Assignment - 5

① Given,

for q-ary codes for limited magnitude error,
non-wrap error model.

a) $Z_8 = \{0,1,2,3,4,5,6,7\}$.

Maximum number of non-systematic codewords can be defined as '$|C|$'.

whereas,

limited magnitude $(l) = 3$.

codeword length $(n) = 2$.

$|C|$ can be defined as,

$$|C| = \left\lfloor \frac{q}{l+1} \right\rfloor^n$$

$$|C| = \left\lfloor \frac{8}{3+1} \right\rfloor^2 \Rightarrow \left\lfloor \frac{8}{4} \right\rfloor^2$$

$$|C| = \lfloor 2 \rfloor^2$$

$$\boxed{|C| = 4}$$

$\therefore$ Maximum number of non-systematic codewords are 4.

b) Given,

No. of information digits (k) = 4.

limited magnitude (l) = 3.

Then,

$$\left|\frac{q}{l+1}\right|^{r} = (l+1)^{k}$$

$$\Rightarrow \left|\frac{8}{3+1}\right|^{r} = (3+1)^{4}$$

$$\Rightarrow |2|^{r} = (4)^{4}$$

$$\Rightarrow 2^{r} = 2^{8} \Rightarrow \boxed{r = 8}$$

$\therefore$ No. of check digits (r) = 8.

c) Given,

Information word = 7246.

we have,

k = 4, q = 8, l = 3, r = 8

$\therefore X \bmod (l+1)$,

$X = 7246$

$\Rightarrow 7246 \bmod (3+1) = 7246 \bmod 4$

$\Rightarrow 3202.$

converting into base form.

$3202 \text{ base } 4 \Rightarrow 3 \times 4^3 + 2 \times 4^2 + 0 + 2 \times 1$

$= 192 + 32 + 2$

$= 226.$

expressing in the radix form, $\frac{r}{l+1} = \frac{8}{4} = 2$ //.

$A = 226$

$A \Rightarrow 2^7 + 2^6 + 2^5 + 2$

$(11100010)_2$

multiplying A with 4, $(l+1)$

$\Rightarrow 44400040.$

$\therefore$ The final code word is $\Rightarrow$ 72464440040.

d) Recieved ~~information~~ code word = 5134. 333 7737

∴ Check bits are: 33377737.

By doing mod 4.

⇒ (33377737) mod 4 ⇒ 4440040 (c').

∴ Now dividing c' with (l+1).

⇒ (4440040) div(4)

⇒ $(1110010)_2$

⇒ $(226)_{10}$.

⇒ moving it to base 4,

$(226)_{10} \to (3202)_4$

Recieved information word = 5134.

⇒ 5134 mod 4 = $(3202)_4$

⇒ Based on the recieved code word,

nearest one will be

5	1	3	4
↓	↓	↓	↓
7	2	4	6

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$\Rightarrow$ The correct information word is 7246.

(2) Given,

Matrix H,

rows $\to r$,

columns $\to n$.

weight of the vectors $\Rightarrow 1, 3, 5, \ldots$

a) Given,

$n = 16$,

$n \leq 2^r - 1$

$\Rightarrow r \geq \log(n+1)$

$\Rightarrow r \geq \log(16+1)$

$r \geq \log 17$

$\boxed{r \geq 5}$.

By constructing H matrix,

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$H = [P_{r \times k} \quad I_{r \times r}]$

$n = k + r \Rightarrow k = n - r,$

$k = 16 - 5 \Rightarrow \boxed{k = 11}$ //.

b) Single error correction and double error detection are possible when,

$$D_{min} \geq t + d + 1.$$

$$\Rightarrow D_{min} \geq 1 + 2 + 1$$

$$\boxed{D_{min} \geq 4}$$

Hence, it means, there's a need of 4 minimum columns to get all errors.

$X' \rightarrow$ recieved code word,

$X \rightarrow$ transmitted word,

$E \rightarrow$ Error vector.

$$\boxed{X' = X + E}$$

where,

$$\begin{aligned} \text{Syndrome}(S) &= X'H^T \\ &= (X+E)H^T \\ &= XH^T + EH^T \\ &= EH^T. \end{aligned}$$

$\boxed{\therefore XH^T = 0}$

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let, there be an error in 4th bit.

$E = (000100 \ldots \ldots 0)\ (1 \times 16)$.

$S = EH^T = (00111) \Rightarrow$ 4th column of H matrix

$\Rightarrow$ error in 4th bit.

$\Rightarrow$ single error can be corrected.

$\therefore$ Assume, error is in 2nd and 7th bit.

$E = (0100000100 \ldots \ldots 0)_{1 \times 16}$.

$S = EH^T = (01101 + 10101)$

$= 11000 \neq 00000$

$\Rightarrow$ error exists.

we have even weight for 11000.

$\therefore$ Hence it doesn't match with any column in H matrix,

Double error detection is not possible

c) Gmatrix = $[I_{k \times k} \; P_{k \times r}]$.

$$G = \begin{bmatrix} & 11100 \\ & 11010 \\ & 11001 \\ I_{11 \times 11} & 10110 \\ & 10101 \\ & 10011 \\ & 01110 \\ & 01101 \\ & 01011 \\ & 00111 \\ & 11111 \end{bmatrix}$$

③ Given,

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

converting into systematic form,

$G = [I_{k \times k} \; P_{k \times r}]$.

