ERROR CORRECTING CODES

ASSIGNMENT – 4

Given,

1.a) no. of data bits = 8,

no. of parity bit = r

$$2^r \geq m+r+1$$

Hence,

if $r=2$,

$$2^2 \geq 8+2+1 \Rightarrow 4 \geq 9 \text{ false,}$$

$r=3$,

$$2^3 \geq 8+3-1 \Rightarrow 8 \geq 10 \text{ false,}$$

$r=4$,

$$2^4 \geq 8+4-1 \Rightarrow 16 \geq 11 \text{ satisfied.}$$

∴ Therefore no. of checkbits 'r' = 4, and 1 more bit for an overall parity for double error detection.

So total number of bits = 13.

b) Given,

information bits = 1011 1110.

P_1	P_2	M_3	P_4	M_5	M_6	M_7	P_8	M_9	M_{10}	M_{11}	M_{12}	P_{13}
		1		0	1	1		1	1	1	0	

$$\Rightarrow P_1 + M_3 + M_5 + M_7 + M_9 + M_{11} = 0$$

$$P_1 + 1 + 0 + 1 + 1 + 1 = 0$$

$$\therefore P_1 = 0$$

$P_2 + m_3 + m_6 + m_7 + m_{10} + m_{11} = 0$

$P_2 + 1 + 1 + 1 + 1 + 1 = 0$

$P_2 = 1$

$P_4 + m_5 + m_6 + m_7 + m_{12} = 0$

$P_4 + 0 + 1 + 1 + 0 = 0$

$P_4 = 0$

$P_8 + m_9 + m_{10} + m_{11} + m_{12} = 0$

$P_8 + 1 + 1 + 1 + 0 = 0$

$P_8 = 1$

$P_0 + P_1 + P_2 + m_3 + P_4 + m_5 + m_6 + m_7 + P_8 + m_9 + m_{10} + m_{11} + m_{12} = 0$

$P_0 + 0 + 1 + 1 + 0 + 0 + 1 + 1 + 1 + 1 + 1 + 1 + 0 = 0$

$P_0 = 0$

final encoded code word =

$\underline{0}\,\underline{1}\,1\,\underline{0}\,0\,1\,1\,\underline{1}\,1\,1\,1\,0\,\underline{0}$

a b

a → check bits

b → parity bit (overall).

c) error code word → 0 1 0 0 0 1 1 1 1 1 1 0 0
error bit.

$P_0 = e_4 e_3 e_2 e_1$

$P_0 = 0+1+0+0+0+1+1+1+1+1+1+0+0$

$\boxed{P_0 = 1}$

$e_1 = P_1 + m_3 + m_5 + m_7 + m_9 + m_{11}$

$e_1 = 0+0+0+1+1+1 = 1$ //

$e_2 = P_2 + m_3 + m_6 + m_7 + m_{10} + m_{11}$

$e_2 = 1+0+1+1+1+1 = 1$ //

$e_3 = P_4 + m_5 + m_6 + m_7 + m_{12}$

$e_3 = 0+0+1+1+0 = 0$ //.

$e_4 = P_8 + m_9 + m_{10} + m_{11} + m_{12}$

$e_4 = 1+1+1+1+0 = 0$ //.

⇒ $P_0 e_4 e_3 e_2 e_1$

1 0011; ⇒ It represents that there is one error and the location of it is

0011 = 3

also known as first information bit.

d) Given code word,

0 1 0 0 0 1 1 1 0 1 1 0 0

error bit

$P_0 =$ sum of all bits. (adding entire codeword).

$= 0$

$e_1 = p_1 + m_3 + m_5 + m_7 + m_9 + m_{11}$

$= 0$

$e_2 = p_2 + m_3 + m_6 + m_7 + m_{10} + m_{11}$

$= 1$

$e_3 = p_4 + m_5 + m_6 + m_7 + m_{12}$.

$= 0$

$e_4 = 1$

$P_0 e_4 e_3 e_2 e_1 = 0\,\underline{1010}$.

$P_0 \Rightarrow$ represents there is a double if $e_4e_3e_2e_1$ is not 0000.

