Decision Models
Lecture 4
Decision Tree

1

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Introduction

- What is involved in making a good decision?
- **Decision theory** is an analytic and systematic approach to the study of decision making
- A “good” decision is one that is based on logic, considers all available data and possible alternatives, and uses a quantitative approach

The Six Steps in Decision Making

1. Clearly define the problem at hand
2. List the possible alternatives
3. Identify the possible outcomes or states of nature
4. List the payoff of each combination of alternatives and outcomes
5. Select one of the mathematical decision theory models
6. Apply the model and make your decision

Case: Rao Lumber Company

- Rao Lumber company is in the lumber business for over 40 years. Management at Rao Lumber is deliberating whether to manufacture and market a new product, backyard sheds. The alternatives are to construct (1) a large new plant, (2) a small plant or (3) no plant at all. There are two possible outcomes: the market for the sheds could be favorable or there could be low demand. The potential profits are as follows: With a favorable market a large facility will result in a net profit of Rs. 200000 whereas if the market is unfavorable there would be a net loss of Rs. 180000. A small plant would result in a net profit of Rs. 100000 in a favorable market but a net loss of Rs. 20000 in a unfavorable market. Doing nothing results in Rs.0 in either market.
- What should be the decision of Rao Lumber management?

Rao Lumber: Decision Making

Define problem	To manufacture or market backyard storage sheds
List alternatives	1. Construct a large new plant 2. A small plant 3. No plant at all
Identify outcomes	The market could be favorable or unfavorable for storage sheds
List payoffs	List the payoff for each state of nature/decision alternative combination
Select a model	Decision tables and/or trees can be used to solve the problem
Apply model and make decision	Solutions can be obtained and a sensitivity analysis used to make a decision

Rao Lumber Company

Step 1 – Define the problem

- Expand by manufacturing and marketing a new product, backyard storage sheds

Step 2 – List alternatives

- Construct a large new plant
- A small plant
- No plant at all

Step 3 – Identify possible outcomes

- The market could be favorable or unfavorable

Rao Lumber Company

Step 4 – List the payoffs

- Identify *conditional values* for the profits for large, small, and no plants for the two possible market conditions

Step 5 – Select the decision model

- Depends on the environment and amount of risk and uncertainty

Step 6 – Apply the model to the data

- Solution and analysis used to help the decision making

Rao Lumber Company

	STATE OF NATURE	
ALTERNATIVE	FAVORABLE MARKET (Rs)	UNFAVORABLE MARKET (Rs)
Construct a large plant	200,000	–180,000
Construct a small plant	100,000	–20,000
Do nothing	0	0

Decision Making Under Risk

- The decision maker *knows* the chance of occurrence or the probability of the various outcomes
- Decision making when there are several possible states of nature and we know the probabilities associated with each possible state
- Most popular method is to choose the alternative with the highest *expected monetary value (EMV)*

EMV (alternative i) = (payoff of first state of nature)
x (probability of first state of nature)
+ (payoff of second state of nature)
x (probability of second state of nature)
+ … + (payoff of last state of nature)
x (probability of last state of nature)

EMV for Rao Lumber

- Each market has a probability of 0.50
- Which alternative would give the highest EMV?
- The calculations are

$$
\begin{aligned}
\text{EMV (large plant)} &= (0.50)(200{,}000) + (0.50)(-180{,}000) \\
&= 10{,}000 \\
\text{EMV (small plant)} &= (0.50)(100{,}000) + (0.50)(-20{,}000) \\
&= 40{,}000 \\
\text{EMV (do nothing)} &= (0.50)(0) + (0.50)(0) \\
&= 0
\end{aligned}
$$

EMV for Rao Lumber

ALTERNATIVE	STATE OF NATURE FAVORABLE MARKET (Rs)	 UNFAVORABLE MARKET (Rs)	EMV (Rs)
Construct a large plant	200,000	–180,000	10,000
Construct a small plant	100,000	–20,000	40,000
Do nothing	0	0	0
Probabilities	0.50	0.50	

Largest EMV

Sensitivity Analysis

- Sensitivity analysis examines how our decision might change with different input data
- For the Rao Lumber example

P = probability of a favorable market

$(1 – P)$ = probability of an unfavorable market

Sensitivity Analysis

EMV(Large Plant) $= 200{,}000P - 180{,}000)(1 - P)$

$= 200{,}000P - 180{,}000 + 180{,}000P$

$= 380{,}000P - 180{,}000$

EMV(Small Plant) $= 100{,}000P - 20{,}000)(1 - P)$

$= 100{,}000P - 20{,}000 + 20{,}000P$

$= 120{,}000P - 20{,}000$

EMV(Do Nothing) $= 0P + 0(1 - P)$

$= 0$

Sensitivity Analysis

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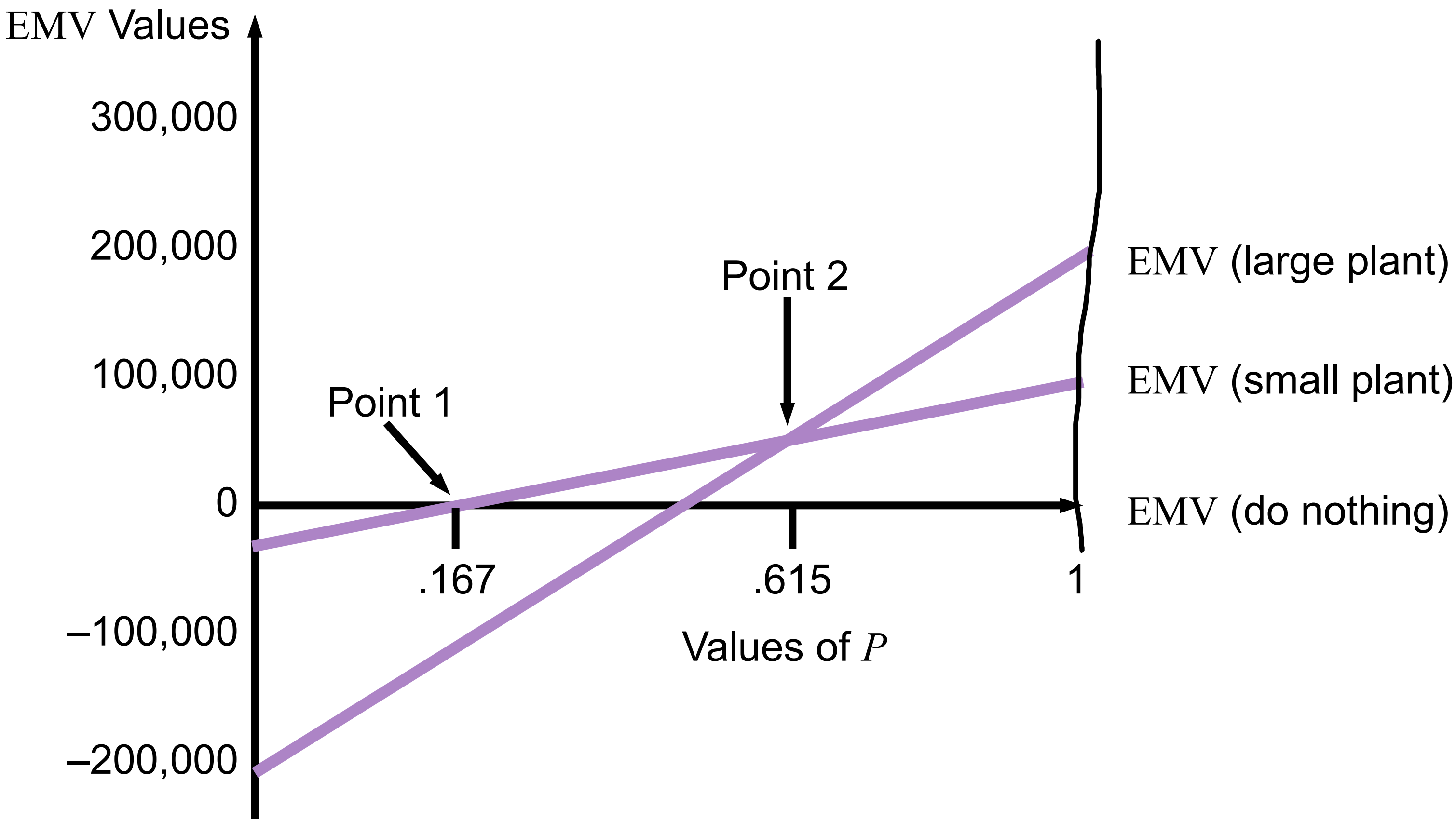


