SALLEN-KEY FILTERS

BY

K.Akhil - EE24BTECH11035
K.Teja Vardhan - EE24BTECH11034

2025-12-18

Contents

1 Objective

1. To design and implement a bandpass filter using separate Sallen-Key Low Pass Filter (LPF) and High Pass Filter (HPF).
2. To analyze and compare the frequency response of LPF, HPF, and the final bandpass filter.
3. To plot the magnitude response (gain vs. frequency) of all three filters.

2 Theory

Bandpass Filter (BPF)

A bandpass filter (BPF) allows frequencies within a specified range while attenuating those outside it. It is constructed using:

- A High Pass Filter (HPF) to remove low-frequency components.
- A Low Pass Filter (LPF) to remove high-frequency components.
- The combined response results in a bandpass characteristic.

Sallen-Key Second-Order Filters

- It is an active filter topology using operational amplifiers.
- Provides a Butterworth, Bessel, or Chebyshev response based on component selection.
- The transfer function is given by:

$$H(s) = \frac{A}{s^2 + \frac{\omega_c}{Q}s + \omega_c^2} \tag{1}$$

  where:

  - ω_c is the cutoff angular frequency.
  - Q is the quality factor.

3 Components & Equipment Required

- Operational Amplifiers (e.g., TL074, TL081, or LM358)
- Resistors: R_1, R_2, R_3, R_4 (in $k\Omega$)
- Capacitors: C_1, C_2, C_3, C_4 (in nF)
- Function Generator
- Oscilloscope or Spectrum Analyzer
- DC Power Supply (+12V)
- Breadboard and connecting wires

4 Circuit Design

High Pass Filter (HPF) Design

- Cutoff frequency f_{c1} (Lower cutoff frequency of BPF).
- Standard Sallen-Key HPF formula:

$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \tag{2}$$

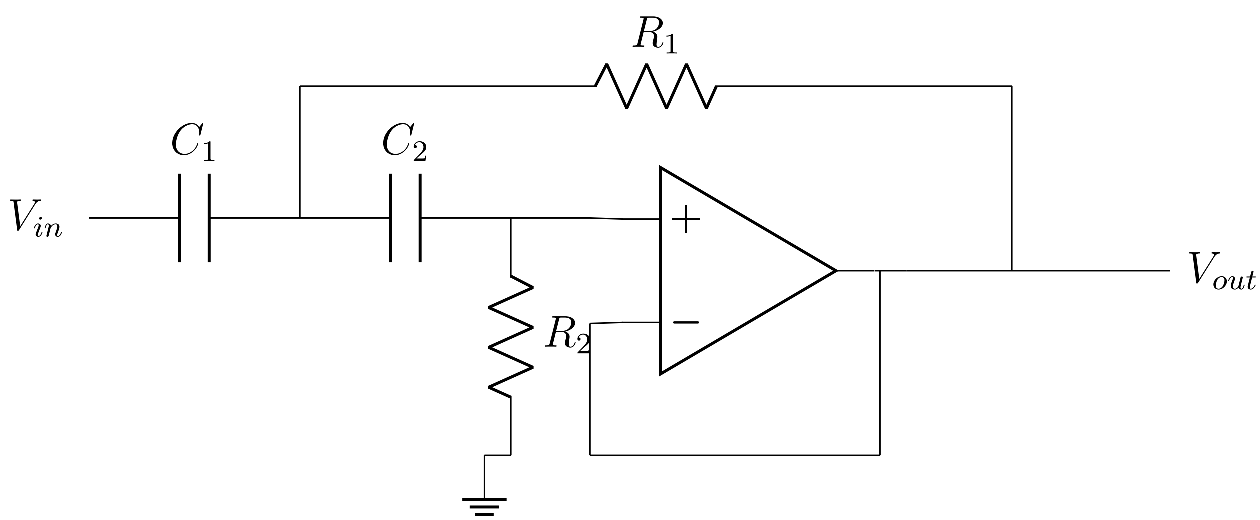


Figure 1: HPF

Low Pass Filter (LPF) Design

- Cutoff frequency f_{c2} (Upper cutoff frequency of BPF).
- Standard Sallen-Key LPF formula:

