Contraintes continues

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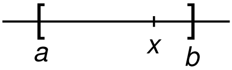
Problèmes continus

- Les variables sont réelles
  - On ne peut pas représenter les réels $\Rightarrow$ nombres flottants
  - Approxime les réels par un intervalle à bornes flottantes



  - Il peut y avoir des problèmes de précision

Arithmétique des intervalles

Opérations arithmétiques

- $[a, b] + [c, d] =$

Exemple

- $[-2, 3] + [2, 4] =$

Arithmétique des intervalles

Opérations arithmétiques

- $[a, b] + [c, d] = [a + c, b + d]$

Exemple

- $[-2, 3] + [2, 4] = [0, 7]$

Arithmétique des intervalles

Opérations arithmétiques

- $[a, b] + [c, d] = [a + c, b + d]$
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Exemple

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Arithmétique des intervalles

Opérations arithmétiques

- $[a, b] + [c, d] = [a + c, b + d]$
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Exemple

- $[-2, 3] + [2, 4] = [0, 7]$
- $[-2, 3] - [2, 4] = [-6, 1]$

Arithmétique des intervalles

Opérations arithmétiques

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- $[a, b] \times [c, d] =$

Exemple

- $[-2, 3] + [2, 4] = [0, 7]$
- $[-2, 3] - [2, 4] = [-6, 1]$
- $[-2, 3] \times [2, 4] =$

Arithmétique des intervalles

Opérations arithmétiques

- $[a, b] + [c, d] = [a + c, b + d]$
- $[a, b] - [c, d] = [a - d, b - c]$
- $[a, b] \times [c, d] = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)]$

Exemple

- $[-2, 3] + [2, 4] = [0, 7]$
- $[-2, 3] - [2, 4] = [-6, 1]$
- $[-2, 3] \times [2, 4] = [-8, 12]$

Arithmétique des intervalles

Opérations arithmétiques

- $[a, b] + [c, d] = [a + c, b + d]$
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Exemple

- $[-2, 3] + [2, 4] = [0, 7]$
- $[-2, 3] - [2, 4] = [-6, 1]$
- $[-2, 3] \times [2, 4] = [-8, 12]$
- $[-2, 3] \div [2, 4] = [-1, 1.5]$

Arithmétique des intervalles

Opérations arithmétiques

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Exercice

- $[-5, 5] + [2, 4] =$
- $[-2, 5] \times [-2, 4] =$
- $[1, 3] \times [-2, 5] - [2, 4] =$
- $[-10, 9] + [-2, 3] \times [-5, 3] - [-1, 6] =$

Arithmétique des intervalles

Opérations arithmétiques

- $[a, b] + [c, d] = [a + c, b + d]$
- $[a, b] - [c, d] = [a - d, b - c]$
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Exercice

- $[-5, 5] + [2, 4] = [-3, 9]$
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Arithmétique des intervalles

Opérations arithmétiques

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- $[-5, 5] + [2, 4] = [-3, 9]$
- $[-2, 5] \times [-2, 4] = [-10, 20]$
- $[1, 3] \times [-2, 5] - [2, 4] = [-6, 15] - [2, 4] =$
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Exercice

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- $[-10, 9] + [-2, 3] \times [-5, 3] - [-1, 6] =$
  $[-10, 9] + [-15, 10] - [-1, 6] =$

Arithmétique des intervalles

Opérations arithmétiques

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- $[-5, 5] + [2, 4] = [-3, 9]$
- $[-2, 5] \times [-2, 4] = [-10, 20]$
- $[1, 3] \times [-2, 5] - [2, 4] = [-6, 15] - [2, 4] = [-10, 13]$
- $[-10, 9] + [-2, 3] \times [-5, 3] - [-1, 6] =$
  $[-10, 9] + [-15, 10] - [-1, 6] = [-25, 19] - [-1, 6] =$

Arithmétique des intervalles

Opérations arithmétiques

- $[a, b] + [c, d] = [a + c, b + d]$
- $[a, b] - [c, d] = [a - d, b - c]$
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- $[-5, 5] + [2, 4] = [-3, 9]$
- $[-2, 5] \times [-2, 4] = [-10, 20]$
- $[1, 3] \times [-2, 5] - [2, 4] = [-6, 15] - [2, 4] = [-10, 13]$
- $[-10, 9] + [-2, 3] \times [-5, 3] - [-1, 6] =$
  $[-10, 9] + [-15, 10] - [-1, 6] = [-25, 19] - [-1, 6] = [-31, 20]$

Évaluer une contrainte

$x \in [-2, 5]$
$y \in [-3, 7]$

$$2x - y \quad = \quad 0$$

Évaluer une contrainte

$x \in [-2, 5]$
$y \in [-3, 7]$

$$\begin{aligned} 2x - y &= 0 \\ 2 \times [-2, 5] - [-3, 7] &= 0 \end{aligned}$$

Évaluer une contrainte

$x \in [-2, 5]$
$y \in [-3, 7]$

$$\begin{aligned} 2x - y &= 0 \\ 2 \times [-2, 5] - [-3, 7] &= 0 \\ [-4, 10] - [-3, 7] &= 0 \end{aligned}$$

Évaluer une contrainte

$x \in [-2, 5]$
$y \in [-3, 7]$

$$\begin{array}{rcl} 2x - y & = & 0 \\ 2 \times [-2, 5] - [-3, 7] & = & 0 \\ [-4, 10] - [-3, 7] & = & 0 \\ [-11, 13] & = & 0 \end{array}$$