Sensitivity Analysis

Point 1:

EMV(do nothing) = EMV(small plant)

$$0 = 120{,}000P - 20{,}000 \qquad P = \frac{20{,}000}{120{,}000} = 0.167$$

Point 2:

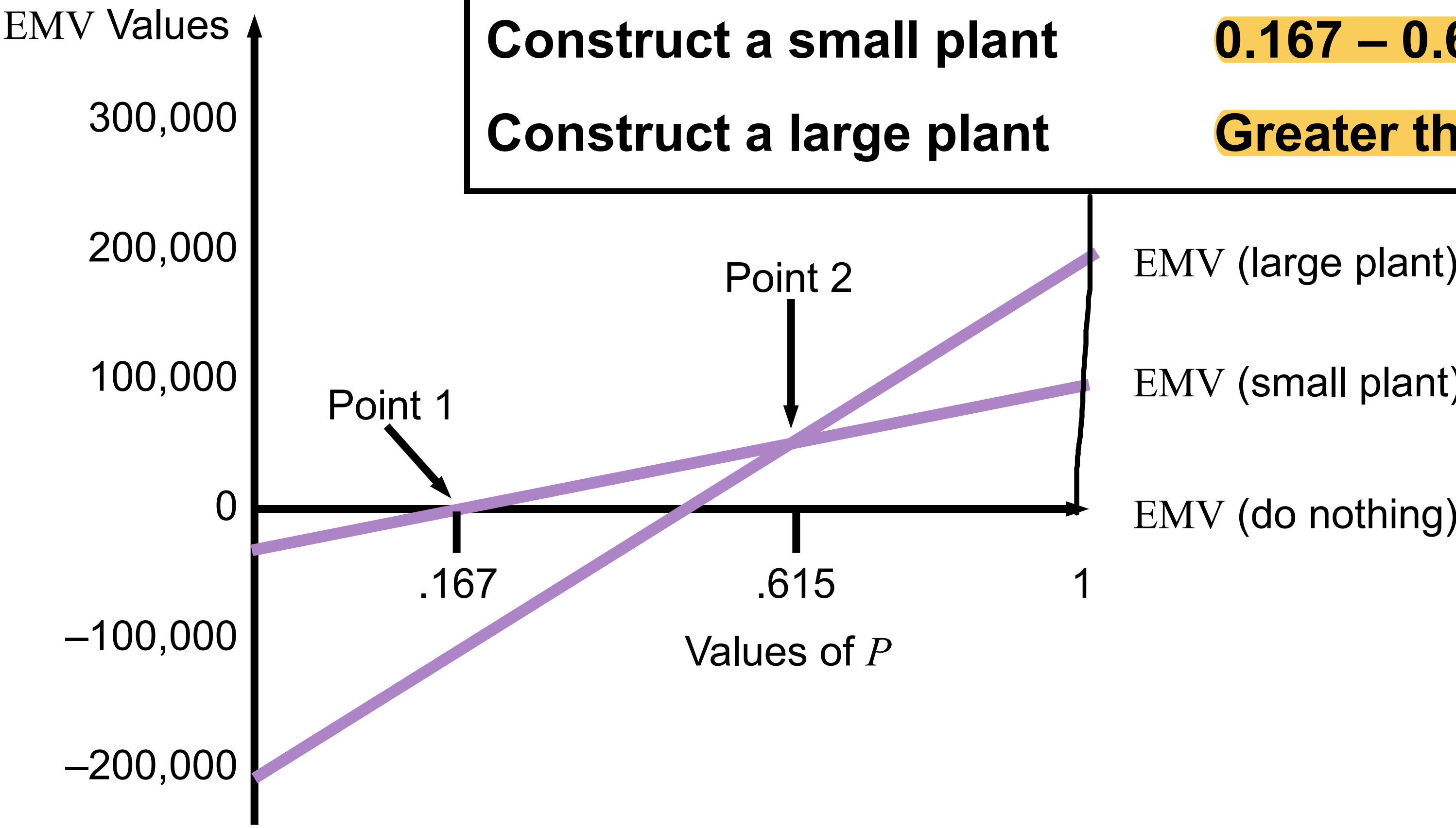
EMV(small plant) = EMV(large plant)

$$120{,}000P - 20{,}000 = 380{,}000P - 180{,}000$$

$$P = \frac{160{,}000}{260{,}000} = 0.615$$

Sensitivity Analysis

BEST ALTERNATIVE	RANGE OF *P* VALUES
Do nothing	Less than 0.167
Construct a small plant	0.167 – 0.615
Construct a large plant	Greater than 0.615



Decision Analysis: Example

- Megley Cheese company is a small manufacturer of several different cheese products. One of the products is a cheese spread that is sold in retail outlets. The production manager at Megley must decide how many cases of cheese spread to manufacture each month. The probability that the demand will be six cases is 0.1, for seven cases is 0.3, for eight cases is 0.5 and for nine cases is 0.1. The cost of every case is $45 and the price that each case earns is $95. However any case not sold by the end of the month is of no value due to spoilage.
- How many cases of cheese should Megley manufacture each month?
- Megley also deliberates on hiring a marketing executive who will give a better idea of the demand. What is the maximum that the management at Megley will be willing to pay to the marketing executive?