$$f_c = \frac{1}{2\pi\sqrt{R_3 R_4 C_3 C_4}} \tag{3}$$

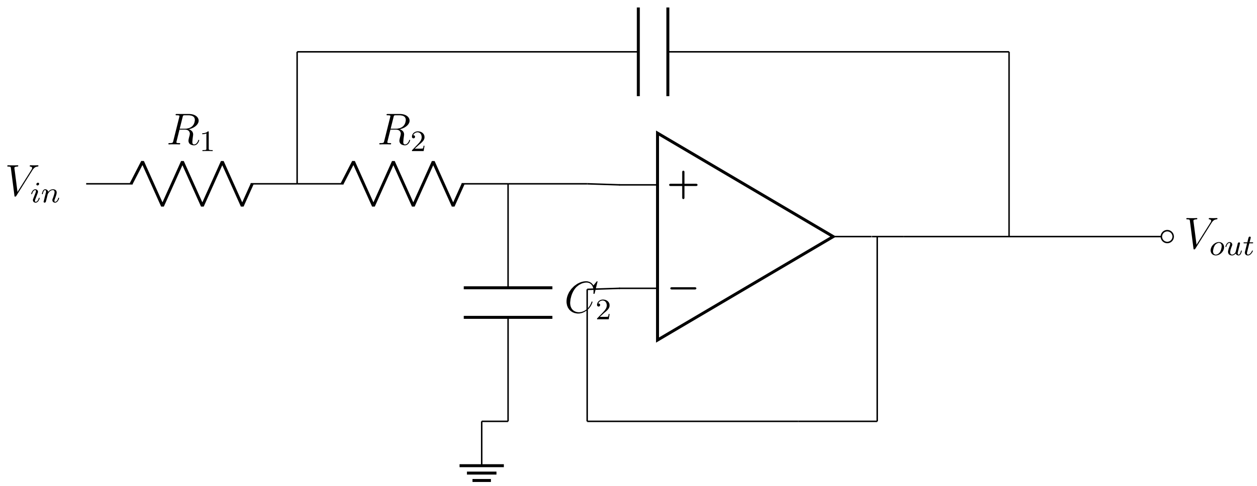


Figure 2: LPF

Bandpass Filter (BPF) Design

- The cascaded connection of the HPF and LPF forms a second-order bandpass filter.
- Bandwidth:
$$BW = f_{c2} - f_{c1} \tag{4}$$
- Center Frequency:
$$f_0 = \sqrt{f_{c1} f_{c2}} \tag{5}$$

5 Procedure

Step 1: HPF Implementation

1. The Sallen-Key HPF circuit was assembled on the breadboard.
2. Used the function generator to apply a sine wave.
3. The input frequency was varied and the output voltage was measured.measured.
4. Recorded gain values for different frequencies are,

V_{in} (V)	V_{out} (V)	Frequency (Hz)	Gain (dB)
5.201	0.216	100	27.63
5.201	1.08	500	13.64
5.201	2.881	1500	5.13
5.201	3.441	2000	3.588
5.201	4.201	3000	1.854
5.201	4.401	5500	0.695

5. Plot of gain vs. frequency (Bode plot),

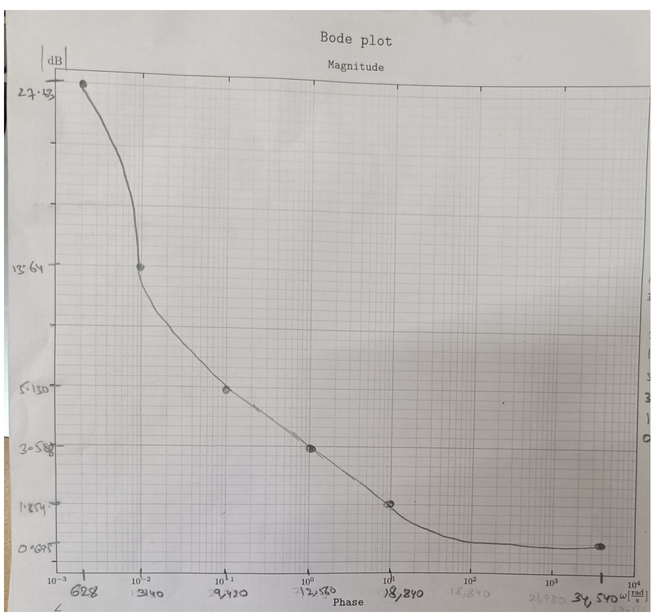


Figure 3

Step 2: LPF Implementation

1. Assembled the Sallen-Key LPF circuit.

2. Repeated the same procedure as in Step 1.

V_{in} (V)	V_{out} (V)	ω ($\frac{rad}{sec}$)	Gain (dB)
5.201	4.041	4960	-1.63
5.201	3.041	9300	-4.63
5.201	1.841	12400	-9.03
5.201	1.041	19840	-13.98