Évaluer une contrainte

$x \in [-2, 5]$
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$$\begin{array}{rcl} 2x - y & = & 0 \\ 2 \times [-2, 5] - [-3, 7] & = & 0 \\ [-4, 10] - [-3, 7] & = & 0 \\ [-11, 13] & = & 0 \end{array}$$

$0 \in$ à l'intervalle résultat $\Rightarrow$ Il existe peut-être une solution

Évaluer une contrainte

$x \in [-2, 5]$
$y \in [-3, 7]$

$$\begin{aligned} 2x - y &= 0 \\ 2 \times [-2, 5] - [-3, 7] &= 0 \\ [-4, 10] - [-3, 7] &= 0 \\ [-11, 13] &= 0 \end{aligned}$$

$0 \in$ à l'intervalle résultat $\Rightarrow$ Il existe peut-être une solution
$0 \notin$ à l'intervalle résultat $\Rightarrow$ Pas de solution

Évaluer une contrainte

Exercice

$x \in [-2, 5]$
$y \in [-3, 7]$
$z \in [4, 9]$

Les contraintes suivantes ont-elles des solutions ?

- $x + y - z = 5$
- $3z \leq 10$
- $x + y + z \geq 10$
- $x \times y + y \times z \neq 0$

Évaluer une contrainte

Exercice

$x \in [-2, 5]$
$y \in [-3, 7]$
$z \in [4, 9]$

Les contraintes suivantes ont-elles des solutions ?

- $x + y - z = 5 \rightarrow 5 \in [-14, 8] \Rightarrow$ peut-être une solution
- $3z \leq 10 \rightarrow 10 < [12, 27] \Rightarrow$ pas de solution
- $x + y + z \geq 10 \rightarrow 10 \in [-1, 21] \Rightarrow$ peut-être une solution
- $x \times y + y \times z \neq 0 \rightarrow [0, 0] \neq [-42, 98] \Rightarrow$ peut-être une solution

Limites

$x \in [-2, 5]$

$x \times x \quad =$

Limites

$x \in [-2, 5]$

$$x \times x \quad = \quad [-2, 5] \times [-2, 5]$$

Limites

$x \in [-2, 5]$

$$\begin{aligned} x \times x &= [-2, 5] \times [-2, 5] \\ &= [-10, 25] \end{aligned}$$

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$$x - x = [-2,5] - [-2,5]$$

Limites

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$$\begin{aligned} x - x &= [-2, 5] - [-2, 5] \\ &= [-7, 7] \end{aligned}$$

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Plus de corrélation entre les différentes occurrences d'une variable

Limites

$x \in [-2, 5]$

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Plus de corrélation entre les différentes occurrences d'une variable

$$x^2 - x =$$

Limites

$x \in [-2, 5]$

$$\begin{aligned} x \times x &= [-2, 5] \times [-2, 5] \\ &= [-10, 25] \end{aligned}$$

$$\begin{aligned} x - x &= [-2, 5] - [-2, 5] \\ &= [-7, 7] \end{aligned}$$

Plus de corrélation entre les différentes occurrences d'une variable

$$x^2 - x = [0, 25] - [-2, 5]$$

Limites

$x \in [-2, 5]$

$$\begin{aligned} x \times x &= [-2, 5] \times [-2, 5] \\ &= [-10, 25] \end{aligned}$$

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Plus de corrélation entre les différentes occurrences d'une variable

$$\begin{aligned} x^2 - x &= [0, 25] - [-2, 5] \\ &= [-5, 27] \end{aligned}$$

Limites

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Plus de corrélation entre les différentes occurrences d'une variable

$$\begin{aligned} x^2 - x &= [0, 25] - [-2, 5] \\ &= [-5, 27] \end{aligned}$$

$$x(x - 1) =$$

Limites

$x \in [-2, 5]$

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$$\begin{aligned} x^2 - x &= [0, 25] - [-2, 5] \\ &= [-5, 27] \end{aligned}$$

$$x(x-1) = [-2, 5] \times [-3, 4]$$

Limites

$x \in [-2, 5]$

$$\begin{aligned} x \times x &= [-2, 5] \times [-2, 5] \\ &= [-10, 25] \end{aligned}$$

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Plus de corrélation entre les différentes occurrences d'une variable

$$\begin{aligned} x^2 - x &= [0, 25] - [-2, 5] \\ &= [-5, 27] \end{aligned}$$

$$\begin{aligned} x(x - 1) &= [-2, 5] \times [-3, 4] \\ &= [-15, 20] \end{aligned}$$

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Plus de corrélation entre les différentes occurrences d'une variable

$$\begin{aligned} x^2 - x &= [0, 25] - [-2, 5] \\ &= [-5, 27] \end{aligned}$$

$$\begin{aligned} x(x - 1) &= [-2, 5] \times [-3, 4] \\ &= [-15, 20] \end{aligned}$$

Dépend de l'écriture (valeur réelle $[-0.25, 20]$)

Consistance

Opérateurs ensemblistes

- $[a, b] \cap [c, d] = [\max(a, c), \min(b, d)]$
- $[a, b] \cup [c, d] = [\min(a, c), \max(b, d)]$

Opérateurs inverses

On considère 3 intervalles u, v et r

- $u + v = r$
  $\Rightarrow u = u \cap r - v$
  $\Rightarrow v = v \cap r - u$
- $u - v = r$
  $\Rightarrow u = u \cap r + v$
  $\Rightarrow v = v \cap u - r$

