

# Hamming Code – Error Detection & Correction

## Introduction to Hamming Code

Hamming code is a linear error-detecting and error-correcting code developed by Richard Hamming in 1950. It is used in digital systems to ensure data reliability in communication and memory storage.

### Features:

✓ Detects up to 2-bit errors and corrects 1-bit errors. ✓ Efficient: Uses a small number of parity bits for error correction. ✓ Widely used in computer memory (ECC RAM), networking, and wireless communication.

## General Formula:

A Hamming code follows the format  $(n, k)$ , where:

- $n$  = Total number of bits (data + parity)
- $k$  = Number of data bits
- $r$  = Number of parity bits (where  $n = k + r$ )

## Common Hamming Code Variants:

♦ **(7,4) Hamming Code** → 4-bit data, 3 parity bits, 7-bit codeword. ♦ **(15,11) Hamming Code** → 11-bit data, 4 parity bits, 15-bit codeword.

## Working of (7,4) Hamming Code

Hamming (7,4) encodes 4-bit data into a 7-bit codeword using 3 parity bits.

### Bit Arrangement in (7,4) Code

- **4 Data Bits:** D1, D2, D3, D4
- **3 Parity Bits:** P1, P2, P4
- **7-bit Codeword:** (P1, P2, D1, P4, D2, D3, D4)

### Encoding Process

Each parity bit is responsible for checking specific bits. The parity bits are calculated as follows:

- $P1 \text{ (bit 1)} = D1 \oplus D2 \oplus D4$
- $P2 \text{ (bit 2)} = D1 \oplus D3 \oplus D4$
- $P4 \text{ (bit 4)} = D2 \oplus D3 \oplus D4$

### ✓ Example (Encoding 1011 in Hamming Code)

#### 📄 Encoding Structure:

Bit Position	P1	P2	D1	P4	D2	D3	D4
Data Value	?	?	1	?	0	1	1
Parity Calc	$D1 \oplus D2 \oplus D4$	$D1 \oplus D3 \oplus D4$	1	$D2 \oplus D3 \oplus D4$	0	1	1
Final Codeword	0	1	1	1	0	1	1

📌 Encoded 7-bit Codeword: 0111011

### 📄 Verilog Code for Hamming (7,4) Encoder & Decoder

#### ♦ Hamming74\_Encoder\_Decoder.v

```
// Hamming Code Encoder (7,4) in Verilog
module HammingEncoder (
    input  [3:0] data_in, // 4-bit input data
    output [6:0] code_out // 7-bit encoded Hamming code
);

    // Parity bit calculations
    assign code_out[0] = data_in[0] ^ data_in[1] ^ data_in[3]; // P1
    assign code_out[1] = data_in[0] ^ data_in[2] ^ data_in[3]; // P2
    assign code_out[2] = data_in[0]; // D1
    assign code_out[3] = data_in[1] ^ data_in[2] ^ data_in[3]; // P4
    assign code_out[4] = data_in[1]; // D2
    assign code_out[5] = data_in[2]; // D3
    assign code_out[6] = data_in[3]; // D4

endmodule

// Hamming Code Decoder (7,4) in Verilog
module HammingDecoder (
    input  [6:0] code_in, // 7-bit received code
    output [3:0] data_out, // 4-bit corrected output
    output reg   error     // Error detection flag
);
    wire p1, p2, p4;
    wire [2:0] parity;
```

```

// Parity check bits
assign p1 = code_in[0] ^ code_in[2] ^ code_in[4] ^ code_in[6];
assign p2 = code_in[1] ^ code_in[2] ^ code_in[5] ^ code_in[6];
assign p4 = code_in[3] ^ code_in[4] ^ code_in[5] ^ code_in[6];

assign parity = {p4, p2, p1};

always @(*) begin
    error = (parity != 3'b000); // If parity is non-zero, error exists
end

// Correct the received code if error exists
reg [6:0] corrected_code;
always @(*) begin
    if (error)
        corrected_code = code_in ^ (1 << (parity - 1));
    else
        corrected_code = code_in;
end

// Extract original data
assign data_out = {corrected_code[6], corrected_code[5], corrected_code[4],
corrected_code[2]};

endmodule

```

#### ♦ **tb\_Hamming74.v (Testbench)**

```

module tb_Hamming74;
    reg [3:0] data_in;
    wire [6:0] code_out;
    wire [3:0] data_out;
    wire error;

    // Instantiate modules
    HammingEncoder enc (.data_in(data_in), .code_out(code_out));
    HammingDecoder dec (.code_in(code_out), .data_out(data_out), .error(error));

    initial begin
        // Test Cases
        data_in = 4'b1011; #10;
        $display("Data: %b, Encoded: %b, Decoded: %b, Error: %b", data_in, code_out,
data_out, error);

        data_in = 4'b1100; #10;
        $display("Data: %b, Encoded: %b, Decoded: %b, Error: %b", data_in, code_out,
data_out, error);
    end
endmodule

```

```
data_in = 4'b0110; #10;
$display("Data: %b, Encoded: %b, Decoded: %b, Error: %b", data_in, code_out,
data_out, error);

data_in = 4'b0001; #10;
$display("Data: %b, Encoded: %b, Decoded: %b, Error: %b", data_in, code_out,
data_out, error);

$stop;
end
endmodule
```

## Applications

- Memory error correction (ECC RAM)
- Wireless communication
- Data transmission protocols

## Conclusion

Hamming codes are essential for reliable data transmission and storage, ensuring data integrity in noisy environments.