Earth

flat-spacetime

$h_{\mu\nu}$

$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$

$= \boxed{\eta_{\mu\nu} + h_{\mu\nu}}$

"Tensor" ?

vector

"circular orbit"

But ?

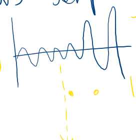
Eccentric

Detector LIGO

$G_{\mu\nu} = T_{\mu\nu}$ General Relativity

↳ GWs templates

Theory →



$10 M_\odot, 10 M_\odot$

Data

pycbc ?

data → Theory

$h(t)$

(t)

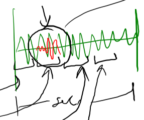
pycbc ?

Detector Data

Noise

Theory

↳ Template



Matched filtered

SNR

Special Rel → General Rel → Grav. Waves

metric

$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$

Tensor

$\begin{bmatrix} -1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{bmatrix}$

Minkowski s-t

$\eta_{\mu\nu}$

S.R.T + Gravity

$ds^2 = g_{\mu\nu}(x,t)\, dx^\mu dx^\nu$

(x) Quadrupole

diff EM vs Grav Ra

$\boxed{h} \sim \frac{d(\text{Quadrupole})}{dt}$

Earth

$\eta_{\mu\nu}$

$h_{\mu\nu}$

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

x) Gravity ≈ curvature of s-t

x) GWs ≈ ripples of s-t

Project

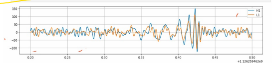
Assuming → circular orbit

1) Eccentric Orbit ?

$P = ?$

II) Look for GW signals in data

circular



Precession

m_1 m_2

Usual setting

h

For eccentric case

h

non-spinning

coupled differential eqⁿ

Task I →

$$\dot{P}_{orb} = -\frac{192\pi}{5c^5}\left(\frac{2\pi G}{P_{orb}}\right)^{5/3}\frac{m_1 m_2}{(m_1+m_2)^{1/3}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)\left(1-e^2\right)^{-7/2}, \quad (2)$$

$$\dot{e} = -\frac{608\pi}{15c^5}\frac{e}{P_{orb}}\frac{m_1 m_2}{(m_1+m_2)^{1/3}}\left(1+\frac{121}{304}e^2\right)\left(1-e^2\right)^{-5/2}. \quad (3)$$

It is important to note that these equations ignore spin effects and higher order effects, such as spin precession or higher order modes. Blanchet integrates analytically to determine that orbital period and eccentricity are related via

$$c_0 P^{19/9} = \frac{e^2}{(1-e^2)^{19/6}}\left(1+\frac{121}{304}e^2\right)^{145/121} \quad (4)$$

$\frac{dP_{orb}}{dt} = f_1(e, P_{orb})$

$\frac{de}{dt} = f_2(e, P_{orb})$

Python

$P_{orb}(t)$, $e(t)$

⇒ Numerical

$\frac{dP_{orb}/dt}{de/dt} = \frac{f_1(e, P_{orb})}{f_2(e, P_{orb})}$

$\frac{dP_{orb}}{de} = \frac{f_1(e, P_{orb})}{f_2(e, P_{orb})}$

into Python

P_{orb}

e

Vs

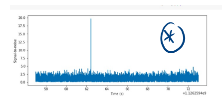
Analytical eqⁿ 4

P_{orb}

e

Run google colab tutorial and get (x) using

Task II →



(x)

Matched-filter

Till now we have ~ 70-90 GW detections

GW150914

GW

Name	Version
GW200311_103121	v1
GW200308_173609	v1
GW200306_093714	v1
GW200302_015811	v1
GW200225_060421	v1
GW200224_222234	v1
GW200220_124850	v1
GW200220_061928	v1
200219_201407	v1
GW200219_094415	v1
GW200216_220804	v1
200214_224526	v2
200214_224526	v1
GW200210_092254	v1

Data

$s(t) = n(t) + h(t)$ → Theory → circular orbit

noise

signal

Matched-Filtering to get

Signal to Noise ratio

noise

thermal

Seismic

$\frac{S}{N} = 10$

$\boxed{S = 10 \times N}$

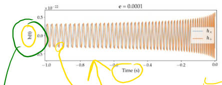
$\frac{S}{N} = 15$ signal is 15 > Noise

GW19..21 ⇒ eccentric waveform

PyCBC

Data-Analysis ⇒ GW searches

modelled



↳ you already have a waveform

↳ you look for waveform in data

10 M☉

10 M☉

Template

$h \sim \frac{\Delta L}{L}$

unmodelled

↳ not based on any particular waveform

↳ look for deviations from noise

⇓

Matched-Filtering ⇒ Search for known waveform in data

⇓

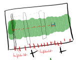
Signal-to-Noise: SNR

Fourier Transform

fun

Method

(i) value of a calculated waveform, at each successive time step is multiplied value of detector data at each successive time step



Σ all results products are totalled

(ii) Process is repeated for each time step ⇒ get a new data stream

(iii) If signal is in the data stream ⇒ Matched-filtering will show this as a signal ⇒ that can be distinguished from random noise fluctuation

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S.N.R ⇒ can be estimated by approx the output of match-filter

*) Define (i) scalar product $(x|y)$ using Fourier transform

$$(x|y) = 4\mathbb{R}\int_0^\infty \frac{\tilde{x}(f)\,\tilde{y}^*(f)}{S_n(f)}\cdot e^{2\pi i f t}\,df$$

$*$ → Complex-conjugate

$\mathbb{R}$ → real part

$\tilde{x}(f), y(f)$ → are the fourier transform of time series data ⇒

$$\tilde{x}(f) = \int_{-\infty}^{\infty} x(t)\cdot e^{-2\pi i f t}\,dt$$

↓ Inverse FT

$$x(t) = \int_{-\infty}^{\infty} \tilde{x}(f)\, e^{2\pi i f t}\,df$$

(ii) $S_n(f)$ ⇒ one-sided power spectral density of detector noise

Assumed $S_n(f) = \text{const} = S_0$

Math : PARSEVAL'S Theorem

$$(x|y) \sim \frac{2}{S_0}\int_0^T x(t)\,y(t)\,dt$$

Data : $s(t) = n(t) + h(t)$

noise signal

The matched filter output of data stream $s(t)$ with a filter template $h_{(temp)}(t)$ is defined using the scalar product

$$(s|h) = 4\mathbb{R}\int_0 \frac{\tilde{s}(f)\,\tilde{h}^*_{temp}(f)}{S_n(f)}\, e^{2\pi i f t}\,df$$

Signal to Noise: $\left(\frac{S}{N}\right) = \rho^2 = (h|h)$

$$\rho^2 \sim \frac{1}{\ldots}\int \dot{h}(t)\,dt$$

$$\sim \frac{1}{\ldots}\int \Phi_{GW}$$

where $\Phi_{GW} = \int \left|\frac{dE_{GW}}{dt}\right| = \ldots \int \left[\dot{h}_+(t)^2 + \dot{h}_\times(t)^2\right]$

$h = \frac{\Delta L}{L}$ → measurement