

① Draw the directed graph for  $R$  where  $A = \{3, 4, 5, 6, 7, 8\}$ ,

$$\forall x, y \in A, x R y \leftrightarrow 2(x-y)$$

$$3 R 3 \leftrightarrow 2(3-3) = 0, 3 \not R 3$$

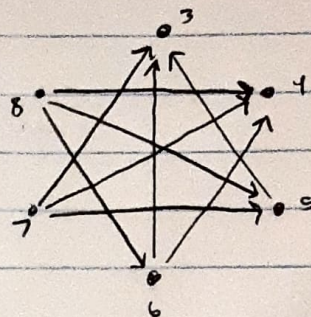
$$3 R 4 \leftrightarrow 2(3-4) = -2, 3 \not R 4$$

$$3 R 5 \leftrightarrow 2(3-5) = -4, 3 \not R 5$$

$$3 R 6 \leftrightarrow 2(3-6) = -6, 3 \not R 6$$

$$3 R 7 \leftrightarrow 2(3-7) = -8, 3 \not R 7$$

$$3 R 8 \leftrightarrow 2(3-8) = -10, 3 \not R 8$$



$$5 R 3 \leftrightarrow 2(5-3) = 4 \in A$$

$$\text{so } 5 R 3$$

$$* \text{ Basically, } x R y \leftrightarrow 2(x-y) \in A.$$

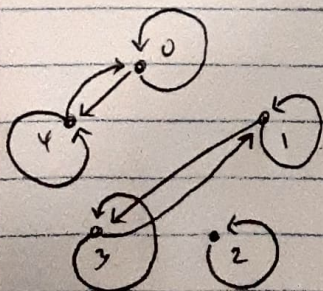
② Reflexive:  $R$  is reflexive  $\leftrightarrow \forall x \in A, x R x$

Symmetric:  $R$  is symmetric  $\leftrightarrow \forall x, y \in A, x R y \rightarrow y R x$

Transitive:  $R$  is transitive  $\leftrightarrow$

$$\forall x, y, z \in A, x R y \wedge y R z \rightarrow x R z$$

③



$$[0] = \{x \in A \mid x R 0\}$$

$$= \{0, 4\}$$

$$[1] = \{x \in A \mid x R 1\}$$

$$= \{1, 3\}$$

$$[2] = \{x \in A \mid x R 2\}$$

$$= \{2\}$$

$$[3] = \{1, 3\}$$

$$[4] = \{0, 4\}$$

Our distinct equivalence classes:

$$\{0, 4\}, \{1, 3\}, \text{ and } \{2\}$$