Decision Trees

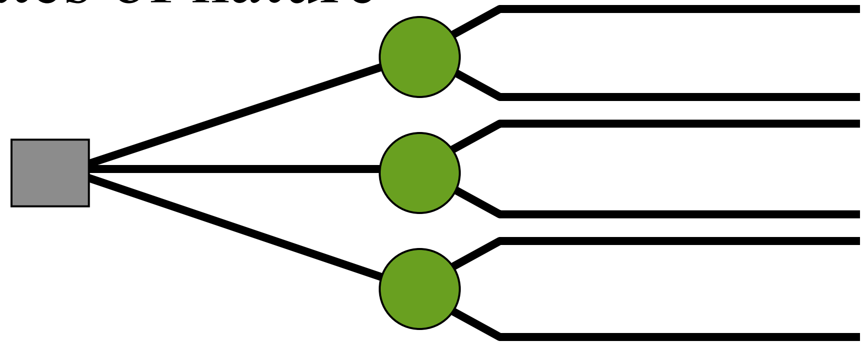
- Any problem that can be presented in a decision table can also be graphically represented in a *decision tree*
- Decision trees are most beneficial when a sequence of decisions must be made
- All decision trees contain *decision points* or *nodes* and *state-of-nature points* or *nodes*
  - A decision node from which one of several alternatives may be chosen
  - A state-of-nature node out of which one state of nature will occur

Five Steps to Decision Tree Analysis

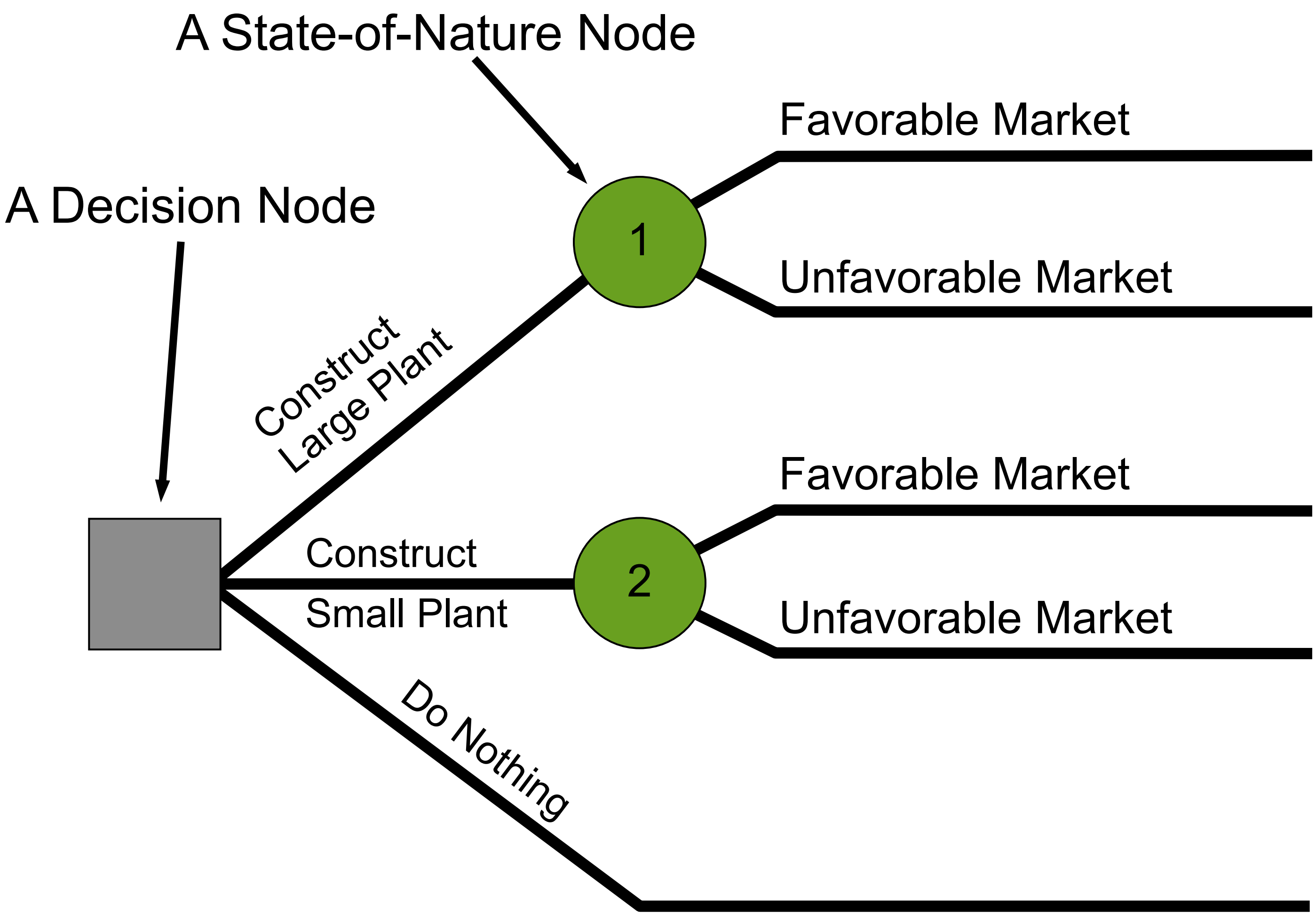
1. Define the problem
2. Structure or draw the decision tree
3. Assign probabilities to the states of nature
4. Estimate payoffs for each possible combination of alternatives and states of nature
5. Solve the problem by computing expected monetary values (EMVs) for each state of nature node