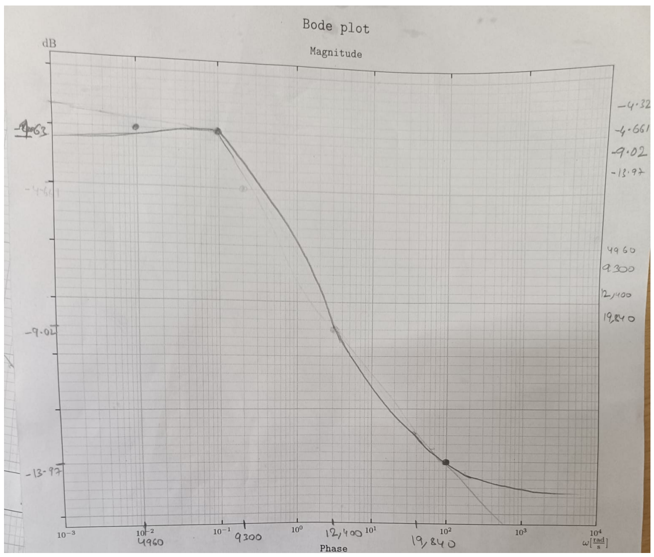


Figure 4

Step 3: BPF Implementation

1. Connected the HPF output to the LPF input.

2. Repeated the measurements.

V_{in} (V)	V_{out} (V)	ω ($\frac{rad}{sec}$)	Gain (dB)
5.201	0.4	942	-22.6
5.201	0.96	1570	-14.67
5.201	4.241	2951.6	-1.812
5.201	5.041	7975.6	-0.28
5.201	2.721	12560	-10.561
5.201	1.681	18840	-15
5.201	0.72	21980	-17.5
5.201	0.42	29516	-22.2

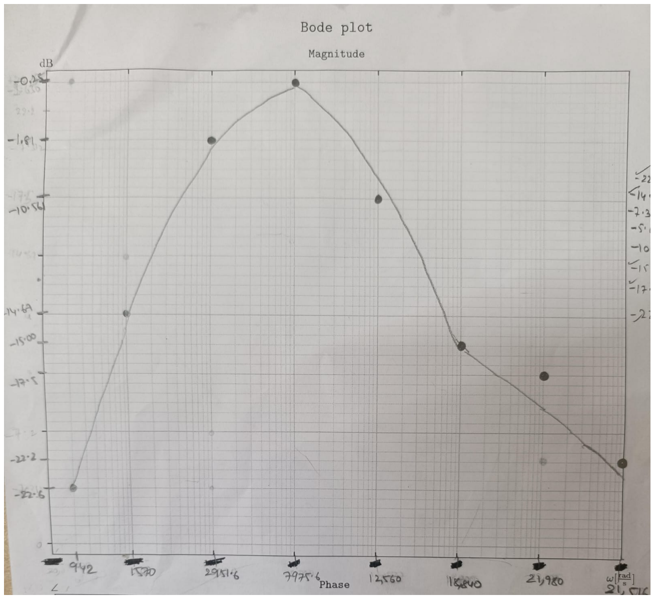


Figure 5

All the input and output pictures given in the following github link:
`https://github.com/AKHIL11035/Electrical-Circuits-Lab/tree/121cfafc034a9359d7555a5bd68139`
`experiment%20-%2006`

6 Mathematical calculation values

Low Pass Filter Parameters

- $R_1 = R_2 = 100\,\mathrm{k}\Omega$
- $C_1 = C_2 = 1\,\mathrm{nF}$

$$
\begin{aligned}
f_{c_{LP}} &= \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \\
&= \frac{1}{2\pi\sqrt{(100 \times 10^3)^2 (1 \times 10^{-9})^2}} \\
&= \frac{1}{2\pi(100 \times 10^3)(1 \times 10^{-9})} \\
&= \frac{1}{2\pi \times 0.1} \\
&= \frac{1}{0.628} \\
&\approx 1591\,\mathrm{Hz}
\end{aligned}
$$

High Pass Filter Parameters

- $R_1 = R_2 = 68\,\mathrm{k\Omega}$
- $C_1 = C_2 = 4.7\,\mathrm{nF}$

$$
\begin{aligned}
f_{c_{HP}} &= \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \\
&= \frac{1}{2\pi\sqrt{(68 \times 10^3)^2 (4.7 \times 10^{-9})^2}} \\
&= \frac{1}{2\pi(68 \times 10^3)(4.7 \times 10^{-9})} \\
&= \frac{1}{2\pi \times 0.3196} \\
&= \frac{1}{2.006} \\
&\approx 498\,\mathrm{Hz}
\end{aligned}
$$

