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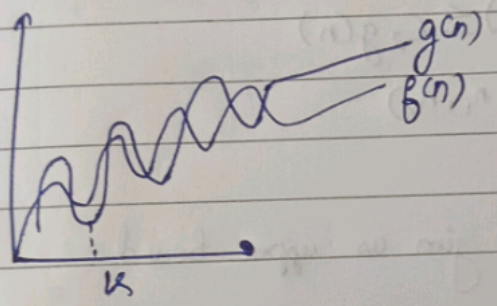
Assignment - 1

→ These notations are used to tell the complexity of an algorithm when the input is very large

→ It describes the algorithm efficiency & performance in a meaningful way. It describes the behaviour of time or space complexity for large instance characteristics

① Big Oh notation - The function $f(n) = O(g(n))$, if & only if there exist a +ve constant c & k such that $f(n) \leq c * g(n)$ for all n, $n \geq k$



$f(n) = O(g(n))$

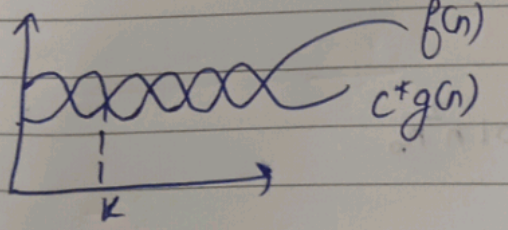
iff

$f(n) \leq c \cdot g(n)$

$\forall\ n \geq n_0$

so constant $c > 0$

② Big Omega Notation → The function $f(n) = \Omega(g(n))$, iff there exists a +ve constant c & k such that $f(n) \geq c * g(n)$ for all n, $n \geq k$



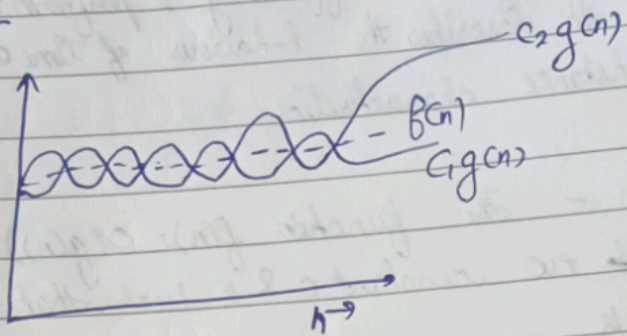
$f(n) = \Omega(g(n))$

iff $f(n) \geq c \cdot g(n)$

$\forall\ n \geq n_0$ and some ~~constant~~ constant $c > 0$

(iii) ~~Big~~ Big theta notation

Similary



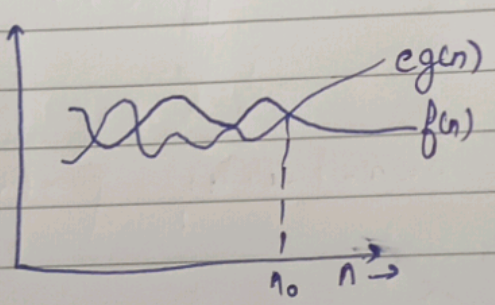
$f(n) = \Theta(g(n))$

iff $c_1 g(n) \leq f(n) \leq c_2 g(n)$

$\forall\ n \geq \max(n_1, n_2)$

(iv) Small-oh notation – O gives us upper bound

$f(n) = O(g(n))$



$f(n) \leq cg(n)$

$\forall\ n > n_0$ & $\forall\ c > 0$

$n = O(n^2)$

$n < 1n^2$

$2n^2$

$0.5n^2$

$n < 0.001 n^2 n_0$

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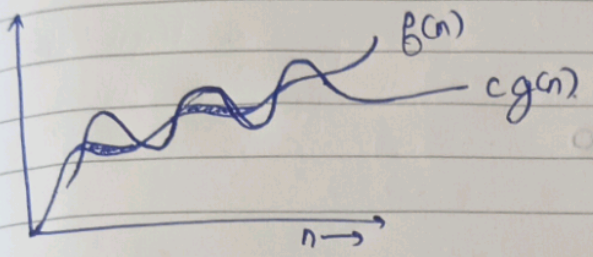
Small- omega (ω)-