Structure of Decision Trees

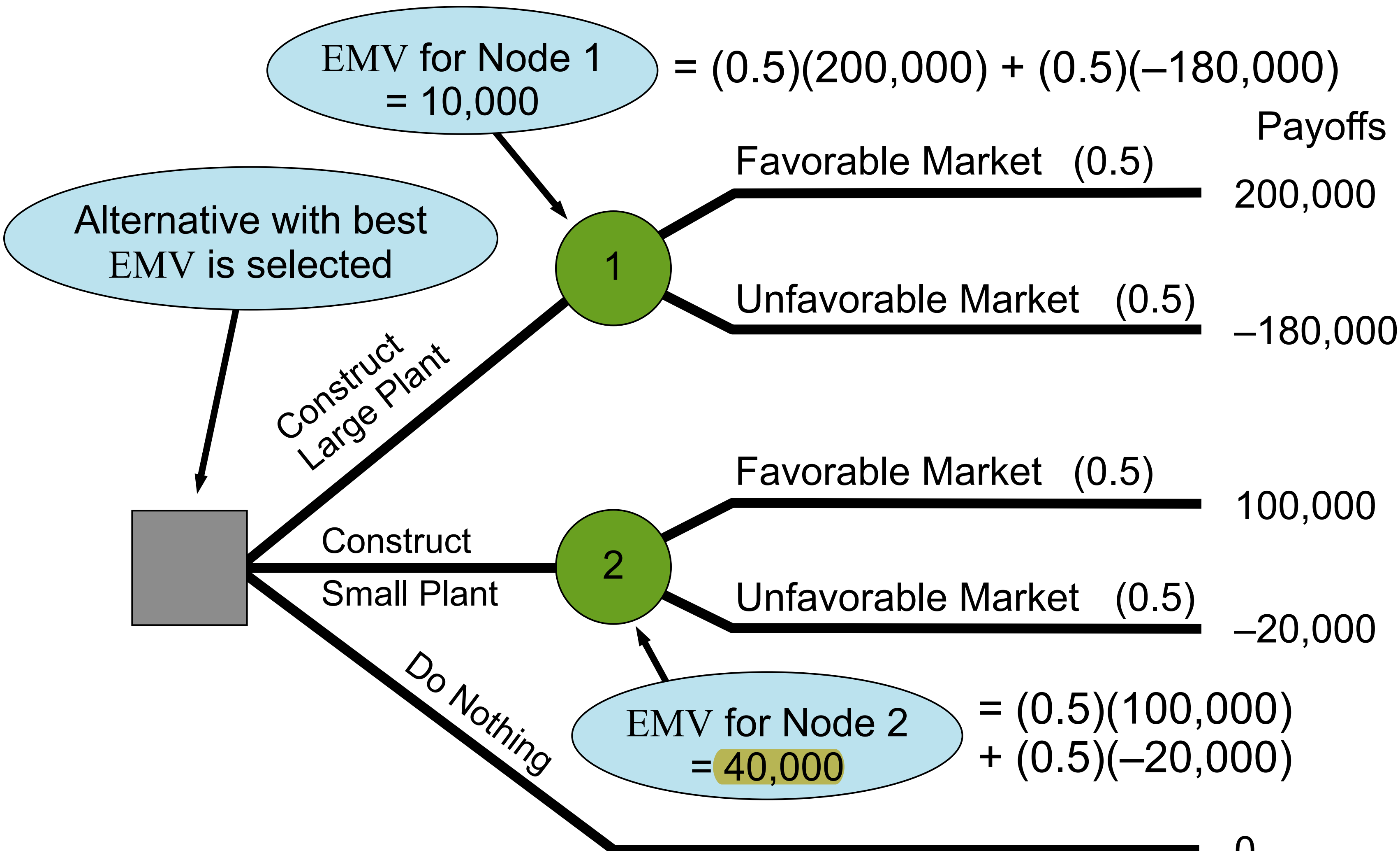
- Trees start from left to right
- Represent decisions and outcomes in sequential order
- Squares represent decision nodes
- Circles represent states of nature nodes
- Lines or branches connect the decisions nodes and the states of nature



Rao Lumber's Decision Tree



Rao Lumber's Decision Tree



Decision Trees: Evaluating Risk Responses in a Project

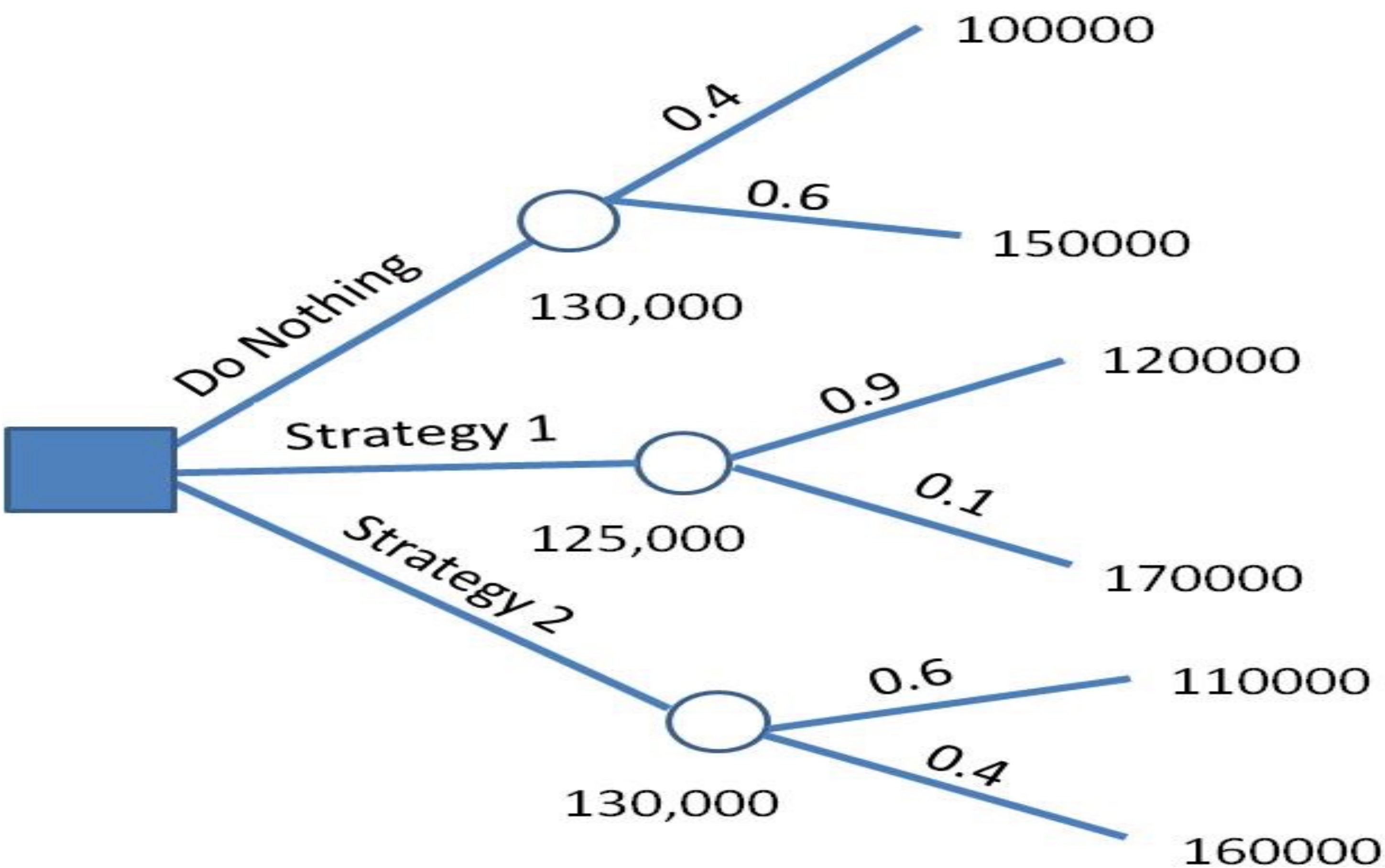
- A Project has a cost estimate of Rs. 100,000 with risk failure likelihood of 0.6 and a risk impact of Rs. 50,000 (i.e., a cost estimate of Rs. 150,000 in case of failure). Two strategies are considered:

  Strategy 1 will cost Rs. 20,000 and will reduce failure likelihood to 0.1 (i.e., increasing cost but decreasing risk)

  Strategy 2 will cost Rs. 10,000 and will reduce failure likelihood to 0.4

  Which strategy should be adopted?

Decision Trees: Evaluating Risk Responses in a Project



- Strategy 1 is the “best” option.

Rao Lumber's Complex Decision Problem

- The management at Rao Lumber has the option of conducting a marketing survey to get a better idea of the future market conditions before they decide on constructing the new pant. The market survey will cost an additional Rs. 10,000. Now the management has to decide whether to do the survey first and then decide on constructing the new plant.

- Multistage decision problem

- Decision Tree for the multi-stage problem is given in the next slide

Rao Lumber's Complex Decision Problem

- Rao Lumber has the opportunity of obtaining a market survey that will give additional information on the probable state of nature. Results of the market survey will likely indicate there is a percent change of a favorable market. Historical data show market surveys accurately predict favorable markets 78 % of the time. Thus

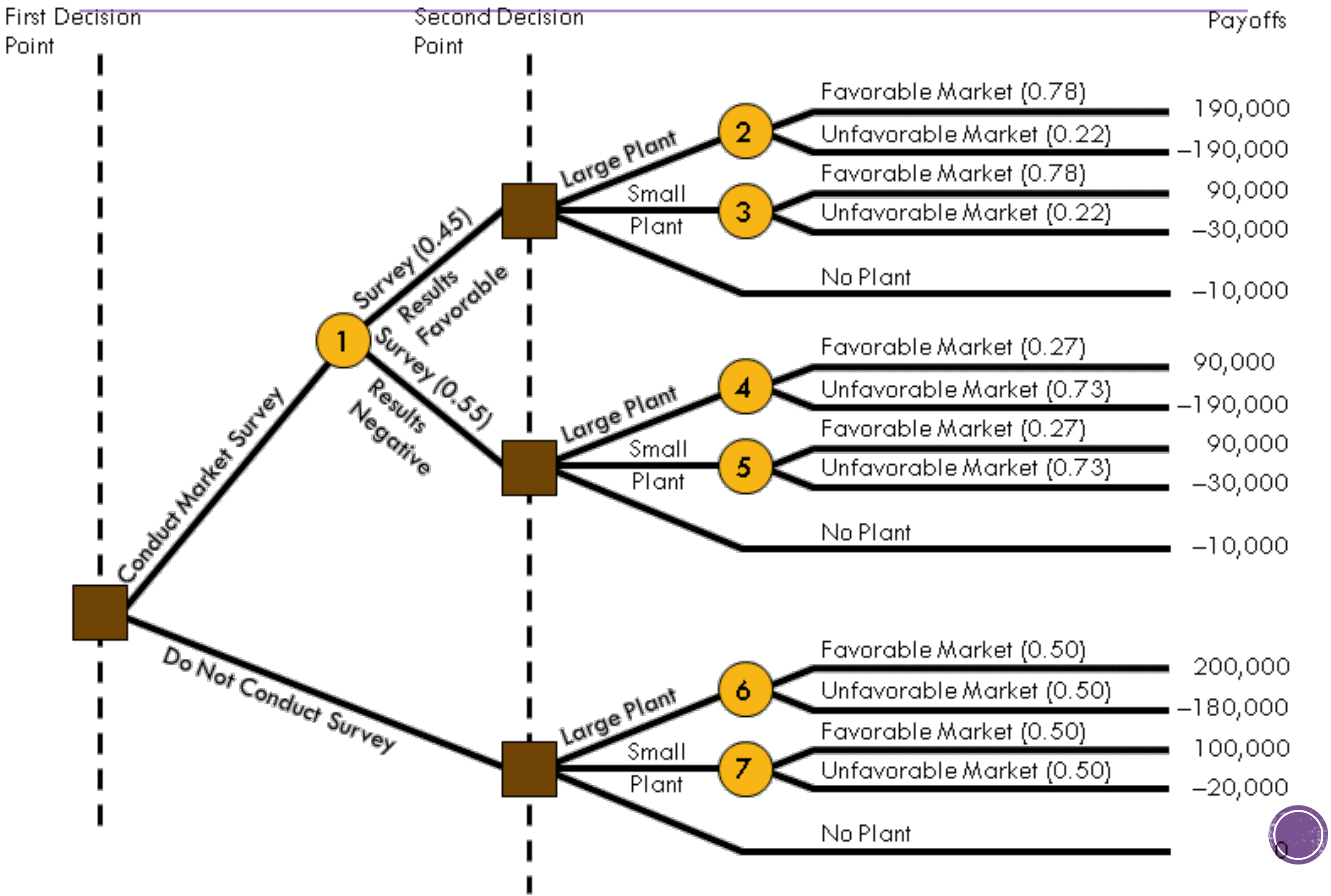
  P(Fav. Mkt | Fav. Survey Results) = .78

- Likewise, if the market survey predicts an unfavorable market, there is a 13 % chance of its occurring.