Band-Pass Filter Center Frequency

$$
\begin{aligned}
f_{c_{Low}} &= f_{c_{HP}} = 498\,\text{Hz} \\
f_{c_{High}} &= f_{c_{LP}} = 1591\,\text{Hz} \\
f_{c_{BP}} &= \sqrt{f_{c_{Low}} \cdot f_{c_{High}}} \\
&= \sqrt{498 \times 1591} \\
&= \sqrt{792318} \\
&\approx 890\,\text{Hz}
\end{aligned}
$$

As, we can observe from the above bode plots , the decay stay starts for cutoff frequencies correctly.

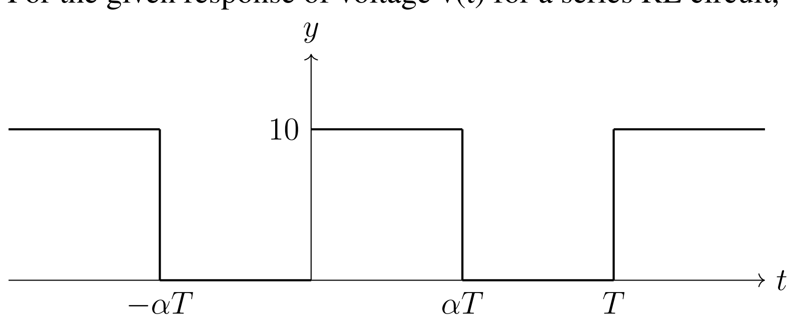
7 Conclusion

- The experiment verifies the cascading method to form a bandpass filter.
- The experimental results matches the theoretical calculations.
- Sallen-Key topology provides good stability and response.

8 Fourier

9 Fourier-Series

For the given response of voltage v(t) for a series RL circuit,



Complex exponential form of Fourier series:

$$g(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$$

To find the Fourier Coefficients,

$$C_k = \frac{1}{T}\int_0^T g(t)e^{-jk\omega_0 t}\,dt \quad \forall k \neq 0$$

$$= \frac{1}{T}\int_0^{\alpha T} 10e^{-jk\omega_0 t}\,dt \quad + \frac{1}{T}\int_{\alpha T}^{T} 0e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{T}\int_0^{\alpha T} 10e^{-jk\frac{2\pi}{T}t}\,dt$$

$$= \frac{10}{T}\int_0^{\alpha T} e^{-jk\frac{2\pi}{T}t}\,dt$$

$$= \frac{10}{T}\cdot \left.\frac{e^{-jk\frac{2\pi}{T}t}}{-jk\frac{2\pi}{T}}\right|_0^{\alpha T}$$

$$= \frac{10}{T}\cdot \frac{1}{-jk\frac{2\pi}{T}}\left(e^{-jk\frac{2\pi}{T}(\alpha T)} - e^{-jk\frac{2\pi}{T}(0)}\right)$$

$$= \frac{10}{T}\cdot \frac{1}{-jk\frac{2\pi}{T}}\left(e^{-jk2\pi\alpha} - 1\right)$$

$$= \frac{10}{-jk2\pi}\left(e^{-jk2\pi\alpha} - 1\right)$$

$$= \frac{5}{jk\pi}(1 - e^{-jk2\pi\alpha})$$

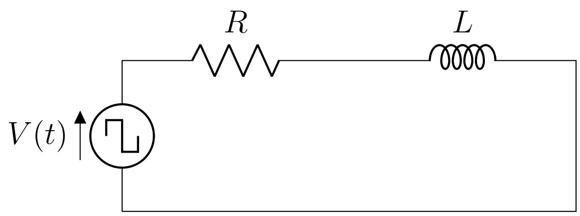
when k = 0

$$C_0 = \frac{1}{T}\int_0^{\alpha T} 10\,dt = 10\alpha$$

Hence,

$$V(t) = 10\alpha + \sum_{k=-\infty}^{\infty} \frac{5}{jk\pi}(1 - e^{-jk2\pi\alpha})e^{jk\omega_0 t} \quad \forall k \neq 0$$

