) = (-5, 5)$? NO, $5 \neq -5$.

10c. Is $(8-9, \sqrt[3]{-1}) = (-1, -1)$? Yes, $8-9 = -1$ and
$\sqrt[3]{-1} = -1$, so they are equal.

10d. Is $(^{-2}/_{-4}, (-2)^3) = (^3/_6, -8)$? Yes, since
$-^2/_{-4} = ^3/_6 = ^1/_2$ and $(-2)^3 = -8$.

Chapter 1 Section 3 Problem 1

1. Let $A = \{2, 3, 4\}$ and $B = \{6, 8, 10\}$ and define a relation R from A to B as follows: for all $(x, y) \in A \times B$, $(x, y) \in R$ means that y/x is an integer.

a. Is $4 R 6$? NO Is $4 R 8$? YES
   Is $(3, 8) \in R$? NO Is $(2, 10) \in R$? YES.
$6/4 = 3/2 \neq$ integer, $8/4 = 2$ ✓, $8/3 \neq$ integer, $10/2 = 5$ ✓

b. Write R as a set of ordered pairs.
$R = \{(2,6), (2,8), (2,10), (3,6), (4,8)\}$.
   For all ordered pairs, y/x is an integer.

c. Write the domain and co-domain of R.
   Domain: $A = \{2, 3, 4\}$
   Co-domain: $B = \{6, 8, 10\}$

d. Draw an arrow diagram for R.

$2 R 6$ $3 R 6$
$2 R 8$ $4 R 8$
$2 R 10$

Chapter 1 section 3 Problem 4

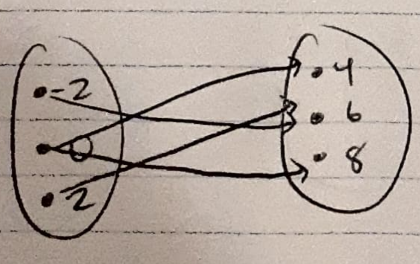
4. Let $G = \{-2, 0, 2\}$ and $H = \{4, 6, 8\}$ and define a relation V from G to H as follows: For all $(x,y) \in G \times H$, $(x,y) \in V$ means that $x-y/4$ is an integer.

a. Is $2 \, V \, 6$? Yes, $2-6/4 = -4/4 = -1$ ✓.
Is $(-2) \, V \, (-6)$? NO, -6 isn't related to -2 by V.
Is $(0,6) \in V$? NO, $0-6/4 = -3/2$ ✗. $(0,6) \notin V$.
Is $(2,4) \in V$? NO, $2-4/4 = 1/2$ ✗. $(2,4) \notin V$.

b. Write V as a set of ordered pairs.
$V = \{(-2,6), (0,4), (0,8), (2,6)\}$
$-2-6/4 = 2$ ✓, $0-4/4 = -1$ ✓, $0-8/4 = -2$ ✓, $2-6/4 = -1$ ✓

c. Write the domain and codomain of V.
Domain: $G = \{-2, 0, 2\}$
Codomain: $H = \{4, 6, 8\}$

d. Draw an arrow diagram for V.



Chapter 1 Section 3 Problem 7

7. Let $A = \{4, 5, 6\}$ and $B = \{5, 6, 7\}$ and define relations R, S, and T from A to B as follows: For all $(x, y) \in A \times B$,

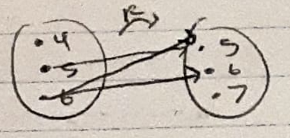
$(x, y) \in R$ means that $x \geq y$.

$(x, y) \in S$ means that $x - y/2$ is an integer.

$T = \{(4,7), (6,5), (6,7)\}$

a. Draw arrow diagrams for R, S, and T.