HC4 principe

Pour une contrainte

Revisiting Hull and Box Consistency [Benhamou et al., 1999]

$$x \in [-2, 5]$$
$$y \in [-2, 5]$$

$$2y + x \leq 2$$

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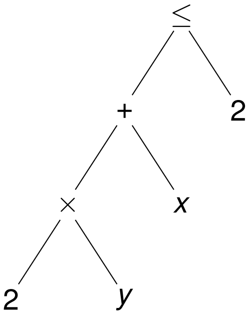

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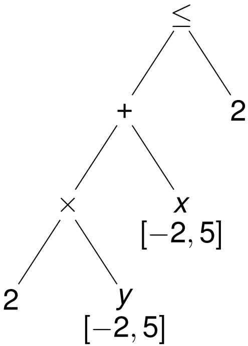

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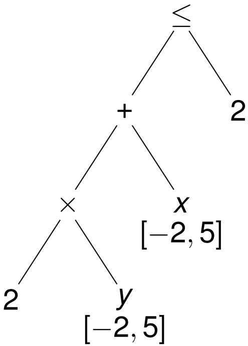
Montée : opérateurs arithmétiques



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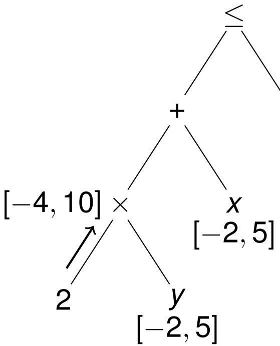

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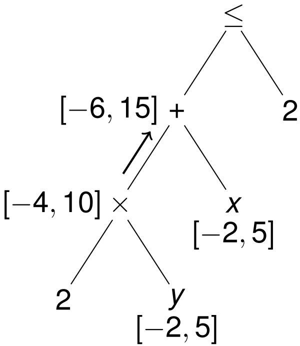

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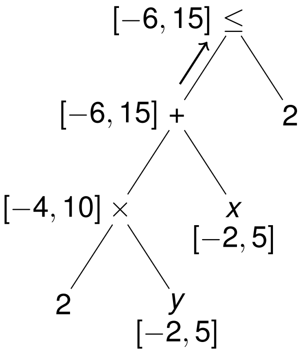

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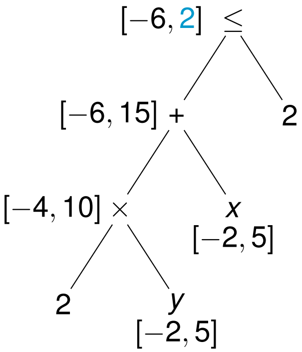

Descente : opérateurs inverses

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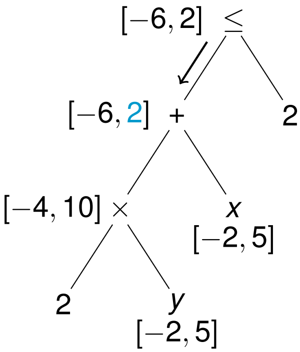

HC4 principe

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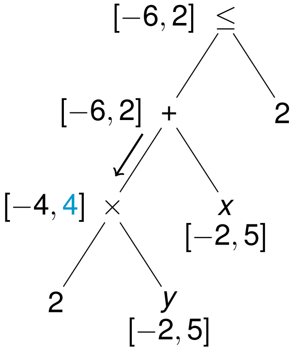
Revisiting Hull and Box Consistency [Benhamou et al., 1999]

$$x \in [-2, 5]$$
$$y \in [-2, 2]$$

$$2y + x \leq 2$$

Descente : opérateurs inverses



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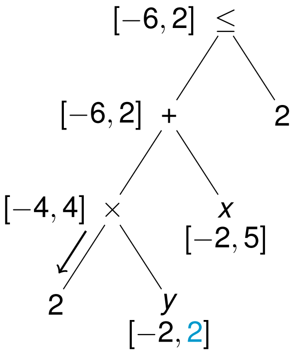


HC4 principe

Pour une contrainte

Exercice

$x \in [-2, 5]$
$y \in [-3, 7]$
$z \in [4, 9]$

Quel est le résultat de la consistance pour chacune des contraintes ?

- $x + y - z = 5$
- $y + z \geq 10$
- $x + 2y \leq 5$

HC4 principe

Pour une contrainte

Exercice

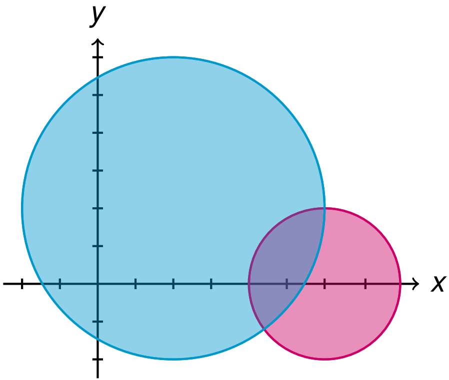
$x \in [-2, 5]$
$y \in [-3, 7]$
$z \in [4, 9]$

Quel est le résultat de la consistance pour chacune des contraintes ?