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$C_3 \Leftrightarrow C_6$

$$G = \begin{bmatrix} \underbrace{1 & 0 & 0}_{I} & \underbrace{1 & 0 & 1}_{P} \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$\because k = 3, r = 3, n = 6$

b) The priority check matrix H for the code, in (a) is

$$H = [P I]_{k \times r} \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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c) Codeword = 110

$$C' = (110)\begin{pmatrix} 100 & 101 \\ 010 & 111 \\ 001 & 110 \end{pmatrix} = (110010)$$

$$C'H^T = (110010)\begin{pmatrix} 101 \\ 111 \\ 110 \\ 100 \\ 010 \\ 001 \end{pmatrix}$$

$$C'H^T = (000)$$

(Null space of H).

(4) Given Z_5,

a) no. of information bits, $k = 8$.

$q = 5$.

$$n \leq \frac{q^r - 1}{q - 1} \qquad (\because n = k + r)$$

$$k+r \le \frac{5^r-1}{4}$$

$$4(k+r) \le 5^r-1$$

$$4(8+r) \le 5^r-1$$

$$32+4r \le 5^r-1$$

$$33+4r \le 5^r$$

$$\therefore 45 \le 125 \qquad \boxed{\therefore r=3}$$

$$H=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 & 4 \end{bmatrix}$$

Shifting columns 1, 2, 7 to form identity matrix.

$$H=\left[\underbrace{\begin{matrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 4 & 1 & 2 & 3 & 4 \end{matrix}}_{P_{3\times 8}}\;\underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I_{3\times 3}}\right]$$

(b) Generator Matrix (G) = $\left[\begin{array}{c|ccc} & 0 & 4 & 4 \\ & 0 & 4 & 3 \\ I_{8\times 8} & 0 & 4 & 2 \\ & 0 & 4 & 1 \\ & 4 & 0 & 4 \\ & 4 & 0 & 3 \\ & 4 & 0 & 2 \\ & 4 & 0 & 1 \end{array}\right]$

c) Recieved codeword$^{(c)}$ = 1 2 0 4 1 1 2 3

$C' = CG \Rightarrow (1\,2\,0\,4\,1\,1\,2\,3)\,G$

$= (1\,2\,0\,2\,1\,1\,2\,3\,3\,3\,3)$

d) $C' = (1\,2\,0\,2\,1\,1\,2\,3\,3\,3\,3)$.

$$C'H^T = (1\,2\,0\,2\,1\,1\,2\,3\,3\,3\,3)\begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 1 & 4 \\ 1 & 0 & 1 \\ 1 & 0 & 2 \\ 1 & 0 & 3 \\ 1 & 0 & 4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = (0\,3\,2)$$

non-zeros $\neq 1$

multiplying by $(3^{-1}) \Rightarrow$

$\Rightarrow (3^{-1})(0\ 3\ 2)$

$\Rightarrow 2(0\ 3\ 2)$

$\Rightarrow (0\ 6\ 4)$

$= (0\ 1\ 4)$.

$= 3 \times c_4$

$\therefore$ Hence error is in 4th bit with a difference of 3.

$C' = C + E \Rightarrow C = C' - E$

$\Rightarrow (1\ 2\ 0\ 2\ 1\ 1\ 2\ 3) -$

$(0\ 0\ 0\ 3\ 0\ 0\ 0\ 0)$

$= (1\ 2\ 0\ 4\ 1\ 1\ 2\ 3)$.

(5) a) Given,

$k = 8,\ q = 7$

considering the elements 1,2,3.

$1 \times 1 = 1,\quad 2 \times 1 = 2,\quad 3 \times 1 = 3$

$-1 = 6,\quad -2 = 5,\quad -3 = 4.$

So, $r=2$ as 1, 2, 3 can be represented in 2 bits.

$r=2,\ k=8,\ n=10.$

b) $$H = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 2 & 3 & 1 & 2 & 3 & 4 & 5 & 6 & 0 & 1 \end{bmatrix}$$

$P_{8\times2}$ $\qquad I_{2\times2}$

c) $G = [\,I_{8\times8} \quad -P_{8\times2}\,]$

$$= \left[\begin{array}{c|cc} & 0 & 4 \\ & 0 & 3 \\ I_{8\times8} & 6 & 6 \\ & 6 & 5 \\ & 6 & 4 \\ & 6 & 3 \\ & 6 & 2 \\ & 6 & 1 \end{array}\right]$$

codeword = $(4\ 2\ 2\ 2\ 1\ 3\ 2\ 1)\ G$

$= (42222\ 132135)$

d) Given,

$c' = 3222213 2135$

$c'H^T = (3222213\ 2135)H^T$

$= \begin{bmatrix} \cancel{\ } \end{bmatrix} . (0,5)$

$= -1(0,5)$

$= (0,2).$

$\begin{bmatrix} 0 \\ 2 \end{bmatrix} \Rightarrow 1^{st}$ column.

Here, magnitude = 1

$c' = C+E \Rightarrow (3222213 2135)$

$- (-1000 0000)$

$= (4222 2132135).$

$\boxed{\text{corrected codeword} = 4222\ 132\ 135.}$

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