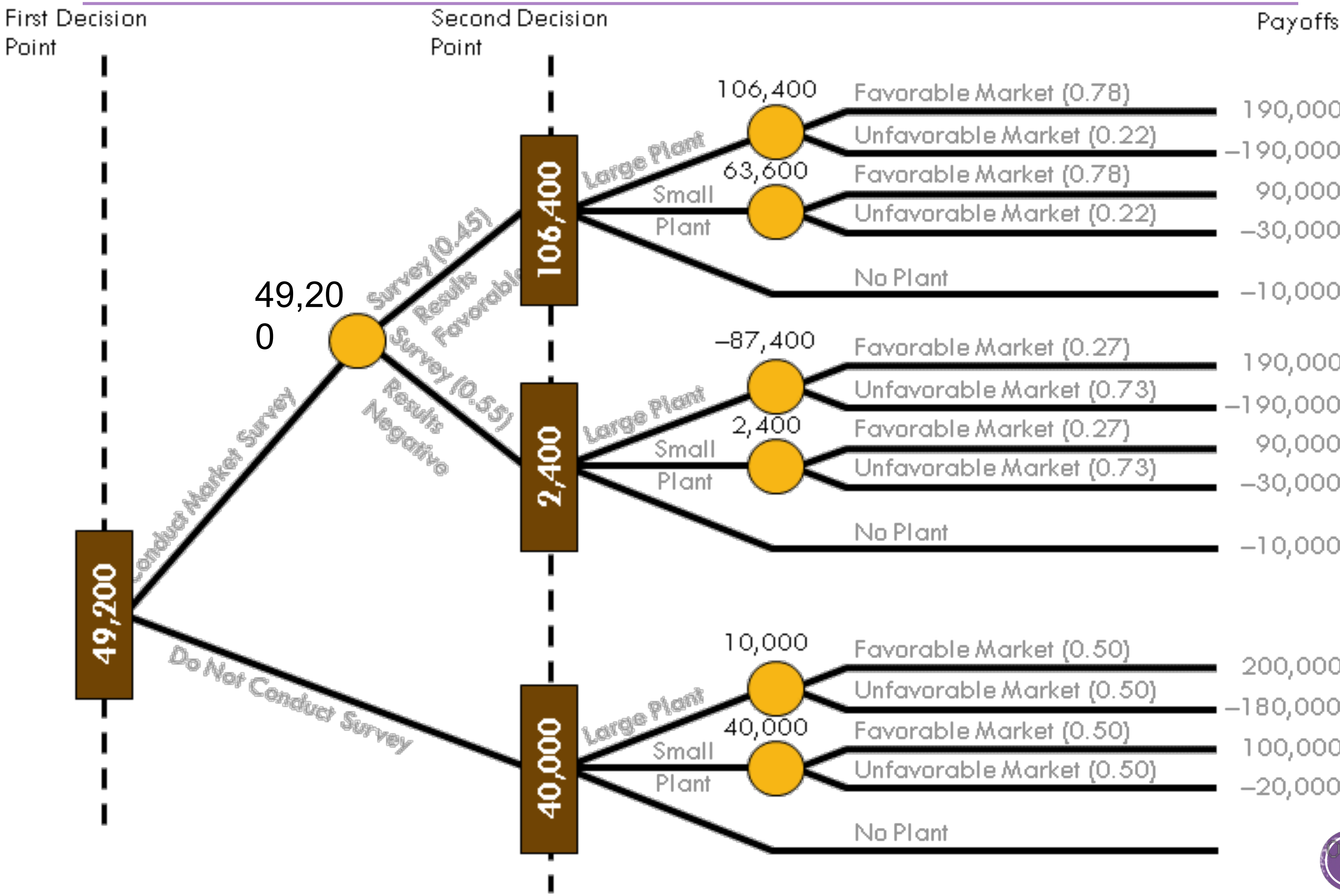
  P(Unfav. Mkt | Unfav. Survey Results) = .13

- Now that we have redefined the problem (Step 1), let's use this additional data and redraw Rao Lumber's decision tree.