- $x + y - z = 5 \rightarrow x \in [2, 5], y \in [4, 7], z \in [4, 7]$
- $y + z \geq 10 \rightarrow x \in [-2, 5], y \in [1, 7], z \in [4, 9]$
- $x + 2y \leq 5 \rightarrow x \in [-2, 5], y \in [-3, 3.5], z \in [4, 9]$

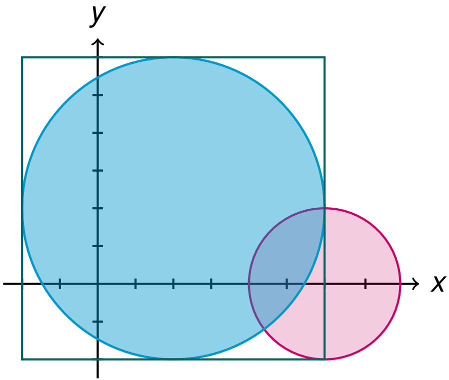
HC4 Principe

Pour plusieurs contraintes



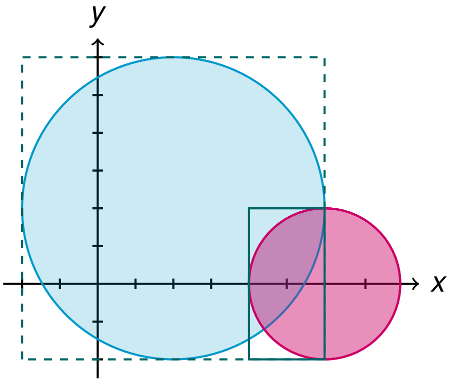
HC4 Principe

Pour plusieurs contraintes



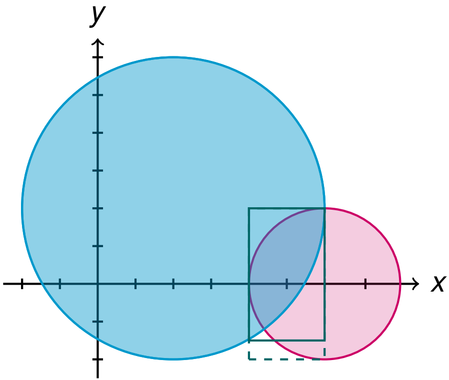
HC4 Principe

Pour plusieurs contraintes



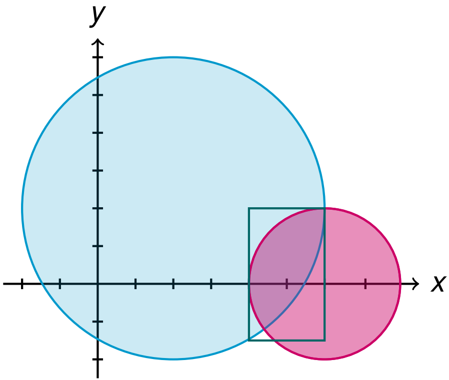
HC4 Principe

Pour plusieurs contraintes



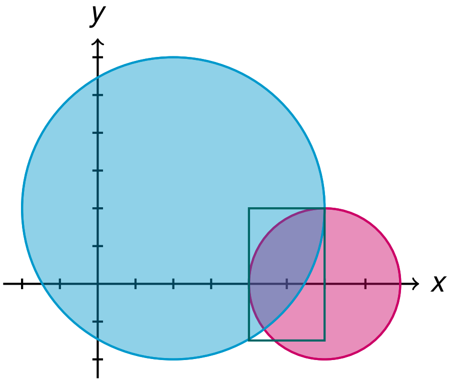
HC4 Principe

Pour plusieurs contraintes



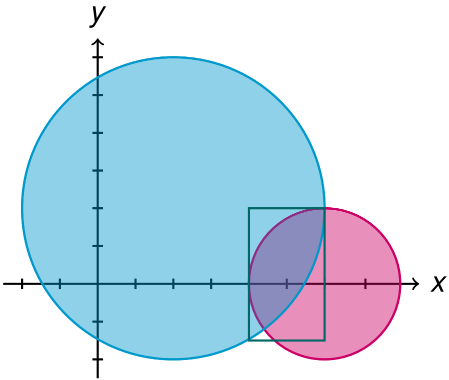
HC4 Principe

Pour plusieurs contraintes



HC4 Principe

Pour plusieurs contraintes



HC4 est généralement rapide mais ne donne pas forcément la plus petite boîte

HC4

Exercice

- $\mathcal{V} = \{x, y\}$
- $D_x = [-2, 5]$
  $D_y = [-3, 3]$
- $C_1 : x - 2y \leq 2$
  $C_2 : x + 2y \leq 2$

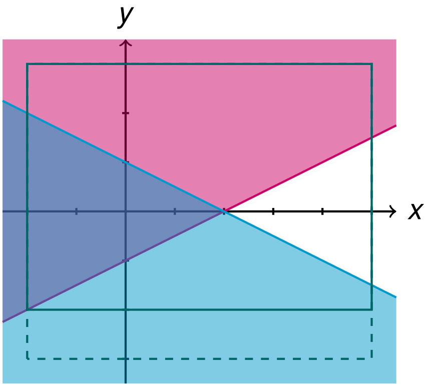
HC4

Exercice

- $\mathcal{V} = \{x, y\}$
- $D_x = [-2, 5]$
  $D_y = [-3, 3]$
- $C_1 : x - 2y \leq 2$
  $C_2 : x + 2y \leq 2$

Solution

- $C_1 : x - 2y \leq 2$
  $\Rightarrow D_x = [-2, 5], D_y = [-2, 3]$



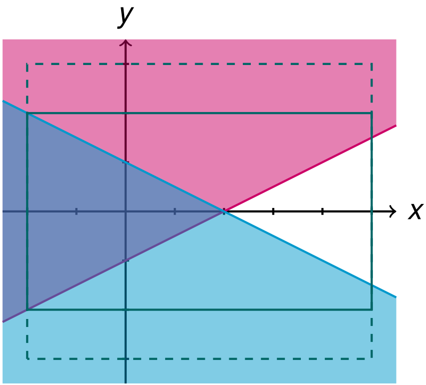
HC4

Exercice

- $\mathcal{V} = \{x, y\}$
- $D_x = [-2, 5]$
  $D_y = [-3, 3]$
- $C_1 : x - 2y \leq 2$
  $C_2 : x + 2y \leq 2$