lower bound

$f(n) = \omega g(n)$

$f(n) > c \cdot g(n)$

$\forall\ n > n_0$ & $\forall\ c > 0$

$n^2 = \omega(n)$

② for (i = 1 to n)
{ i = i * 2
}

time complexity of a loop means no. of times it has runs

i	1	2	4	8	16	32	----	2^k
value	2^1	2^2	2^3	2^4	2^5	2^6	----	n

$i = 1, 2, 4, 8, 16, 32, \ldots, 2^k$ this means k ti

i.e, $2^k = n$

$k \log_2 2 = \log_2 n$

$k = \log n$

$\log_2 2 = 1$

T.C = O(log n)

(3) $T(n) = \begin{cases} 3T(n-1), & n>0 \\ 1 \end{cases}$

By forward substitution

$T(n) = 3T(n-1)$

$T(0) = 3(T(-1)) = 0$

$T(1) = 3T(0) = 3$

$T(2) = 3T(1)$

$= 3*3$

$= 9$

$T(3) = 3T(3-1)$

$= 3T(2)$

$= 3*3^2$

$= 3^3$

$\vdots$

$T(n) = 3^n$

$\therefore$

$\boxed{T.C = O(3^n)}$

(4) $T(n) = \begin{cases} 2T(n-1)-1, & n>0 \\ 1 \end{cases}$

By forward substitution

$T(0) = 1$

$T(1) = 2T(1-1)-1$

$= 2^1-1$

$T(2) = 2T(2-1)-1$

$= 2^2-2^1-1$

$T(3) = 2T(3-1)-1$

$= 2^3-2^2-2^1-1$

$\vdots$

$= 2^n - 2^{n-1} - 2^{n-2} - 2^{n-3} - \cdots - 2^2 - 2^1 - 2^0$

$= 2^n - (2^n - 1)$

$= 2^n - 2^n + 1$

$= 1$

T.C = O(1)

⑤
```
int i=1, s=1;
while (s<=n)
{
    i++;
    s=s+i;
    printf("#");
}
```

The value of "i" increase by one for each iteration. The value contained in "s" at the i^{th} iteration is the sum of the first "i" +ve integers. If k is the total no. of iterations taken by any program then while loop terminate if:

$1 + 2 + 3 + \cdots + k$

$\Rightarrow [k(k+1)/2] > n$

So, $k = O(\sqrt{n})$

$\therefore$ T.C = $O(\sqrt{n})$

⑥
```
void function (int n)
{
    int i, count=0
    for (i=1; i<=n; i++)
    {
        count++;
    }
```

O(n) - T.C

(7) void function (int n)
{
int j, k, i, count = 0;
for (i = n/2; i <= n; i++) O(n)
for (j = 1; j <= n; j = j * 2) O(log n)
for (k = 1; k <= n; k = k * 2) O(log n)
count++;
}

T.C = logn * logn
= O(n log²n)
∴ T.C = O(n log²n)

(8) function (int n)
{
if (n == 1)
return 1;
for (i = 1 to n) O(n)
{
for (j = 1 to n) O(n)
{
printf ("*");
}
}
function (n-3);
}

T.C = O(n²)

(9) void function (int n)
{
for (i = 1 to n {} O(n)
for (j = 1; j <= n; j = j + 1) { O(n) O(n) * O(n)
printf ("*");
}

T.C = $O(n^2)$

(10) for the function, n^k & c^n, what is the asymptotic notations blw these functions

Assume, that $k \geq 1$ & $c > 1$ are constants. Find out the value of c & n_0 for which relation holds.

n^k is $O(c^n)$