Rao Lumber's Complex Decision Tree



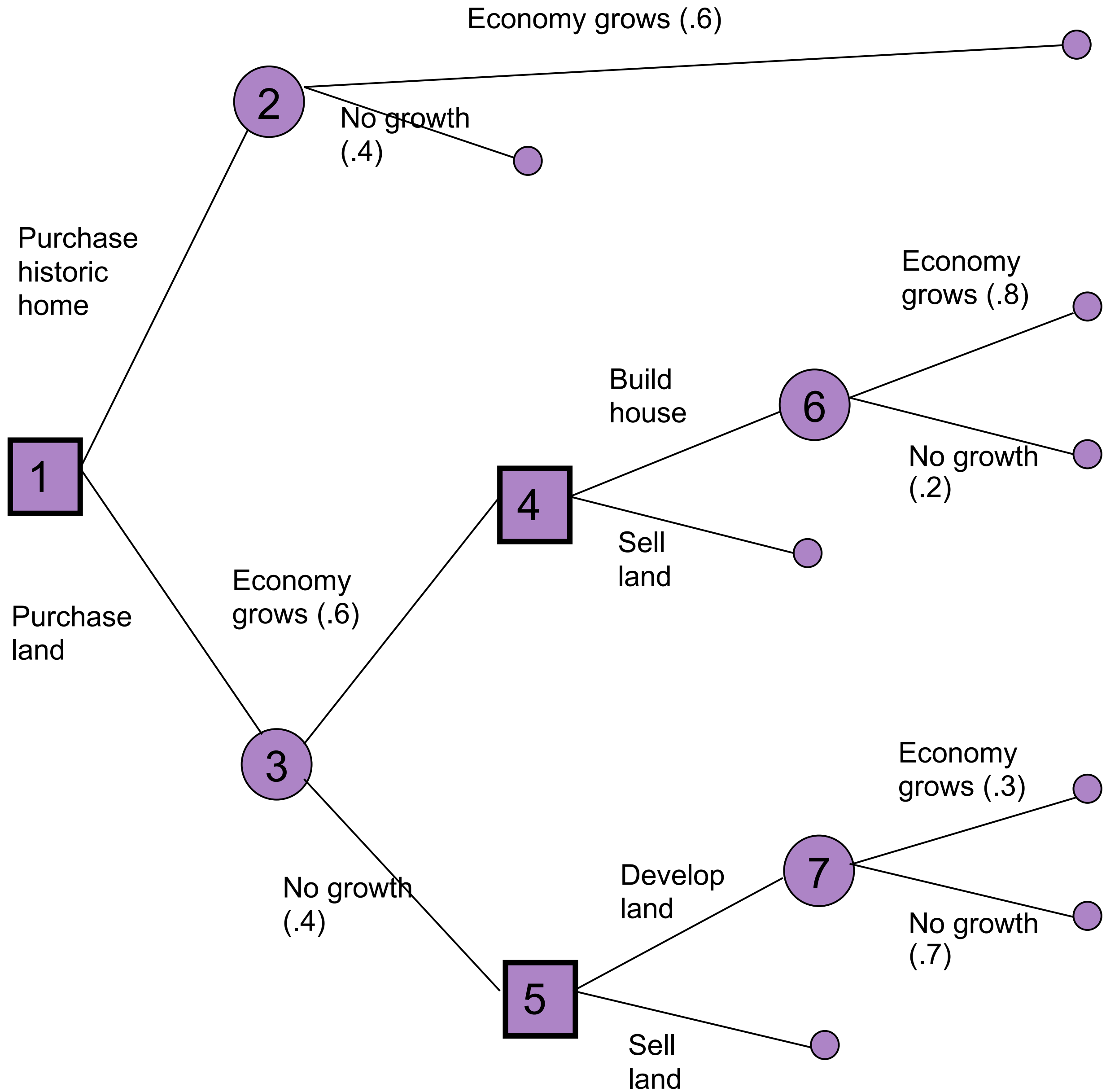
Rao Lumber's Complex Decision Tree



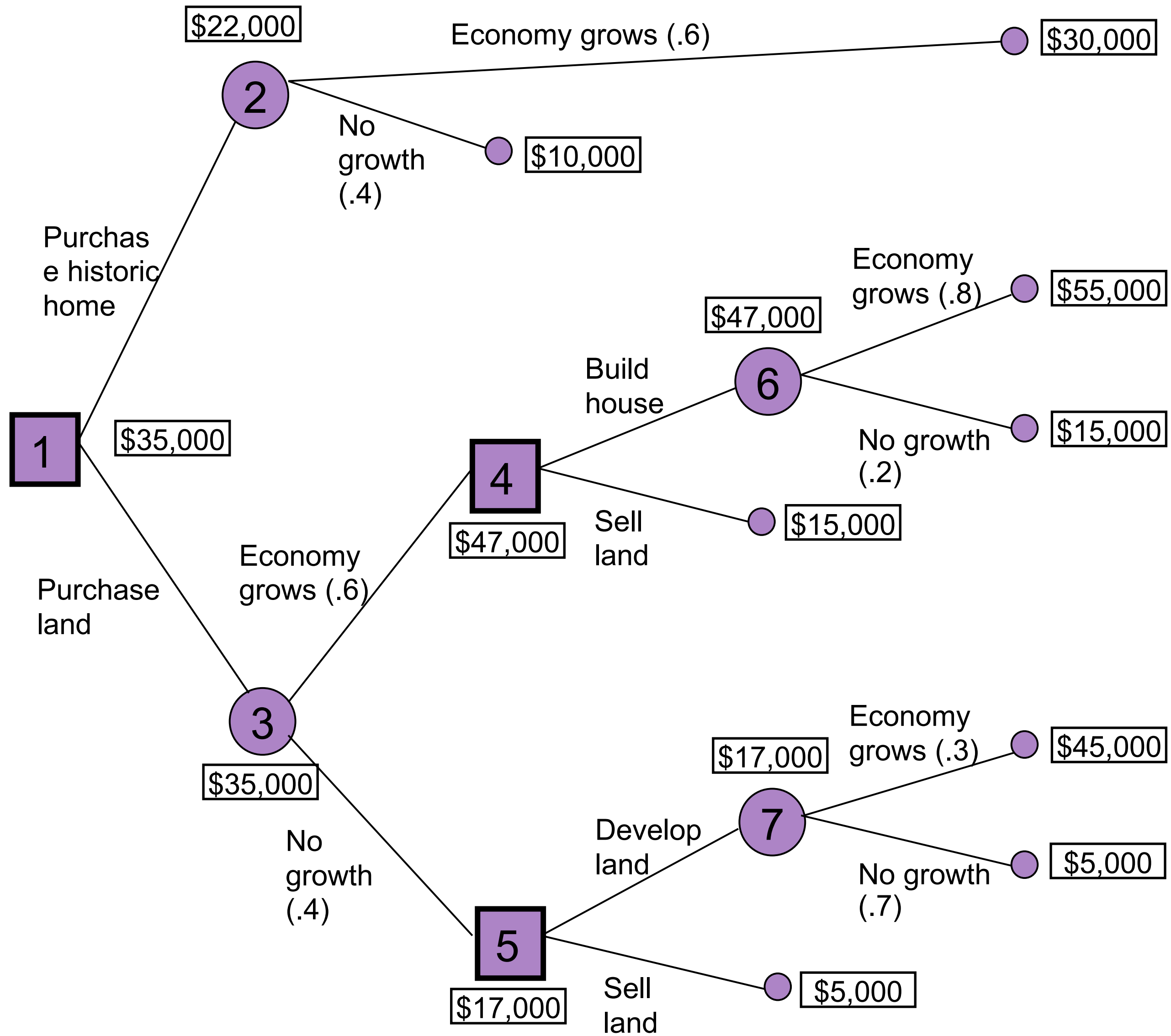
Example

- Leo can purchase a historic home for $200,000 or land in a growing area for $50,000. There is a 60% chance the economy will grow and a 40% change it will not. If it grows, the historic home will appreciate in value by 15% yielding a $30,00 profit. If it does not grow, the profit is only $10,000. If Leo purchases the land he will hold it for 1 year to assess the economic growth. If the economy grew during the first year, there is an 80% chance it will continue to grow. If it did not grow during the first year, there is a 30% chance it will grow in the next 4 years. After a year, if the economy grew, Leo will decide either to build and sell a house or simply sell the land. It will cost Leo $75,000 to build a house that will sell for a profit of $55,000 if the economy grows, or $15,000 if it does not grow. Leo can sell the land for a profit of $15,000. If, after a year, the economy does not grow, Leo will either develop the land, which will cost $75,000, or sell the land for a profit of $5,000. If he develops the land and the economy begins to grow, he will make $45,000. If he develops the land and the economy does not grow, he will make $5,000.

Solution


Economy grows (.6)
2
No growth
(.4)
Purchase
historic
home
1
Economy
grows (.8)
Build
house
6
No growth
(.2)
4
Sell
land
Economy
grows (.6)
Purchase
land
3
Economy
grows (.3)
Develop
land
7
No growth
(.4)
No growth
(.7)
5
Sell
land

Solution



$22,000
2
Economy grows (.6)
$30,000
No growth (.4)
$10,000
Purchase historic home
1
$35,000
Economy grows (.8)
$55,000
$47,000
6
Build house
$15,000
No growth (.2)
4
$47,000
Sell land
$15,000
Economy grows (.6)
Purchase land
3
$35,000
Economy grows (.3)
$45,000
$17,000
7
Develop land
$5,000
No growth (.7)
No growth (.4)
5
$17,000
Sell land
$5,000