$e_4e_3e_2e_1 =$ 1st error location added to 2nd error location.

$= 0011 + 1001$

$= 1010$

(2) Show that code C is capable or correcting or less $\Leftrightarrow$ minimum distance of code is $e+1$.

Sol:

considering,

codewords with length 3 and

$e = 1$.

Hence, minimum distance = 2.

let, the codewords are,

$\{000, 011, 101, 110\}$.

→ If the sender sends a codeword = 011.
and the reciever secieved the codeword = 0_1

→ considering the known correct bits there is only one match from the codewords.

⇒ 0_1 011

→ Let's consider codewords with length = 3 and $e = 2$, and minimum distance = 2.
codewords,

→ $\{000, 011, 101, 110\}$.

If the sender sends = 000
the reciever gets a code = 0 _ _ (last 2 errors)
→ If we consider the correct bits to match, the correct data from the crosswords then we have 2 codewords.

0 _ _ → [000, 011].

from the above example, we can prove it.

→ A code C is capable of correcting e or less erasures iff the minimum distances of the codewords is e+1.

3. Consider, a codeword with t=1 and e=1
minimum distance ⇒ 2t+e+1 = 4.
the codewords are

{0000, 1111}.

Sender sends the 0000.
If the reciever gets = 10_0.
and when the recieved codeword is cross checked with all the code words then the codeword with the least distance is 0000.

⇒ So, if the minimum distance of the codewords is 2t+e+1 from the code can correct with 't' and 'e' errors.

Let's consider the codewords with $t=1$ and $e=1$.

minimum distance = 4.

codeword = {1010, 0001}

If the sender sends = 1010

reciever gets = 00_0

$\therefore$ D(00_0, 1010) = 1 | D(00_0, 0001) = 1

(without considering).

$\therefore$ least distance can be {1010, 0001} either one of them.

→ But can't identify which code does it belongs to.

$\therefore$ From the above example, we can prove that 'c' codewords with 't' errors can be corrected if the minimum distance of codewords is $2t+e+1$.

~~$\therefore$ This is not a codeword in the set.~~
~~this can be corrected,~~
~~erasure of the distance is $2t+e+1$.~~

(4) Given,

Let x' be from the X set formed from $1 \to 0$.
and Y' be from the Y set, similarly.

$\therefore$ $N(X,x') \leq t$ X can be converted to X'
$N(Y,Y') \leq t$ by applying 1 to 0's but
$N(x',X) = 0$ 0's can't be changed back to
$N(Y',Y) = 0$ 1's.

$D_{max}(X,Y) = \max[N(X,Y), N(Y,X)]$.
The above statement is written using the triangle's theorem.

$$D_{max}(X,Y) \leq \max(N(X,X') + N(X',Y), N(Y,X') + N(X',X))$$

$$\leq \max[(t + N(X',Y') + 0), t + N(Y',X') + 0]$$

$$\leq t + \max[N(X',Y'), N(Y',X')]$$

removing constant 't'.

$D_{max}(X,Y) \leq t + D_{max}(X',Y')$

Given,

$D_{max}(X,Y) = t+1.$

Substituting,

$\cancel{t}+1 \leq \cancel{t} + D_{max}(X',Y').$

$\boxed{D_{max}(X',Y') \geq 1}$

This proves that there is minimum distance of 1 or greater than 1 with my circle in the sets formed by X and Y.

5 Given,

X and Y are vectors, with length n.

Assuming,

$X = (4,0,5,3)$

$Y = (0,3,2,4)$

$D_{max} = 4$

$\therefore D_{max} = |x_i - y_i| = 4$

minimum D_{max},

distance of code = $l+1$.

$\forall\ x_i y_i \in C$

$D_{max}(X,Y) \geq l+1$.