Now given a series RL circuit



By applying loop law

$$L\frac{di(t)}{dt} + Ri(t) = V(t)$$

$$\frac{di(t)}{dt} + R\frac{i(t)}{L} = \frac{V(t)}{L}$$

But ,

Let $g(x) = g_1(x) + g_2(x)$,

$$L[y_1] = g_1(x), \quad L[y_2] = g_2(x).$$

By linearity, the solution to $L[y] = g(x)$ is

$$y(x) = y_1(x) + y_2(x).$$

For individual harmonics (when k $\neq$ 0)

$$\frac{di(t)}{dt} + R\frac{i(t)}{L} = \frac{C_k e^{jk\frac{2\pi}{T}t}}{L}$$

The integration factor is $e^{\int \frac{R}{L}dt}$ = $e^{\frac{R}{L}t}$

$$e^{\frac{R}{L}t}\frac{di(t)}{dt} + e^{\frac{R}{L}t}\frac{R}{L}i(t) = e^{\frac{R}{L}t}\frac{C_k e^{jk\frac{2\pi}{T}t}}{L}$$

$$\frac{d}{dt}\left(i(t)e^{\frac{R}{L}t}\right) = \frac{C_k e^{(jk\frac{2\pi}{T}+\frac{R}{L})t}}{L}$$

$$i(t)e^{\frac{R}{L}t} = \int \frac{C_k e^{(jk\frac{2\pi}{T}+\frac{R}{L})t}}{L} dt$$

$$i(t)e^{\frac{R}{L}t} = \frac{C_k}{L(jk\frac{2\pi}{T}+\frac{R}{L})} e^{(jk\frac{2\pi}{T}+\frac{R}{L})t}$$

$$i(t) = \frac{C_k}{L\left(jk\frac{2\pi}{T}+\frac{R}{L}\right)} e^{jk\frac{2\pi}{T}t}$$

$$i_k = C_k(\frac{T}{jk2\pi L + RT})$$

when k = 0

$$V_0 = 10\alpha$$

$$i_0 = V_0/R$$

$$i_0 = \frac{10\alpha}{R}$$

For homogeneous part

$$\frac{di(t)}{dt} + R\frac{i(t)}{L} = 0$$
$$i(t) = Ae^{-\frac{R}{L}t}$$

Overall equation

$$i(t) = Ae^{-\frac{R}{L}t} + \frac{10\alpha}{R} + \sum_{k=-\infty}^{\infty} i_k e^{jk\omega_0 t} \quad \forall k \neq 0$$

Assuming initial conditions as i(0) = 0

$$A = -\frac{10\alpha}{R} - \sum_{k=-\infty}^{\infty} i_k$$

Current response

$$i(t) = (-\frac{10\alpha}{R} - \sum_{k=-\infty}^{\infty} i_k)e^{-\frac{R}{L}t} + \frac{10\alpha}{R} + \sum_{k=-\infty}^{\infty} i_k e^{jk\omega_0 t} \quad \forall k \neq 0$$

By using trignometric cofficients method

$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cos(n\omega t) + b_n sin(n\omega t)$$

For a_0

$$a_0 = \frac{1}{T}\int_0^T v(t)dt$$
$$a_0 = \frac{1}{T}\int_0^{\alpha T} 10dt$$
$$a_0 = 10\alpha$$

For a_n

$$a_n = \frac{2}{T}\int_0^T v(t)cos(n\omega t)dt$$

$$a_n = \frac{2}{T}\int_0^{\alpha T} 10cos(n\omega t)dt$$

$$a_n = \frac{20}{T}\int_0^{\alpha T} cos(n\omega t)dt$$

$$\int_0^{\alpha T} \cos(n\omega t)dt = \left[\frac{\sin(n\omega t)}{n\omega}\right]_0^{\alpha T}$$

$$= \frac{\sin(n\omega\alpha T)}{n\omega} - \frac{\sin(0)}{n\omega}$$

$$= \frac{\sin(n\omega\alpha T)}{n\omega}$$

$$a_n = \frac{20}{T} \cdot \frac{\sin(n\omega\alpha T)}{n\omega}$$

$$= \frac{20\sin(n\omega\alpha T)}{n\omega T}$$

$$a_n = \frac{10}{\pi n}\sin(2\pi n\alpha)$$

$$a_n = \frac{20}{\pi n}\sin(\pi n\alpha)\cos(\pi n\alpha)$$

For b_n

$$b_n = \frac{2}{T}\int_0^T v(t)\sin(n\omega t)dt$$

$$b_n = \frac{2}{T}\int_0^{\alpha T} 10\sin(n\omega t)dt$$

$$b_n = \frac{20}{T}\int_0^{\alpha T} \sin(n\omega t)dt$$

$$\int_0^{\alpha T} \sin(n\omega t)dt = \left[-\frac{\cos(n\omega t)}{n\omega}\right]_0^{\alpha T}$$

