

Graphs – Connected Components

1. Introduction

A **Connected Component** in a graph is a **set of vertices where each vertex is reachable from every other vertex** in the same set.

Connected components help us understand how a graph is **partitioned into independent parts**.

This concept is especially important in **undirected graphs**.

2. What is a Connected Component?

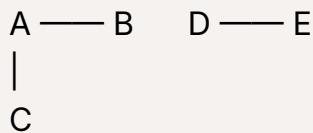
In an **undirected graph**:

- A connected component is a **maximal group of connected vertices**
- There is **at least one path** between every pair of vertices in the component

If a graph has only **one connected component**, it is called a **connected graph**.

3. Example of Connected Components

Graph:



Connected Components:

1. {A, B, C}
 2. {D, E}
- 👉 Total connected components = 2
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4. Connected vs Disconnected Graph

Type	Description
Connected Graph	All vertices are reachable
Disconnected Graph	Graph has multiple components

5. Why Connected Components are Important?

Connected components are used to:

- Identify isolated groups in a network
 - Analyze social networks
 - Find disconnected systems
 - Solve graph connectivity problems
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6. How to Find Connected Components

Connected components can be found using:

- **Breadth First Search (BFS)**
- **Depth First Search (DFS)**

The idea is to:

- Start traversal from an unvisited vertex
 - Mark all reachable vertices as visited
 - Repeat for remaining unvisited vertices
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7. Logic to Find Connected Components (Plain English)

1. Initialize all vertices as unvisited
2. Set component count = 0
3. For each vertex:
 - If unvisited:

- Perform DFS/BFS from that vertex
 - Mark all reachable vertices as visited
 - Increment component count
4. Continue until all vertices are visited
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8. Visualization of Process

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Start DFS from A → visits A, B, C → Component 1  
Start DFS from D → visits D, E → Component 2
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9. Time and Space Complexity

Aspect	Complexity
Time Complexity	$O(V + E)$
Space Complexity	$O(V)$

Where:

- V = number of vertices
 - E = number of edges
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10. Connected Components in Directed Graphs

- For **directed graphs**, connected components are classified as:
 - **Strongly Connected Components (SCC)**
 - **Weakly Connected Components**

(Handled using advanced algorithms like Kosaraju and Tarjan.)

11. Applications of Connected Components

- Social network analysis

- Network connectivity
 - Image segmentation
 - Clustering problems
 - Detecting isolated systems
 - Graph partitioning
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12. Advantages

- Simple to compute
 - Helps analyze graph structure
 - Works efficiently with BFS/DFS
 - Essential for graph algorithms
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13. Limitations

- Does not give shortest path
 - Less meaningful in dense graphs
 - Directed graphs need special handling
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14. Summary

- Connected component is a group of reachable vertices
 - Used mainly in undirected graphs
 - Found using BFS or DFS
 - Helps identify disconnected parts of a graph
 - Time complexity is $O(V + E)$
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