Solution

- $C_1 : x - 2y \leq 2$
  $\Rightarrow D_x = [-2, 5], D_y = [-2, 3]$
- $C_2 : x + 2y \leq 2$
  $\Rightarrow D_x = [-2, 5], D_y = [-2, 2]$



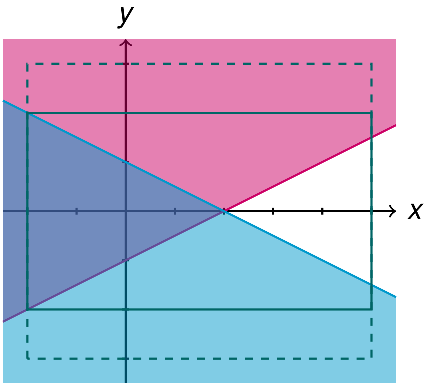
HC4

Exercice

- $\mathcal{V} = \{x, y\}$
- $D_x = [-2, 5]$
  $D_y = [-3, 3]$
- $C_1 : x - 2y \leq 2$
  $C_2 : x + 2y \leq 2$

Solution

- $C_1 : x - 2y \leq 2$
  $\Rightarrow D_x = [-2, 5], D_y = [-2, 3]$
- $C_2 : x + 2y \leq 2$
  $\Rightarrow D_x = [-2, 5], D_y = [-2, 2]$
- $C_1 : x - 2y \leq 2$
  $\Rightarrow D_x = [-2, 5], D_y = [-2, 2]$



HC4

Exercice

- $\mathcal{V} = \{x, y\}$
- $D_x = [0, 8]$
  $D_y = [-1, 3]$
- $C_1 : x + 4y = 8$
  $C_2 : x + 2y = 6$

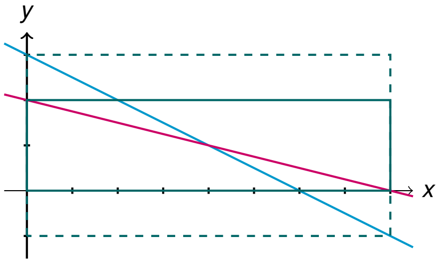
HC4

Exercice

- $\mathcal{V} = \{x, y\}$
- $D_x = [0, 8]$
  $D_y = [-1, 3]$
- $C_1 : x + 4y = 8$
  $C_2 : x + 2y = 6$

Solution

- $C_1 : x + 4y = 8$
  $\Rightarrow D_x = [0, 8], D_y = [0, 2]$



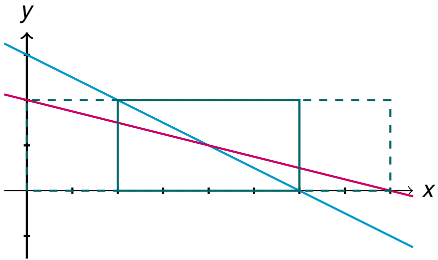
HC4

Exercice

- $\mathcal{V} = \{x, y\}$
- $D_x = [0, 8]$
  $D_y = [-1, 3]$
- $C_1 : x + 4y = 8$
  $C_2 : x + 2y = 6$

Solution

- $C_1 : x + 4y = 8$
  $\Rightarrow D_x = [0, 8], D_y = [0, 2]$
- $C_2 : x + 2y = 6$
  $\Rightarrow D_x = [2, 6], D_y = [0, 2]$



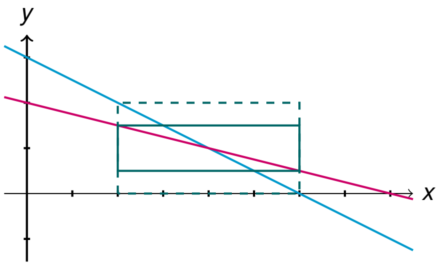
HC4

Exercice

- $\mathcal{V} = \{x, y\}$
- $D_x = [0, 8]$
  $D_y = [-1, 3]$
- $C_1 : x + 4y = 8$
  $C_2 : x + 2y = 6$

Solution

- $C_1 : x + 4y = 8$
  $\Rightarrow D_x = [0, 8], D_y = [0, 2]$
- $C_2 : x + 2y = 6$
  $\Rightarrow D_x = [2, 6], D_y = [0, 2]$
- $C_1 : x + 4y = 8$
  $\Rightarrow D_x = [2, 6], D_y = [0.5, 1.5]$



HC4

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- $\mathcal{V} = \{x, y\}$
- $D_x = [0, 8]$
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- $C_1 : x + 4y = 8$
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Solution

- $C_1 : x + 4y = 8$
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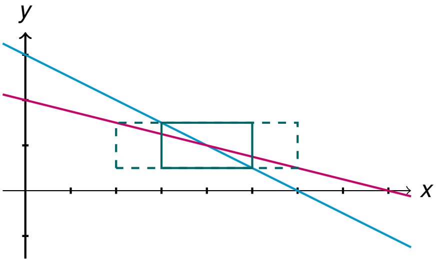
  $\Rightarrow D_x = [2, 6], D_y = [0, 2]$
- $C_1 : x + 4y = 8$
  $\Rightarrow D_x = [2, 6], D_y = [0.5, 1.5]$
- $C_2 : x + 2y = 6$
  $\Rightarrow D_x = [3, 5], D_y = [0.5, 1.5]$



