SM402 Basic Computational Topology

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Group-3
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Problem Statement

Given any input simplicial complex (up to 3-dimensional), compute β2 using the boundary matrix method.

Theory

Betti numbers are used to distinguish topological figures on the basis of the connectivity of the n-dimensional simplicial complexes. A *p*th Betti number refers to the number of *p*th dimensional holes.

0 ⇒ Number of connected components.

1 ⇒ Number of tunnels (One-Dimensional holes).

2 ⇒ Number of voids (Two-Dimensional holes).

$H_p(K) = Ker(\partial_p) / Im(\partial_{(p+1)})$

$p = \dim(H_p(K)) = \dim(Ker(\partial_p)) - \dim(Im(\partial_{(p+1)}))$

Algorithm

1. $H_2(K) = Ker(\partial_2) / Im(\partial_3)$
2. The formula used to calculate β_2 is :

$$\beta_2 = \dim(H_2(K)) = \dim(Ker(\partial_2)) - \dim(Im(\partial_3))$$

3. We take the values of the vertices, edges, faces, and tetrahedrons from the given ".gts" file and compute the ∂_2 and ∂_3 matrices.

4. We then compute the rank of ∂2 and ∂3 matrices. These are the values of dim(Im(∂2)) and dim(Im(∂3)) respectively.
5. From the Rank Nullity theorem, we know that dim(C2(K)) = dim(Ker(∂2)) + dim(Im(∂2)). So from this, we get the dim(Ker(∂2)) value, and then we can find the value of 2 for the given figure using the formula mentioned in (2).

Implementation

1. Extract the number of vertices, edges, faces, and tetrahedrons. And then extract the edges, faces, and tetrahedrons from the given ".gts" file.

2. Compute the ∂2 and ∂3 matrices using the vertices, edges, faces, and tetrahedrons data obtained in step-I.

3. Find the ranks of ∂2 and ∂3 matrices.

4. Compute the value of 2 using the formula.

System Requirements

Pre-installed *numpy* and *python3* are required to run the program.

To install the required libraries, execute the following command in the terminal

- pip3 install numpy

Steps To Run The Program

To run the program, execute the following command in the terminal

- python3 betti2.py

Then enter the name of the ".gts" file that is to be taken as the input.

Output

- Single Tetrahedron:

```
Enter the filename: testcase1.gts

Number of vertices: 4
Number of edges: 6
Number of faces: 4
Number of tetrahedrons: 1

Image of Delta_2 =  3
Kernel of Delta_2 =  1
Image of Delta_3 =  1

+-------------+
|betti_2 =  0 |
+-------------+

execution time: 0.0012545585632324219
```

- Double Tetrahedron:

```
Enter the filename: testcase2.gts

Number of vertices: 5
Number of edges: 9
Number of faces: 7
Number of tetrahedrons: 2

Image of Delta_2 =  5
Kernel of Delta_2 =  2
Image of Delta_3 =  2

+-------------+
|betti_2 =  0 |
+-------------+

execution time: 0.0019342899322509766
```

- Cone

```
Enter the filename: cone.gts
no of vertices are: 22
no of edges are: 60
no of faces are: 40
no of tetrahedrons are: 0

Image of betti_2 =  39
Kernel of betti_2 =  1
Image of betti_3 =  0

+-------------+
|betti_2 =  1 |
+-------------+

execution time: 0.004691123962402344
```

- Cube

```
Enter the filename: cube.gts
no of vertices are: 8
no of edges are: 18
no of faces are: 12
no of tetrahedrons are: 0

Image of betti_2 =  11
Kernel of betti_2 =  1
Image of betti_3 =  0

+-------------+
|betti_2 =  1 |
+-------------+

execution time: 0.0013797283172607422
```

- Sphere5

```
Enter the filename: sphere5.gts
no of vertices are: 252
no of edges are: 750
no of faces are: 500
no of tetrahedrons are: 0

Image of betti_2 =  499
Kernel of betti_2 =  1
Image of betti_3 =  0

+-------------+
|betti_2 =  1 |
+-------------+

execution time: 4.948351144790649
```

- Icosa

```
akhil@akhil-VirtualBox:~/Desktop/Semester-4/Topo Proj
Enter the filename: icosa.gts
no of vertices are: 12
no of edges are: 30
no of faces are: 20
no of tetrahedrons are: 0

Image of betti_2 =  19
Kernel of betti_2 =  1
Image of betti_3 =  0

+-------------+
|betti_2 =  1 |
+-------------+

execution time: 0.002256155014038086
```

References

1. http://gts.sourceforge.net/samples.html

2. https://numpy.org/doc/stable/reference/generated/numpy.linalg.matrix_rank.html#:~:text=Rank%20of%20the%20array%20is,that%20are%20greater%20than%20tol.&text=Input%20vector%20or%20stack%20of%20matrices.&text=Threshold%20below%20which%20SVD%20values,tol%20is%20set%20to%20S.

3. https://en.wikipedia.org/wiki/Rank%E2%80%93nullity_theorem#:~:text=The%20rank%E2%80%93nullity%20theorem%20is,the%20dimension%20of%20its%20kernel).