Chapter 3 Homework Cover Page:

* Problems Done:

3.1 : 4, 8, 12, 16, 20, 24, 28, 32 (ALL)

3.2 : 6, 12, 18, 24, 30, 36, 42, 48 (ALL)

3.3 : 10, 20, 30, 40, 50, 60 (ALL)

3.4 : 4, 8, 12, 16, 20, 24, 28, 32 (ALL)

* Problems Not Done: NONE.

Ch. 3 Sect. 1 Prob. 4

Let $Q(n)$ be the predicate "$n^2 \leq 30$."

a) Write $Q(2)$, $Q(-2)$, $Q(7)$, and $Q(-7)$ and indicate which are true or false.

$Q(2)$ = "$2^2 \leq 30$" = $4 \leq 30$ TRUE

$Q(-2)$ = "$-2^2 \leq 30$" = $4 \leq 30$ TRUE

$Q(7)$ = "$7^2 \leq 30$" = $49 \leq 30$ False

$Q(-7)$ = "$-7^2 \leq 30$" $49 \leq 30$ False

b) Find the truth set of $Q(n)$ if the domain of n is $\mathbb{Z}$.

Truth set = $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$, since the square of any of these ≤ 30.

c) If the domain is the set $\mathbb{Z}^+$, what is the truth set?

Truth set = $\{1, 2, 3, 4, 5\}$.

$Q(0) = 0 \leq 30$ ✓	
$Q(1) = 1 \leq 30$ ✓	$Q(-1) = 1 \leq 30$ ✓
$Q(2) = 4 \leq 30$ ✓	$Q(-2) = 4 \leq 30$ ✓
$Q(3) = 9 \leq 30$ ✓	$Q(-3) = 9 \leq 30$ ✓
$Q(4) = 16 \leq 30$ ✓	$Q(-4) = 16 \leq 30$ ✓
$Q(5) = 25 \leq 30$ ✓	$Q(-5) = 25 \leq 30$ ✓
$Q(6) = 36 \leq 30$ ✗	$Q(-6) = 36 \leq 30$ ✗

Ch. 3 Sect. 1 Prob. 8

Let $B(x)$ be "$-10 < x < 10$". Find the truth set of $B(x)$ for the following domains.

a) $\mathbb{Z}$: Truth set $= \{-9, -8, \ldots, -1, 0, 1, \ldots 8, 9\}$.

b) $\mathbb{Z}^+$: Truth set $= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

c) The set of all even integers:

Truth set $= \{-8, -6, -4, -2, 0, 2, 4, 6, 8\}$.

Ch. 3 Sect. 1 Prob. 12

Find a counterexample for the following:

$\forall$ real numbers x and y, $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$.

If we let $x = 3$ and $y = 6$, then we get

$\sqrt{3+6} = 3$ and $\sqrt{3} + \sqrt{6} \approx 4.18$

Since $3 \neq 4.18$, we've shown that the statement is false using a counterexample.

Ch. 3 Sect. 1 Prob. 16

Rewrite each of the following in the form:

$\forall$ ___ x, ___ .

a) All dinosaurs are extinct. $\forall$ dinosaurs x, x is extinct.

b) Every real number is positive, negative, or zero.

$\forall$ real numbers x, x is positive, negative, or zero.

c) No irrational numbers are integers.

$\forall$ irrational numbers x, x is not an integer.

d) No logicians are lazy.

$\forall$ logicians x, x is not lazy.

e) The number 2,147,581,953 is not equal to the square of any integer.

$\forall$ integers x, x^2 is not equal to 2,147,581,953.

f) The number -1 is not equal to the square of any real number.

$\forall$ real numbers x, x^2 is not equal to -1.

Ch. 3 Sect. 1 Prob. 20

Rewrite the statement informally in atleast 2 different ways w/o using variables or $\forall$ or the words "for all".

$\forall$ real numbers x, if x is positive, then the square root of x is positive.

① The square root of any positive real number is positive.

② If a real number is positive, then its square root is positive.

Ch. 3 Sect. 1 Prob. 24

Rewrite the following in the 2 forms:

① "∃ ___ x such that ___."

② ∃x such that ___ and ___."

a) Some hatters are mad.

① ∃ a hatter x such that x is mad.

② ∃x such that x is a hatter and x is mad.

b) Some questions are easy.

① ∃ a question x such that x is easy.

② ∃x such that x is a question and x is easy.

Ch. 3 Sect. 1 Prob. 28

Let the domain of x be the set D of objects discussed in mathematics courses, and let Real(x) be "x is a real number," Pos(x) be "x is positive real number," Neg(x) be "x is a negative real number," and Int(x) be "x is an integer."