HC4

Exercice

- $\mathcal{V} = \{x, y\}$
- $D_x = [0, 8]$
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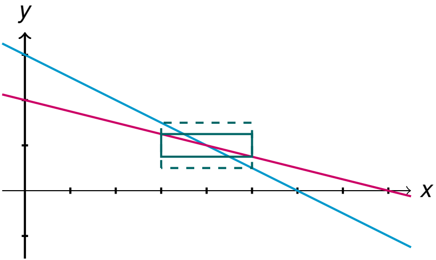
- $C_1 : x + 4y = 8$
  $C_2 : x + 2y = 6$

Solution

- $C_1 : x + 4y = 8$
  $\Rightarrow D_x = [0, 8], D_y = [0, 2]$
- $C_2 : x + 2y = 6$
  $\Rightarrow D_x = [2, 6], D_y = [0, 2]$
- $C_1 : x + 4y = 8$
  $\Rightarrow D_x = [2, 6], D_y = [0.5, 1.5]$
- $C_2 : x + 2y = 6$
  $\Rightarrow D_x = [3, 5], D_y = [0.5, 1.5]$

Solution

- $C_1 : x + 4y = 8$
  $\Rightarrow D_x = [3, 5], D_y = [0.75, 1.25]$



HC4

Exercice

- $\mathcal{V} = \{x, y\}$
- $D_x = [0, 8]$
  $D_y = [-1, 3]$
- $C_1 : x + 4y = 8$
  $C_2 : x + 2y = 6$

Solution

- $C_1 : x + 4y = 8$
  $\Rightarrow D_x = [0, 8], D_y = [0, 2]$
- $C_2 : x + 2y = 6$
  $\Rightarrow D_x = [2, 6], D_y = [0, 2]$
- $C_1 : x + 4y = 8$
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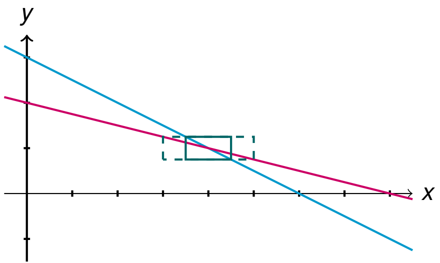
- $C_2 : x + 2y = 6$
  $\Rightarrow D_x = [3, 5], D_y = [0.5, 1.5]$

Solution

- $C_1 : x + 4y = 8$
  $\Rightarrow D_x = [3, 5], D_y = [0.75, 1.25]$
- $C_2 : x + 2y = 6$
  $\Rightarrow D_x = [3.5, 4.5], D_y = [0.75, 1.25]$



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- $D_x = [0, 8]$
  $D_y = [-1, 3]$
- $C_1 : x + 4y = 8$
  $C_2 : x + 2y = 6$

Solution

- $C_1 : x + 4y = 8$
  $\Rightarrow D_x = [0, 8], D_y = [0, 2]$
- $C_2 : x + 2y = 6$
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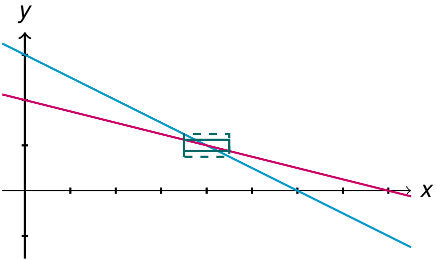
  $\Rightarrow D_x = [3, 5], D_y = [0.5, 1.5]$

Solution

- $C_1 : x + 4y = 8$
  $\Rightarrow D_x = [3, 5], D_y = [0.75, 1.25]$
- $C_2 : x + 2y = 6$
  $\Rightarrow D_x = [3.5, 4.5], D_y = [0.75, 1.25]$
- $C_1 : x + 4y = 8$
  $\Rightarrow D_x = [3.5, 4.5], D_y = [0.875, 1.125]$



HC4

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- $D_x = [0, 8]$
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Solution

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- $C_1 : x + 4y = 8$
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- $C_2 : x + 2y = 6$
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Solution

- $C_1 : x + 4y = 8$
  $\Rightarrow D_x = [3, 5], D_y = [0.75, 1.25]$
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  $\Rightarrow D_x = [3.5, 4.5], D_y = [0.75, 1.25]$
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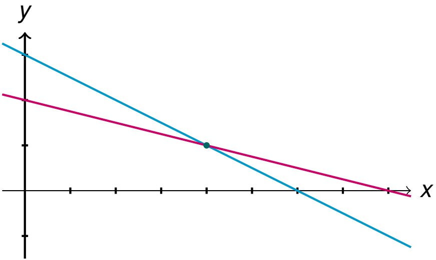
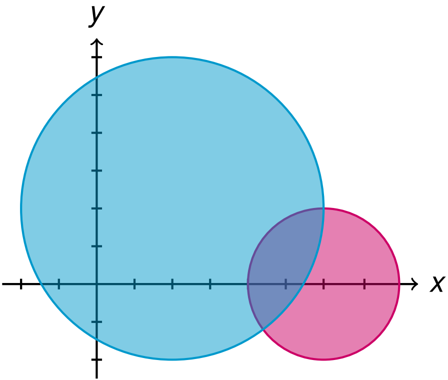
  ...