$\therefore \max|x_i - y_i| \geq l+1$ and $S_x \cap S_y = \phi$.

if $l=2$,

set formed for

$X = (4,0,5,3)$	$Y = (0,3,2,4)$
$(2,0,5,3)$	$(-2,3,2,4)$
$(4,0,5,1)$	$(0,1,2,4)$
$(4,-2,5,3)$	$(0,3,0,4)$
$(4,0,5,3)$	$(0,3,0,2)$

The closest that these 2 sets can get $X' = (2,0,5,3)$ and $Y' = (-2,3,2,4)$. but, both the sets never overlap at all because there are no matches.

minimum of D_{max} can be calculated from $X' = (2,0,5,3)$ and $Y' = (0,3,2,2) = 3$.

where initially we had $l=2$ and now, the final result is 3.
which proves $l+1$ is the minimum distance.

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6 a) ~~Giving~~ Given,
code C,
no. of rows $\rightarrow m$
no. of columns $\rightarrow n$.

$\Rightarrow$

1	1	1	1	P_7 ↙ Parity bits
1	1	1	1	P_6
1	1	1	1	P_5
1	1	1	1	
↗ P_1	P_2	P_3	P_4	

Parity bits

⇓ one bit flip.

1	1	1	1	P_7
1	0	1	1	P_6 ← error bit
1	1	1	1	P_5
1	1	1	1	
P_1	P_2	P_3	P_4	

↑ error bit (under P_2)

⇒ P_2 and P_6 are in error so that means the bit in the P_2 column and P_6 row is the error.

(∴ Parity single error correction is possible.)

1	1	1	1	P_7
1	1	1	1	P_6
1	1	1	1	P_5
1	1	1	1	
P_1	P_2	P_3	P_4	

⇓ 2 bit flips.

0	0	1	1	P_7
1	1	1	1	P_6
1	1	1	1	P_5
1	1	1	1	
P_1	P_2	P_3	P_4	

∴ Here 2 bits which are in error from columns or rows.

Then we can prove that the data has double error flips (P_1P_7, P_2P_7).

b) If the matrix of data with 2d having dimensions of $m \times n$.

Total 3 bit flips are counted as

$$^{m\times n}C_3.$$

→ If there are 3 bit flip in a same row then it can be corrected. So, the possibilities $= m.\,^{n}C_3$.

Similarly for the same column, the no. of possibilities are $= n.\,^{m}C_3$.

% of 3 errors is calculated as,

$$\frac{^{m\times n}C_3 - n.\,^{m}C_3 - m.\,^{n}C_3}{^{m\times n}C_3} \times 100. \quad \text{— (1)}$$

For instance,

assume $m=3$ and $n=3$.

$$\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & \end{array} \quad \begin{array}{ccc|c} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & \end{array} \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

all are the flips that can be corrected.

Using the equation (1).

$$\frac{3\times3_{C_3} - 3.{}^{3}C_3 - 3^{3}C_3}{3\times3_{C_3}} \times 100$$

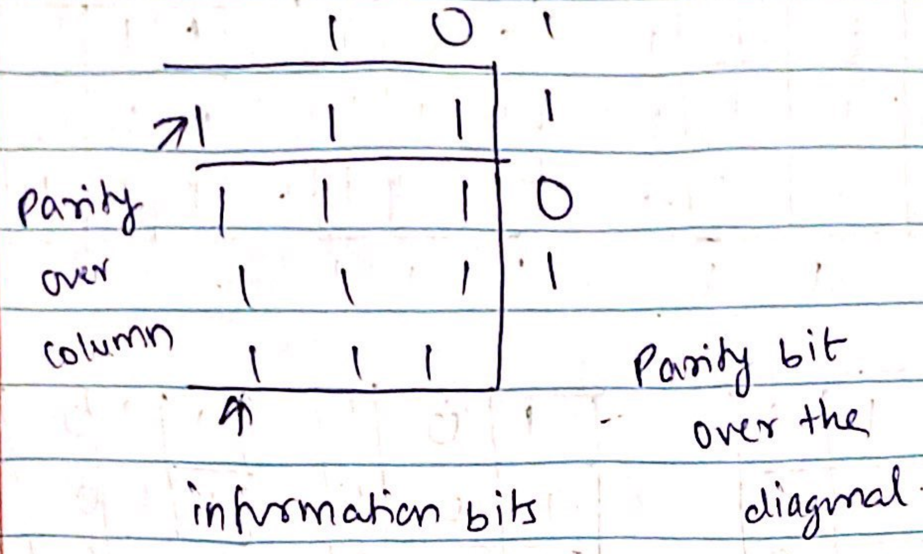
$$\Rightarrow \frac{9_{C_3} - 3.(1) - 3.(1)}{9_{C_3}} \times 100$$