Rewrite each w/o using quantifiers or variables.
Indicate which are true and which are false.

a) Pos(0): 0 is a positive real number. FALSE
0 is neither positive nor negative.

b) $\forall x$, Real(x) $\wedge$ Neg(x) $\rightarrow$ Pos(−x): TRUE
If a real number is negative, then its negative is positive. $-(-x) = x$ which is positive.

c) $\forall x$, Int(x) $\rightarrow$ Real(x): TRUE
If a number is an integer, then it's a real number.
$\mathbb{Z} \subseteq \mathbb{R}$ so all integers are real numbers.

d) $\exists x$ such that Real(x) $\wedge$ ~Int(x): TRUE
There exists a real number that is not an integer.
For ex: $1/2$ is a real number that's not an integer.

Ch. 3 Sect. 1 Prob. 32

Let R be the domain of the predicate variable x. Which of the following are true or false. Give counter examples for those that are false.

a) $x > 2 \Rightarrow x > 1$ TRUE

b) $x > 2 \Rightarrow x^2 > 4$ TRUE

c) $x^2 > 4 \Rightarrow x > 2$ False

Let $x = -3$, then $(-3)^2 = 9 > 4$

but $-3 \not> 2$, so it's false.

d) $x^2 > 4 \Leftrightarrow |x| > 2$ TRUE

Ch 3 Sect 2 Prob. 6

Write a negation for the following:

a) Sets A and B don't have any points in common.

b) Towns P and Q aren't connected by any road on the map.

$\sim(\forall x \text{ in } D,\ Q(x)) \equiv \exists x \text{ in } D$ such that $\sim Q(x)$.

For a):

There is atleast one point A and B have in common.

For b):

There is atleast one road that connects P and Q on the map.

Ch. 3 Sect. 2 Prob. 12

Determine whether the proposed negation is correct.

If not, write a correct negation.

Statement: The product of any irrational number and any rational number is irrational.

Proposed Negation: The product of any irrational number and any rational number is rational,

$\sim(\forall x \text{ in } D,\ Q(x)) \equiv \exists x \text{ in } D \text{ such that } \sim Q(x)$.

The proposed negation is incorrect because its form is incorrect. It should be:

There is atleast one irrational number and rational number whose product is rational.

Ch. 3 Sect. 2 Prob. 18

Write a negation.

$\forall x \in R$, if $x(x+1) > 0$ then $x > 0$ or $x < -1$.

$\sim(\forall x$ in D, if $P(x)$ then $Q(x)) \equiv \exists x$ in D s.t. $P(x)$ and $\sim Q(x)$.

Negation:

$\exists x \in R$ such that $x(x+1) > 0$ and

$x \leq 0$ and $x \geq -1$.

$\sim Q(x) = \sim(x > 0$ or $x < -1)$

$= \sim(x > 0) \wedge \sim(x < -1)$ DeMorgan's Laws

$= x \leq 0 \wedge x \geq -1$.

Ch. 3 Sect. 2 Prob. 24

Rewrite each pair in if-then and indicate the logical relationship between them.

a) All the children in Tom's family are female. ①

All the females in Tom's family are children. ②

①: If a member of Tom's family is a child, then they are female.

②: If a member of Tom's family is a female, then they are a child.

② is the converse of ①.

b) All the integers that are greater than 5 and end in 1, 3, 7, or 9 are prime. ①

All the integers that are greater than 5 and are prime end in 1, 3, 7, or 9. ②

①: If an integer is greater than 5 and ends in 1, 3, 7, or 9 then it is prime.

②: If an integer is greater than 5 and is prime, then it ends in a 1, 3, 7, or 9.

② is the converse of ①.

Ch. 3 Sect. 2 Prob. 30

Write the converse, inverse, and contrapositive. Indicate which are true and which are false. Give a counterexample for those that are false.

Statement: $\forall$ integers a, b, and c, if $a-b$ is even and $b-c$ is even, then $a-c$ is even. TRUE.

If $a-b$ is even, then it must equal $2*$ some integer n.

If $b-c$ is even, then it must equal $2*$ some integer m.

$\rightarrow$ $a-c = (a-b)+(b-c) = 2n+2m = 2\underbrace{(n+m)}_{\text{integer}} = \text{EVEN}$.

Converse: $\forall$ integers a, b, and c, if $a-c$ is even, then $a-b$ is even and $b-c$ is even. FALSE.