Shaving

Consistency Techniques for Numeric CSPs [Lhomme, 1993]



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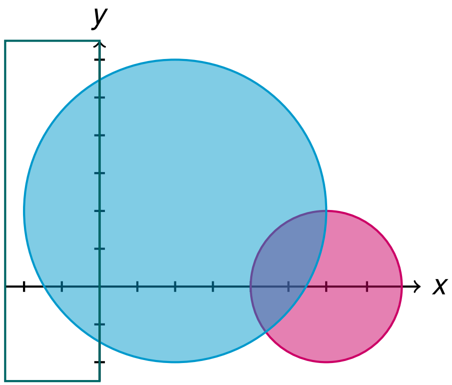

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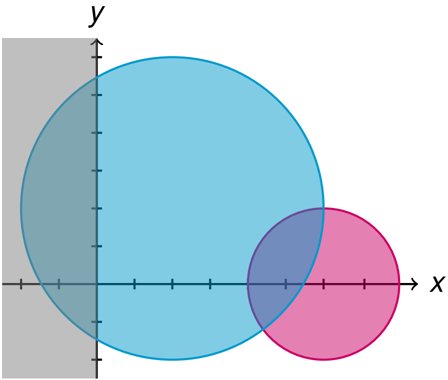

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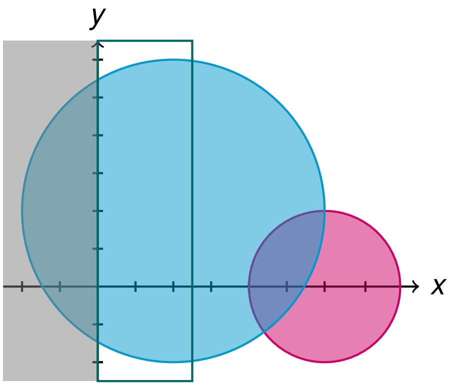

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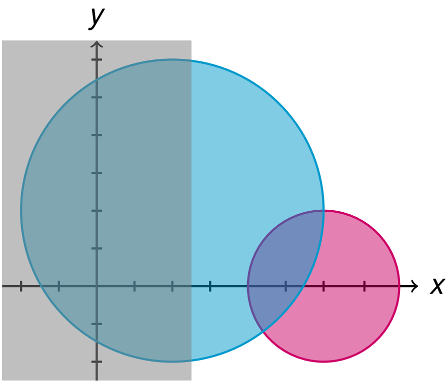

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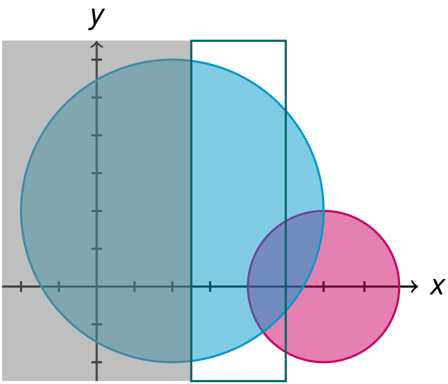

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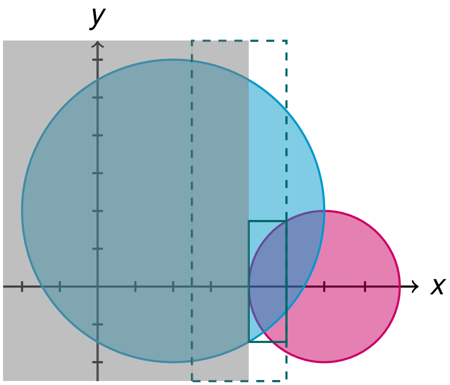

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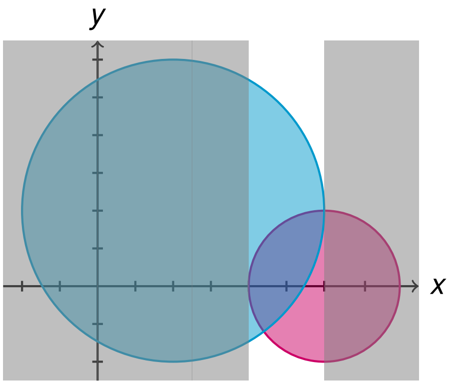

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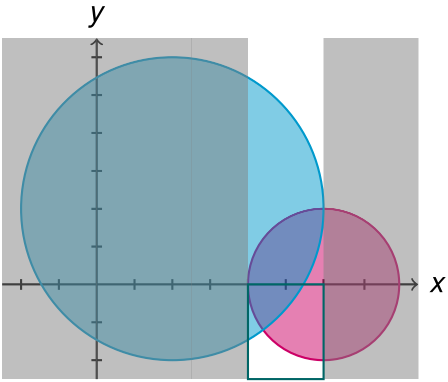

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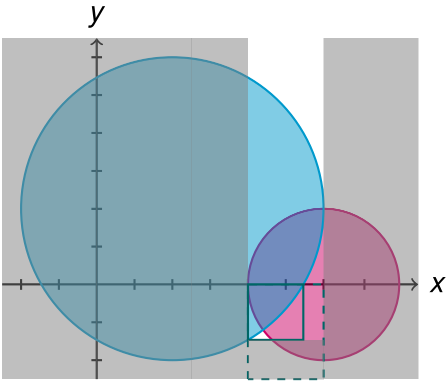

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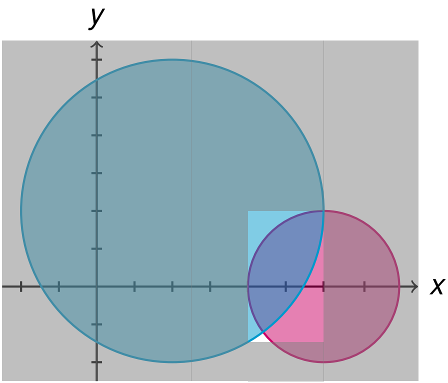

