Graphs – Connected Components

1. Introduction

A **Connected Component** in a graph is a **set of vertices where each vertex is reachable from every other vertex** in the same set.

Connected components help us understand how a graph is **partitioned into independent parts**.

This concept is especially important in **undirected graphs**.

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2. What is a Connected Component?

In an **undirected graph**:

- A connected component is a **maximal group of connected vertices**
- There is **at least one path** between every pair of vertices in the component

If a graph has only **one connected component**, it is called a **connected graph**.

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3. Example of Connected Components

Graph:

```
A —— B     D —— E
|
C
```

Connected Components:

1. {A, B, C}
2. {D, E}

👉 Total connected components = **2**

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4. Connected vs Disconnected Graph

Type	Description
Connected Graph	All vertices are reachable
Disconnected Graph	Graph has multiple components

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5. Why Connected Components are Important?

Connected components are used to:

- Identify isolated groups in a network
- Analyze social networks
- Find disconnected systems
- Solve graph connectivity problems

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6. How to Find Connected Components

Connected components can be found using:

- **Breadth First Search (BFS)**
- **Depth First Search (DFS)**

The idea is to:

- Start traversal from an unvisited vertex
- Mark all reachable vertices as visited
- Repeat for remaining unvisited vertices

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7. Logic to Find Connected Components (Plain English)

1. Initialize all vertices as unvisited
2. Set component count = 0
3. For each vertex:
    - If unvisited:

- Perform DFS/BFS from that vertex
    - Mark all reachable vertices as visited
    - Increment component count
4. Continue until all vertices are visited

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8. Visualization of Process

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Start DFS from A → visits A, B, C → Component 1
Start DFS from D → visits D, E → Component 2
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9. Time and Space Complexity

Aspect	Complexity
Time Complexity	O(V + E)
Space Complexity	O(V)

Where:

- V = number of vertices
- E = number of edges

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10. Connected Components in Directed Graphs

- For **directed graphs**, connected components are classified as:
    - **Strongly Connected Components (SCC)**
    - **Weakly Connected Components**

(Handled using advanced algorithms like Kosaraju and Tarjan.)

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11. Applications of Connected Components

- Social network analysis

- Network connectivity
- Image segmentation
- Clustering problems
- Detecting isolated systems
- Graph partitioning

12. Advantages

- Simple to compute
- Helps analyze graph structure
- Works efficiently with BFS/DFS
- Essential for graph algorithms

13. Limitations

- Does not give shortest path
- Less meaningful in dense graphs
- Directed graphs need special handling

14. Summary

- Connected component is a group of reachable vertices
- Used mainly in undirected graphs
- Found using BFS or DFS
- Helps identify disconnected parts of a graph
- Time complexity is $O(V + E)$