Let $a=4$, $b=1$, $c=2$. Then

$a-c = 4-2 = 2$ even

$a-b = 4-1 = 3$ odd

$b-c = 1-2 = -1$ odd

$\Big\}$ NOT EVEN

Inverse: $\forall$ integers a, b, and c, if $a-b$ and $b-c$ are odd, then $a-c$ is odd. FALSE.

Let $a=3$, $b=2$, $c=1$. Then

$a-c = 3-1 = 2$ even $\rightarrow$ NOT ODD

$a-b = 3-2 = 1$ odd

$b-c = 2-1 = 1$ odd

Contrapositive: $\forall$ integers a, b, and c, if $a-c$ is odd, then $a-b$ and $b-c$ are odd. TRUE.

Ch. 3 Sect. 2 Prob. 36

If $P(x)$ is a predicate and the domain of x is the set of all real numbers, let R be "$\forall x \in \mathbb{Z}, P(x)$," let S be "$\forall x \in \mathbb{Q}; P(x)$," and let T be "$\forall x \in \mathbb{R}, P(x)$."

a) Find a definition for $P(x)$ (don't use $x \in \mathbb{Z}$) so that R is true and both S and T are false.

Let $P(x)$ be $4x \neq 1$, then R is true and both S and T are false.

b) Find a definition for $P(x)$ (don't use $x \in \mathbb{Q}$) so that both R and S are true and T is false.

Let $P(x) = 4x \neq \sqrt{3}$, then R and S are true and T is false.

There's no integer or rational number such that $4x = \sqrt{3}$, so $4x \neq \sqrt{3}$ is true for R and S.

However, there is a real number such that $4x = \sqrt{3}$ if we let $x = \sqrt{3}/4$, so $4x \neq \sqrt{3}$ is false for T.

Ch. 3 Sect. 2 Prob. 42

Rewrite in if-then form.

"Passing a comprehensive exam is a necessary condition for obtaining a master's degree."

Answer:

If you don't pass a comprehensive exam, then you can't obtain a master's degree.

Ch. 3 Sect. 2 Prob. 48

A frequent-flyer club brochure states, "You may select among carriers only if they offer they offer the same lowest fare." Assuming that "only if" has its formal, logical meaning, does this statement guarantee that if 2 carriers offer the same lowest fare, the customer will be free to choose between them?

only if : If $\sim s$ then $\sim r$ or If r then s.

It would have the meaning:

"If carriers don't offer the same lowest fare, then you may not select among them."

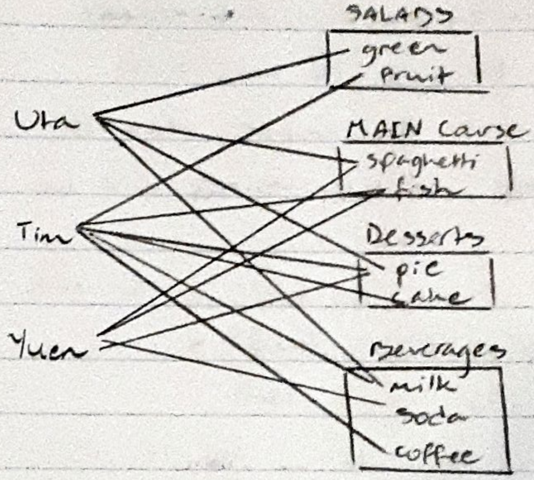
This statement doesn't guarantee what happens if the same lowest fare IS provided.

NO, it doesn't gurrantee the ability to choose.

Ch. 3 Sect. 3 Prob. 10

Determine whether each is true or false.

Ex. 3.3.3



a) $\forall$ students S, $\exists$ a dessert D s.t. S chose D. True (Everyone got a dessert)

b) $\forall$ students S, $\exists$ a salad T s.t. S chose T. False, Yuen didn't choose any salad.

c) $\exists$ a dessert D s.t. $\forall$ students S, S chose D. True, everyone chose pie.

d) $\exists$ a beverage B s.t. $\forall$ students D, D chose B. False, there isn't a beverage that everyone chose.

e) $\exists$ an item I s.t $\forall$ students S, S did not choose I. False, all items have been chosen.

f) $\exists$ a station Z s.t. $\forall$ students S, $\exists$ an item I s.t. S chose I from Z. True, every student chose pie from the Dessert station.

Ch. 3 Sect. 3 Prob. 20

Refer to the Tarski world Figure 3.3.1. In each pair of statements, the order of the quantifiers is reversed but everything else is the same. Determine whether the statements have the same or opposite truth values.

a) 1 For all squares y there is a triangle x s.t. x and y have different color.

2 There is a triangle x s.t. for all squares y, x and y have different colors.

1 is true since for any square, there is a triangle of a different color.