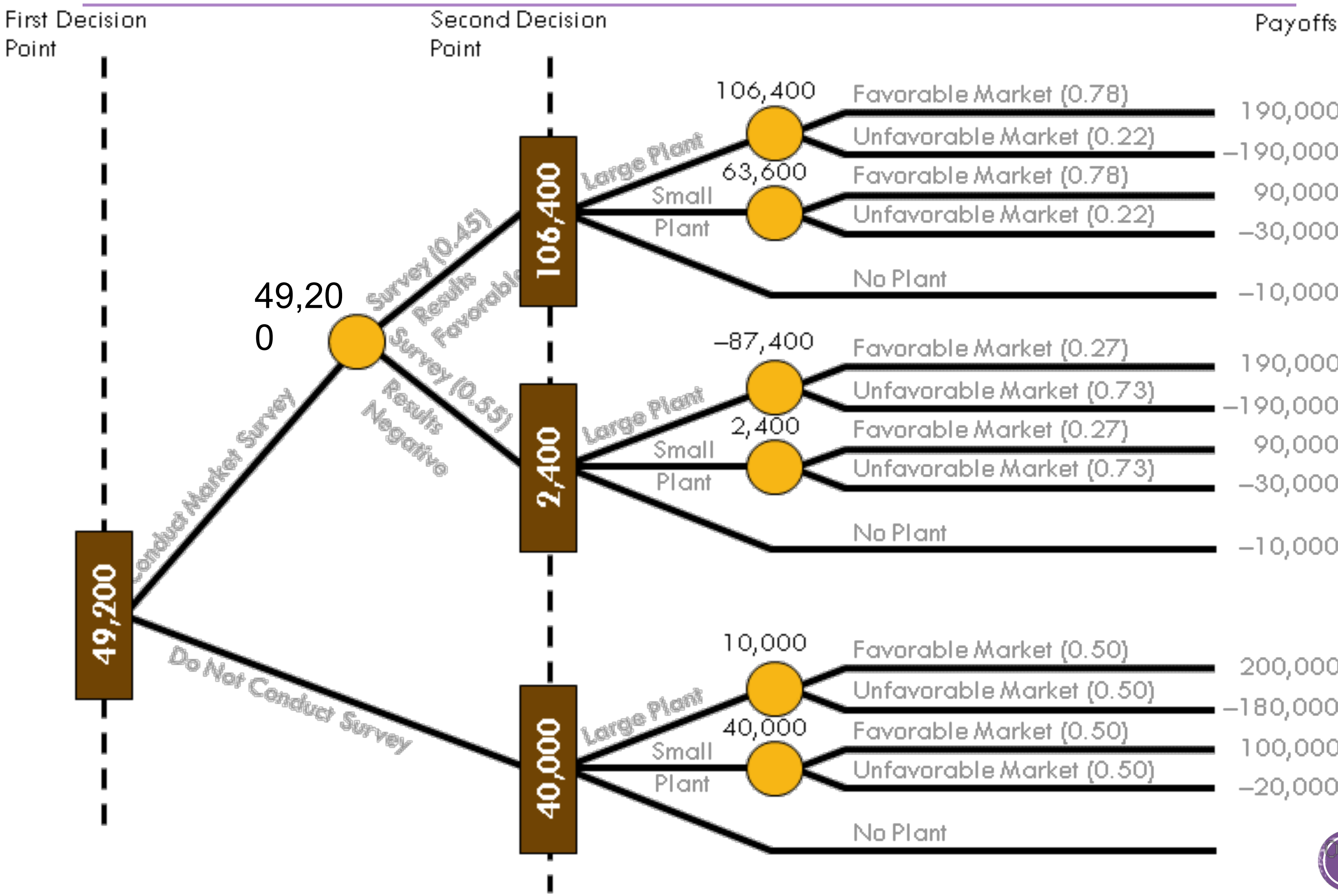
Bayesian Analysis

- Probability values can be computed from other data using Bayes' theorem

- Bayes' theorem incorporates initial estimates and information about the accuracy of the sources.

- It also allows the revision of initial estimates based on new information.

Rao Lumber's Complex Decision Tree



Calculating Revised Probabilities

- In the Rao Lumber case we used these four conditional probabilities:

$$P\text{ (favorable market(FM) | survey results positive)} = 0.78$$
$$P\text{ (unfavorable market(UM) | survey results positive)} = 0.22$$
$$P\text{ (favorable market(FM) | survey results negative)} = 0.27$$
$$P\text{ (unfavorable market(UM) | survey results negative)} = 0.73$$

- But how were these calculated?
- The prior probabilities of these markets are:

$$P\text{ (FM)} = 0.50$$
$$P\text{ (UM)} = 0.50$$

Calculating Revised Probabilities

- Through discussions with experts Rao Lumber management has learned the information in the table below.
- We can use this information and Bayes' theorem to calculate posterior probabilities.

RESULT OF SURVEY	STATE OF NATURE: FAVORABLE MARKET (FM)	STATE OF NATURE: UNFAVORABLE MARKET (UM)
Positive (predicts favorable market for product)	P (survey positive | FM) = 0.70	P (survey positive | UM) = 0.20
Negative (predicts unfavorable market for product)	P (survey negative | FM) = 0.30	P (survey negative | UM) = 0.80

Calculating Revised Probabilities

- Recall Bayes' theorem:

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B \mid A) \times P(A) + P(B \mid A') \times P(A')}$$

where

$$A, B = \textbf{any two events}$$
$$A' = \textbf{complement of } A$$

For this example, A will represent a favorable market and B will represent a positive survey.

Calculating Revised Probabilities

- $P\,(FM \mid \text{survey positive})$

$$= \frac{P(\text{survey positive} \mid FM) \times P(FM)}{P(\text{survey positive} \mid FM) \times P(FM) + P(\text{survey positive} \mid UM) \times P(UM)}$$

$$= \frac{(0.70)(0.50)}{(0.70)(0.50) + (0.20)(0.50)} = \frac{0.35}{0.45} = 0.78$$

- $P\,(UM \mid \text{survey positive})$

$$= \frac{P(\text{survey positive} \mid UM) \times P(UM)}{P(\text{survey positive} \mid UM) \times P(UM) + P(\text{survey positive} \mid FM) \times P(FM)}$$

$$= \frac{(0.20)(0.50)}{(0.20)(0.50) + (0.70)(0.50)} = \frac{0.10}{0.45} = 0.22$$

Calculating Revised Probabilities

- $P\ (FM \mid \text{survey negative})$

$$= \frac{P(\textbf{survey negative} \mid FM) \times P(FM)}{P(\textbf{survey negative} \mid FM) \times P(FM) + P(\textbf{survey negative} \mid UM) \times P(UM)}$$

$$= \frac{(0.30)(0.50)}{(0.30)(0.50) + (0.80)(0.50)} = \frac{0.15}{0.55} = 0.27$$

- $P\ (UM \mid \text{survey negative})$

$$= \frac{P(\textbf{survey negative} \mid UM) \times P(UM)}{P(\textbf{survey negative} \mid UM) \times P(UM) + P(\textbf{survey negative} \mid FM) \times P(FM)}$$

$$= \frac{(0.80)(0.50)}{(0.80)(0.50) + (0.30)(0.50)} = \frac{0.40}{0.55} = 0.73$$

Reference

- Render, Stair, and Hanna, Quantitative Analysis For Management