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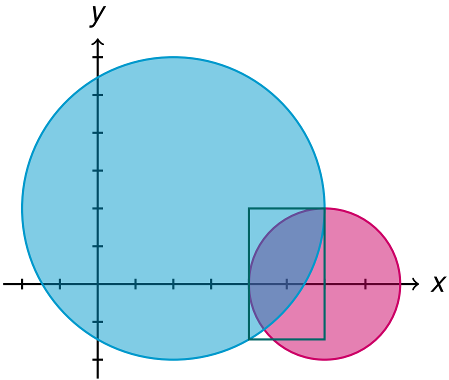
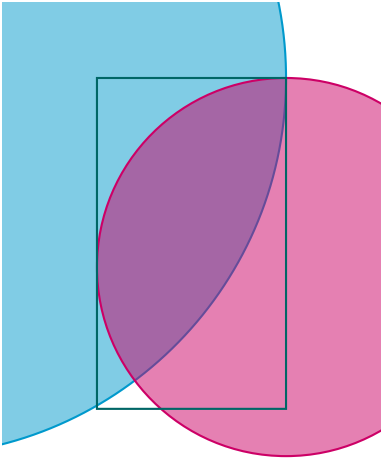
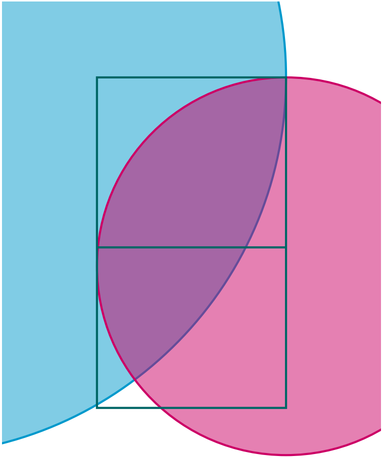


Résolution



Résolution



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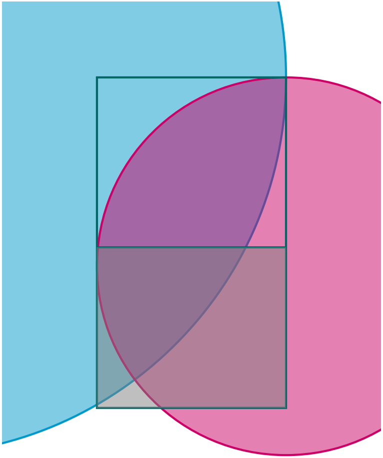

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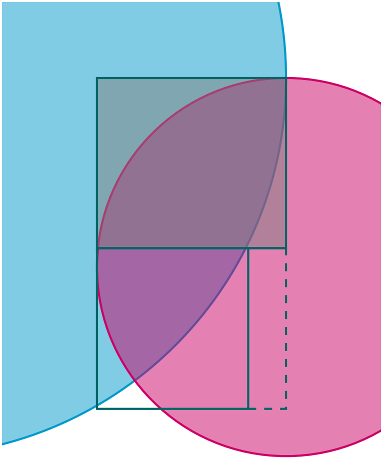

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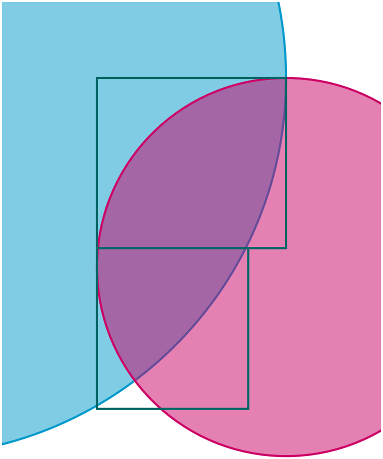

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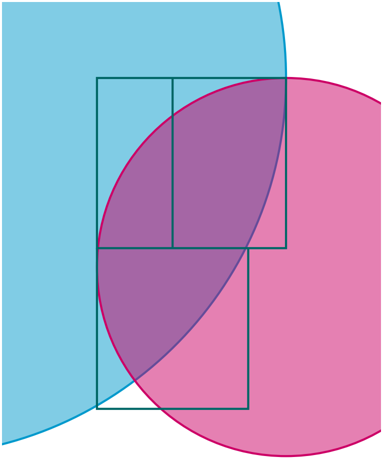

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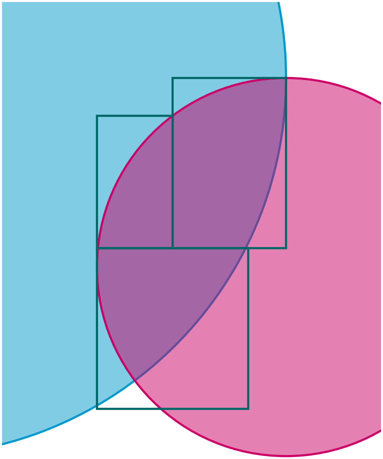

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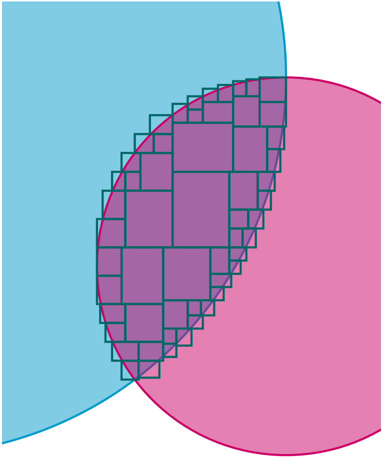

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