2 is false because there isn't a triangle that is a different color to all of the squares.

So 1 and 2 have opposite truth values.

b) 1 For all circles y there is a square x s.t. x and y have the same color.

2 There is a square x s.t. for all circles y, x and y have the same color.

1 is true since there is a square of the same color for any circle,

2 is false because there isn't a square that is the same color of all circles.

So 1 and 2 have opposite truth values.

Ch. 3 Sect. 3 Prob. 30

write a new statement by interchanging the symbols $\forall$ and $\exists$ and state which is true: the given statement, the new statement, neither, or both.

Given: $\exists x \in \mathbb{R}$ s.t. $\forall y \in \mathbb{R}^-$, $x > y$.

True, because there's atleast one x such that $x > y$ for all $y \in \mathbb{R}^-$. For ex. $x = 1$ (positive).

New: $\forall x \in \mathbb{R}$, $\exists y \in \mathbb{R}^-$ s.t. $x > y$.

False, there is never a y in $\mathbb{R}^-$ that All x is greater than. It would be true if $x > 0$, but there's always an $x \in \mathbb{R}$ that can be smaller than a y we choose. So, there isn't a y that would make $x > y$ true for all x.

Given → True

New → False

Ch. 3 Sect. 3 Prob. 40

In informal speech, most sentences of the form "There is ___ every ___" are intended to be understood as meaning "$\forall$ ___ $\exists$ ___," even though the existential quantifier "there is" comes before the universal quantifier "every". Rewrite the following using quantifiers and variables.

a) There is a sucker born every minute.

$\forall$ minutes m, $\exists$ a sucker s s.t. s was born in minute m.

b) There is a time for every purpose under heaven.

$\forall$ purposes p, $\exists$ a time t s.t. t is a purpose p under heaven.

Ch. 3 Sect. 3 Prob. 40

Refer to the Tarski world Figure 3.3.1.
Indicate whether true or false, write the statement using formal logical notation, and write the negation using formal logical notation. (Formal notation is found in Ex. 3.3.10).

For every object x there is an object y s.t.
if $x \neq y$ then x and y have different colors.

① FALSE, because there are objects that are different but have the same colors.
For ex: triangle A and square B are both blue, so there isn't a square that is a different color of all triangles.

② $\forall x(\exists y(x \neq y \rightarrow \sim \text{SameColor}(x,y))$

$p \rightarrow q$

③ $\sim(\forall x(\exists y(x \neq y \rightarrow \sim \text{SameColor}(x,y)))$
$\equiv \exists x(\forall y(x \neq y \wedge \text{SameColor}(x,y))$

$p \rightarrow q \equiv \sim p \vee q$
$\sim(p \rightarrow q) \equiv p \wedge \sim q$

Ch. 3 Sect 3 Prob. 60

Find the answers Prolog would give for the following questions in Ex. 3.3.11.

a. ? isabove (w_1, g)

b. ? color (w_2, blue)

c. ? isabove (X, b_1)

a. Is w_1 above g? NO, g is above w_1.

b. Is w_2 blue? NO, w_2 is white.

c. For which blocks is the predicate "X is above b_1" true? $X = g$, since g is the only block above b_1.

Ch. 3 Sect. 4 Prob. 4

Use universal instantiation or universal modus ponens to fill in the conclusion.

$\forall$ real numbers r, a, and b, if r is positive, then $(r^a)^b = r^{ab}$.

r = 3, a = 1/2, and b = 6 are particular real numbers such that r is positive

$\therefore (3^{1/2})^6 = 3^3 = 27$.

Universal Instantiation: If a property is true of everything in a set, then it is true of any particular thing in the set.

Ch. 3 Sect. 4 Prob. 8

State whether valid or invalid. Justify.

All freshmen must take writing.

Caroline is a freshmen,

$\therefore$ Caroline must take writing.

VALID by Universal Modus Ponens

Universal Modus Ponens:

$\forall x$, if $P(x)$ then $Q(x)$

$P(a)$ for a particular a

$\therefore$ $Q(a)$ for a particular a.

Ch. 3 Sect. 4 Prob. 12

State whether valid or invalid.

All honest people pay taxes.

Darth is not honest.

$\therefore$ Darth does not pay his taxes.

If $P(x)$ = x is honest and $Q(x)$ = x pays taxes,

then we have

$\forall x$, if $P(x)$ then $Q(x)$

$\sim P(x)$

$\therefore \sim Q(x)$

This is INVALID by inverse error.

Ch. 3 Sect 4 Prob. 16

State whether valid or invalid.