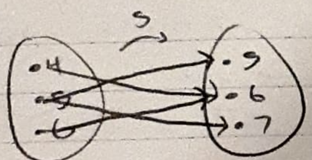
$R = \{(5,5), (6,5), (6,6)\}$ $\quad 5 \geq 5,\ 6 \geq 5,\ 6 \geq 6$ ✓



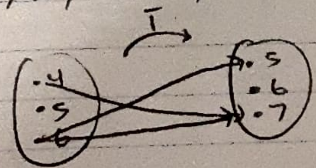
$S = \{(4,6), (5,5), (5,7), (6,6)\}$

$4-6/2 = -2/2 = -1$ ✓, $5-5/2 = 0$ ✓, $5-7/2 = -2/2 = -1$ ✓,

$6-6/2 = 0/2 = 0$ ✓.



$T = \{(4,7), (6,5), (6,7)\}$



b. Indicate whether any of the relations R, S, and T are functions.

R is not a function because 4 has no element in B and 6 has 2 elements in B.

S is not a function because 5 has 2 elements in B.

T is not a function because 5 has no element in B and 6 has 2 elements.

* None are functions *

Chapter 1 Section 3 Problem 10

10. Find 4 relations from $\{a,b\}$ to $\{x,y\}$ that are not functions from $\{a,b\}$ to $\{x,y\}$.

① Find the cartesian product,

$\{a,b\} \times \{x,y\} = \{(a,x),(a,y),(b,x),(b,y)\}$.

② $R_1 = \{(a,x),(a,y)\}$ is not a function because if $(a,x) \in R$ and $(a,y) \in R$, then x would need to equal y, but $x \neq y$.

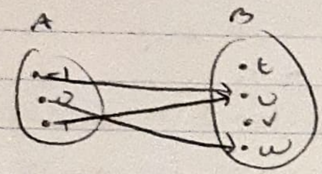
$R_2 = \{(b,x),(b,y)\}$ is not a function because if $(b,x) \in R$ and $(b,y) \in R$, then x would need to equal y, but $x \neq y$.

$R_3 = \{(a,x)\}$ is not a function because b would also need an element in $\{x,y\}$. Only a has an element, so R_3 is not a function.

$R_4 = \{(b,x)\}$ is not a function because a would also need an element in $\{x,y\}$, but only b has an element.

Chapter 1 Section 3 Problem 13

13. Let $A = \{-1, 0, 1\}$ and $B = \{t, u, v, w\}$. Define a function $F: A \to B$ by the following arrow diagram.



a. Write the domain and codomain of F.

Domain : $A = \{-1, 0, 1\}$

Codomain: $B = \{t, u, v, w\}$

b. Find $F(-1)$, $F(0)$, and $F(1)$.

$F(-1) = u$ $\quad (-1, u)$

$F(0) = w$ $\quad (0, w)$

$F(1) = u$ $\quad (1, u)$

Chapter 1 Section 3 Problem 16

16. Let f be the squaring function defined in Example 1.3.6. Find $f(-1)$, $f(0)$, and $f(1/2)$.

$f: \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^2$ for all real numbers x.

$f(-1) = (-1)^2 = 1$ $(-1, 1)$

$f(0) = (0)^2 = 0$ $(0, 0)$

$f(1/2) = (1/2)^2 = 1/4$ $(1/2, 1/4)$

Chapter 1 Section 3 Problem 19

19. Define functions f and g from $\mathbb{R}$ to $\mathbb{R}$ by the following formulas: for all $x \in \mathbb{R}$,

$$f(x) = 2x \quad \text{and} \quad g(x) = \frac{2x^3+2x}{x^2+1}$$

Does $f = g$? Explain.

* For 2 functions f and g to be equal, $f(x) = g(x)$ for all $x \in \mathbb{R}$.

(1) Factor $g(x) = \dfrac{2x^3+2x}{x^2+1} \Rightarrow \dfrac{2x(x^2+1)}{x^2+1}$

$\Rightarrow$ $g(x)$ simplifies to $g(x) = 2x$

(2) Since $f(x) = 2x$ and $g(x) = 2x$, $f(x) = g(x)$ for all $x \in \mathbb{R}$, so $f = g$ by the equality of functions definition.