$$= -\frac{\cos(n\omega\alpha T)}{n\omega} + \frac{\cos(0)}{n\omega}$$

$$= \frac{1-\cos(n\omega\alpha T)}{n\omega}$$

$$b_n = \frac{20}{T}\cdot\frac{1-\cos(n\omega\alpha T)}{n\omega}$$

$$= \frac{20(1-\cos(n\omega\alpha T))}{n\omega T}$$

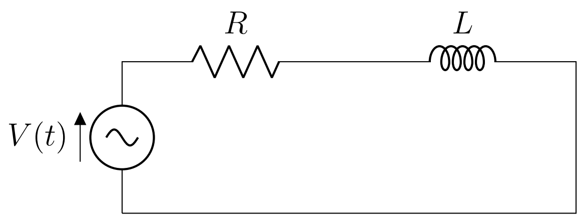
$$b_n = \frac{10}{\pi n}(1-\cos(2\pi n\alpha))$$

$$b_n = \frac{20}{\pi n}\sin^2(\pi n\alpha)$$

Hence ,

$$v(t) = 10\alpha + \frac{20}{\pi n}\sin(\pi n\alpha)\cos(\pi n\alpha) + \frac{20}{\pi n}\sin^2(\pi n\alpha)$$

Now given a series RL circuit



By applying loop law

$$L\frac{di(t)}{dt} + Ri(t) = V(t)$$

$$\frac{di(t)}{dt} + R\frac{i(t)}{L} = \frac{V(t)}{L}$$

But ,

$$\text{Let } g(x) = g_1(x) + g_2(x),$$

$$L[y_1] = g_1(x), \quad L[y_2] = g_2(x).$$

By linearity, the solution to $L[y] = g(x)$ is

$$y(x) = y_1(x) + y_2(x).$$

For i_a

$$\frac{di(t)}{dt} + R\frac{i(t)}{L} = \frac{V(t)}{L}$$

$$\frac{di(t)}{dt} + R\frac{i(t)}{L} = \frac{\frac{20}{\pi n}\sin(\pi n\alpha)\cos(\pi n\alpha)cos(n\omega_0 t)}{L}$$

$$I.F = e^{\int \frac{R}{L}t}$$

$$ie^{\int \frac{R}{L}t} = \frac{20}{\pi n}\sin(\pi n\alpha)\cos(\pi n\alpha)\int e^{\frac{R}{L}t}cos(n\omega_0 t)$$

$$i_a = \frac{20}{\pi n}\sin(\pi n\alpha)\cos(\pi n\alpha)\frac{1}{\frac{R}{L}^2 + (n\omega_0)^2}(\frac{R}{L}cos(n\omega_0 t) + n\omega_0 sin(n\omega_0 t))$$

For i_b

$$\frac{di(t)}{dt} + R\frac{i(t)}{L} = \frac{V(t)}{L}$$

$$\frac{di(t)}{dt} + R\frac{i(t)}{L} = \frac{20}{\pi n}sin^2(n\pi\alpha)sin(n\omega_0 t)$$

$$i_b = \frac{20}{\pi n}\sin(\pi n\alpha)\cos(\pi n\alpha)\frac{1}{\frac{R}{L}^2 + (n\omega_0)^2}(\frac{R}{L}sin(n\omega_0 t) - n\omega_0 cos(n\omega_0 t) + n\omega_0 e^{\frac{-R}{L}t}$$

$i_0 = \frac{10\alpha}{R}$

$$i(t) = i_0 + i_a + i_b$$

$$i(t) = \frac{10\alpha}{R} + \sum_{n=1}^{\infty}\left[\frac{20}{\pi n}\sin(\pi n\alpha)\cos(\pi n\alpha)\frac{1}{\frac{R^2}{L^2} + (n\omega_0)^2}\left(\frac{R}{L}\cos(n\omega_0 t) + n\omega_0\sin(n\omega_0 t)\right)\right]$$

$$+ \sum_{n=1}^{\infty}\left[\frac{20}{\pi n}\sin^2(\pi n\alpha)\frac{1}{\frac{R^2}{L^2} + (n\omega_0)^2}\left(\frac{R}{L}\sin(n\omega_0 t) - n\omega_0\cos(n\omega_0 t) + n\omega_0 e^{-\frac{R}{L}t}\right)\right]$$