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Analysis of Transient Response in LC Circuits.

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2025-12-18

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1 Introduction

An LC circuit, consisting of an inductor (L) and a capacitor (C), is fundamental in electronics. Its transient response, the reaction to sudden changes in input, is critical for applications such as filters, oscillators, and tuners. This document summarizes the theoretical and experimental methods to analyze the transient response, determine the natural frequency (ω_n), and calculate the damping ratio (ξ).

2 Theoretical Analysis

2.1 Differential Equation of an LC Circuit

Using Kirchhoff's Voltage Law (KVL), the voltage across the inductor and capacitor in series sums to zero:

$$L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt = 0.$$

Differentiating with respect to time yields the second-order differential equation:

$$\frac{d^2i(t)}{dt^2} + \frac{1}{LC}i(t) = 0.$$

2.2 Natural Frequency (ω_n)

The natural frequency is the frequency at which the circuit oscillates in the absence of damping:

$$\omega_n = \frac{1}{\sqrt{LC}}.$$

2.3 Damping Ratio (ξ)

In real circuits, resistance (R) introduces damping. The damping ratio is:

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}.$$

It determines the rate of oscillation decay:

- $\xi < 1$: Underdamped (oscillations decay slowly).
- $\xi = 1$: Critically damped (fastest return to equilibrium without oscillation).
- $\xi > 1$: Overdamped (slow return without oscillation).

2.4 Quality Factor (Q)

The quality factor relates to the damping ratio:

$$Q = \frac{1}{2\xi}.$$

A high Q indicates low damping and sustained oscillations.

3 Experimental Procedure

3.1 Experimental Setup

1. Connect the capacitor to a DC voltage supply.
2. Place a switch between the capacitor and the inductor.
3. Connect the oscilloscope probe across the capacitor.
4. Set the oscilloscope to single-shot capture mode.

3.2 Procedure

1. Charge the capacitor fully using the DC voltage supply.
2. Once the capacitor is charged, disconnect it from the DC supply.
3. Set the oscilloscope to trigger on the rising edge of the signal.
4. Close the switch to connect the inductor to the charged capacitor.
5. Observe the captured LC oscillation waveform on the oscilloscope screen.

3.3 Oscilloscope Settings

1. Set the oscilloscope to single-shot capture mode.
2. Adjust the time base to capture the full oscillation period.
3. Set the voltage/division to accommodate the expected signal amplitude.
4. Configure the trigger to capture the start of the oscillation.

3.4 Observations

Upon closing the switch, you will observe:

1. A damped sinusoidal waveform captured in single-shot mode.
2. The initial amplitude is almost just less than DC supply voltage.
3. The oscillation frequency determined by L and C values:
$$f = \frac{1}{2\pi\sqrt{LC}}$$
4. Gradual decay of oscillations due to circuit resistance.

4 Calculation

Taken values of $L = 2.2mH$ and $C = 35\mu F$

The Time period(T) in Oscilloscope for the transient response of above L and C is $1.76\mu S$

4.1 Damped frequency(ω_d)

$$\omega_d = \frac{2\pi}{T}$$
$$\omega_d = \frac{2\pi}{1.76 \times 10^{-6}}$$
$$\omega_d = 3,570,796.33 rad/s$$

4.2 Natural frequency(ω_n)

$$\omega_n = \frac{1}{\sqrt{LC}}$$

on calculating,

$$\omega_n = 3,603,749.85 rad/s$$

4.3 Damping ratio(ζ)

$$\zeta = \sqrt{1 - (\frac{\omega_d}{\omega_n})^2}$$

on calculating we get,

$$\zeta = 0.1366$$

The logarithmic decrement is given by:

$$\delta = \frac{1}{n} \ln \left(\frac{A_k}{A_{k+n}} \right) \tag{1}$$

where:

- A_k is the amplitude at a certain peak.
- A_{k+n} is the amplitude after n cycles.
- n is the number of cycles between the two chosen peaks.

Calculation of Damping ratio through δ Given:

$$A_k = 3.761 \text{ V},$$
$$A_{k+1} = 0.0163 \text{ V}.$$

Using the logarithmic decrement formula:

$$\delta = \ln \left(\frac{A_k}{A_{k+1}} \right) \tag{2}$$

Substituting the values:

$$\delta = \ln \left(\frac{3.761}{0.0163} \right) = 5.444 \tag{3}$$

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \tag{4}$$

Substituting $\delta = 5.444$:

$$\zeta = \frac{5.444}{\sqrt{4\pi^2 + (5.444)^2}} = 0.1366 \tag{5}$$

Hence, the value of ζ calculated through different formulae is same.

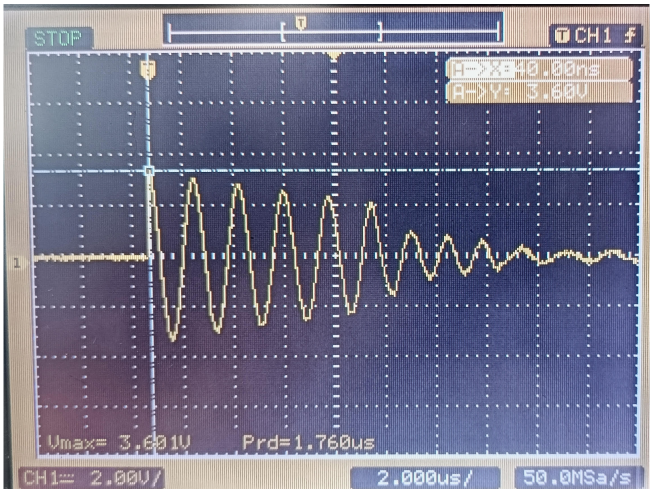

Figure 1: Transient response

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Finding the plot of the above through in python,

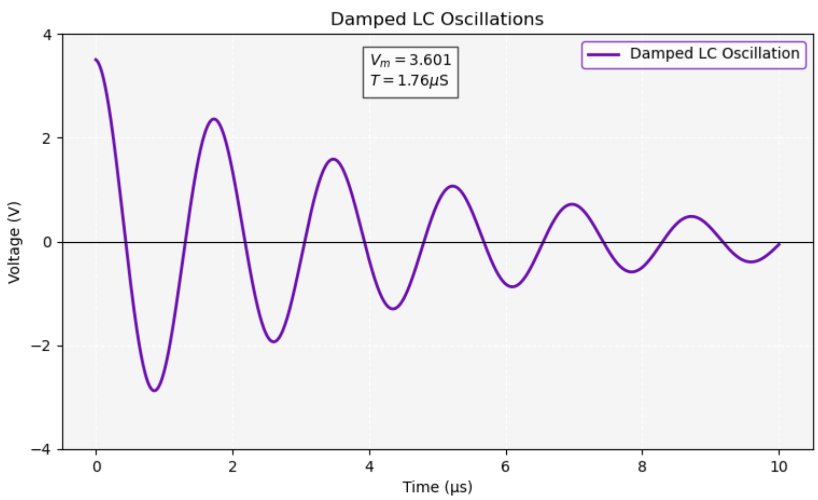

Figure 2: python graph

5.1 Applications

LC circuits are used in:

- Filters (low-pass, high-pass, band-pass, band-stop).
- Oscillators for signal generation.
- Tuners in radio frequency selection.

5.2 Limitations

- Real-world circuits have parasitic elements (e.g., stray capacitance, resistance).
- Nonlinearities in components can complicate analysis.

6 References

- Online Websites
- AI Suggestions