$$\Rightarrow \frac{78}{84} \times 100$$

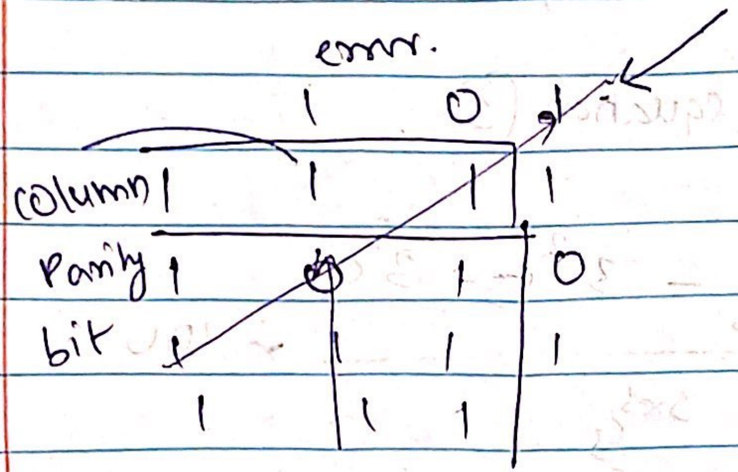
$$\Rightarrow 92.85\%.$$

$\therefore$ Proving the % equation is satisfied.

7



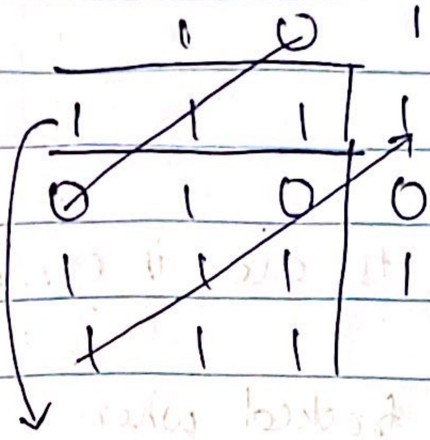
→ If there is one flip (bit diagonal parity) error.



This proves that there is an error that particular diagonal and column

→ That bit in the particular location can be flipped get the correct data

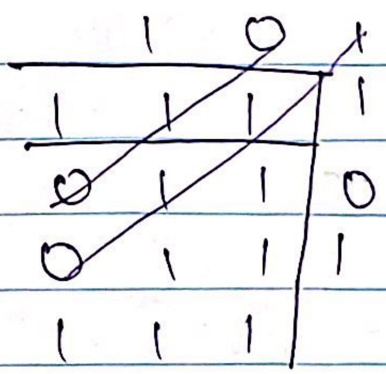
If there are two bit flips.



→ diagonal parity bit error (tracks all error locations).

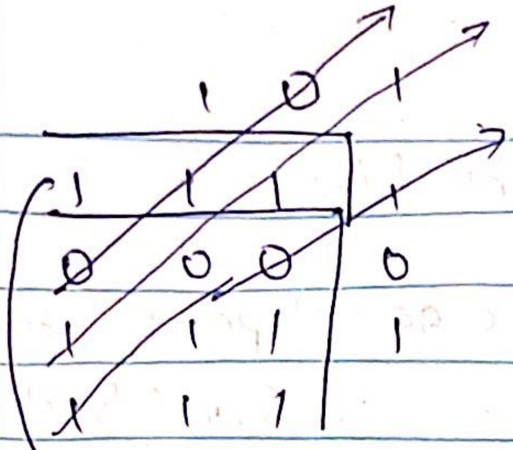
column Parity bit error

→ Double error can also be detected when the error is in the same error.



→ diagonal parity error.

→ These are no. parity bits having error in the column parity. So, this cannot correct the data bits error when in same column.



diagonal parity error.

all the parity bits are in error.

→ every error bit is tracked when all error bits are shown in the same row.

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