If a number is even, then twice that number is even.
The number $2n$ is even, for a particular number n.
$\therefore$ the particular number n is even.

Let $P(x)$ = x is even and $Q(x)$ = $2x$ is even.
Then, we have

$\forall x$, if $P(x)$ then $Q(x)$
$Q(x)$
$\therefore P(x)$.

This is INVALID by the converse error.

Ch. 3 Sect. 4 Prob. 20

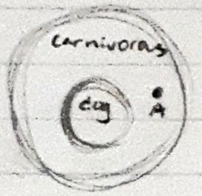
a) Use a diagram to show that the following argument can have true premises and a false conclusion.

All dogs are carnivorous.

Aaron is not a dog.

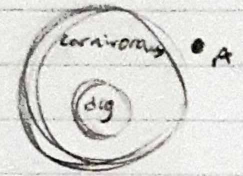
∴ Aaron is not carnivorous.

Either:



Aaron is not a dog but he is carnivorous.

Or:



Aaron is not a dog and he isn't carnivorous.

Either case can be true, but in case 1, the premises are true and the conclusion is false, so the argument isn't valid.

b) What can you conclude about the validity / invalidity of the following argument form? Explain how the result from a) leads to this conclusion.

$\forall x$, if $P(x)$ then $Q(x)$

$\sim P(a)$ for a particular a.

$\therefore \sim Q(a)$.

INVALID by inverse error.

This is the same form of argument we see in part a), which we've shown to be invalid, so this form is invalid in general.

Let $P(x)$ = x is a dog. $Q(x)$ = x is carnivorous. Then,

$\forall x$, if $P(x)$ then $Q(x)$.

$\sim P(a)$ for a particular a.

$\therefore \sim Q(a)$. Inverse error.

Ch. 3 Sect. 4 Prob. 24

Indicate whether valid / invalid. Draw a diagram.

No vegetarians eat meat.

All vegans are vegetarian.

∴ no vegans eat meat.

Let $P(x)$ = x is vegetarian.

$Q(x)$ = x is vegan.

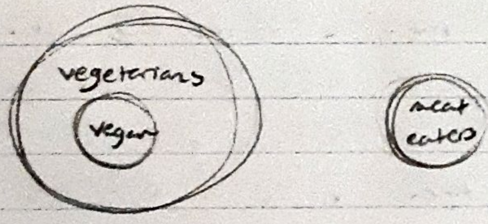
$R(x)$ = x does not eat meat.

Then,

$\forall x$, if $P(x)$ then $R(x)$.

$\forall x$, if $Q(x)$ then $P(x)$.

∴ if $Q(x)$ then $R(x)$.



So, this argument is VALID.

Ch. 3 Sect. 4 Prob. 28

Reorder the premises to show that the conclusion follows as a valid consequence of the premises. It may be helpful to rewrite the statements in if-then form and replace some by their contrapositives.

1. Every object that is to the right of all the blue objects is above all the triangles.
2. If an object is a circle, then it is to the right of all the blue objects.
3. If an object is not a circle, then it is not gray.

$\therefore$ all gray objects are above all triangles.

Rewritten:

1. $\forall x$, if x is to the right... then x is above...
2. $\forall x$, if x is a circle then x is to the right...
3. $\forall x$, if x is not a circle, then it is not gray.

If we take the contrapositive of 3, we get

3. $\forall x$, if x is gray, then x is a circle.

Finally:

Contrapositive: 3. If x is gray, then x is a circle.
2. If x is a circle, then x is to the right of...
1. If x is to the right of... then x is above all...

$\therefore$ If x is gray then it is above all triangles.

Ch. 3 Sect. 4 Prob. 32

Same instructions as #28 on the previous page.

1. When I work a logic example w/o grumbling, you may be sure it is one I understand.
2. The arguments in these examples are not arranged in regular order like the ones I am used to.
3. No easy examples make my head ache.
4. I can't understand examples if the arguments are not arranged in regular order like the ones I am used to.
5. I never grumble at an example unless it gives me a headache.

∴ These examples are not easy.

REWRITTEN:

1. If I work on an example w/o grumbling, then I understand it.
2. If the args are in these examples, then they aren't arranged...
3. If my head hurts, then the example is not easy.
4. If the args aren't arranged... then I can't understand them.
5. If I grumble then it gives me a headache.

Finally,

2. If args are in these exs, then they aren't arranged...
4. If the args aren't arranged... then I don't understand.
1. (Contrapositive) If I don't understand then I grumble.
5. If I grumble then it gives me a headache.
3. If my head hurts, then the examples aren't easy.

∴ These examples are not easy.