CHAPTER 1 - SETS

A well defined collection of objects/facts. The numbers constituting a set is called elements/members of the set. The symbol '$\in$' represents element of/member of. Sets are generally represented by Capital letters and elements are denoted by small letters. E.g.: A = set of natural numbers less than 10. There are two methods to represent a set. They are: Roster Method (Tabular method or Listing method) and Set-builder method (Rule method or Property method). In roster method, elements are written one by one, separated by commas and enclosed between braces or curly brackets { }. E.g.: A = {1,2,3,4,5,6,7,8,9}. But in set builder form, the elements of a set are described by their characterizing property. E.g.: A = {x: x is a natural number <10} or A = {x/x is a natural number<10} or $A=\{x:n,n\in N,n<10\}$.

Letter denoted by set:

N	Set of natural numbers	Z	Set of integers
Z^+	Set of all positive integers	Z^-	Set of –ve integers
Q	Set of all rational numbers	Q^+	Set of all positive rational numbers
R	Set of all real numbers	R^+	Set of all positive real numbers
C	Set of all complex numbers	$\overline{Q}$ or T	Set of irrational numbers

<u>Notations commonly used in sets</u>

: (or) /	Such that	$\subseteq$	Proper subset
=	Equal sets	$\in$	Element of
$\not\subset$	not subset of	$\approx$	Equivalent sets
$\notin$	Not an element of	$\supset$	Superset
U	Universal set	ϕ (or) { }	Null set
$\subset$	Subset	A' or A^c	Compliment of a set A
n(A)	No. of element of set A	$\cap$	Intersection
$\cup$	Union	– (or) \	Difference of sets
Δ	Symmetric difference of sets		

Types of sets

1.	Null set	{ }
2.	Singleton set	{ 5 }
3.	Finite set	{1, 2, 3, … , 100}
4.	Infinite set	{1, 2, 3, … }
5.	Equivalent sets	n(A) = n(B)
6.	Equal sets	Elements of both the sets are same (A = B)
7.	Disjoint sets	Two or more sets having different elements.

Subset: Consider the sets A={1,2,3} and B={2,3}. Here every element of B is an element of A. $\therefore$ B is known as subset of A, denoted by $B\subset A$ and A is known as super set of B, denoted by $A\supset B$.

Note:

1. Number of subsets if a set has n elements = 2^n
2. $n[P(A)] = 2^n$ where $n = n(A)$
3. No. of proper subsets = $2^n - 1$
4. Every set is a subset of itself.
5. ϕ is a subset of every set.
6. If A and B are disjoint sets, a) $A \cap B = \phi$ b) $A - B \neq B - A$
7. The number of elements of a power set = No. of subsets.

No. of elements of a set	**No. of subsets**
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
$\vdots$	$\vdots$
n	2^n

E.g.:

1. $A = \phi$

   Subsets : ϕ
2. $A = \{\phi\}$

   Subsets : $\{\phi\}, \phi$
3. $A = \{1, 2\}$

   Subsets : $\{1, 2\}, \{1\}, \{2\}, \phi$
4. $A = \{1, 2, \{3\}\}$

   Subsets : $\{1, 2, \{3\}\}$ $\{1, 2\}, \{1, \{3\}\}, \{2, \{3\}\}, \{1\}, \{2\}, \{\{3\}\}, \phi$
5. $A = \{1, 2, 3, 4\}$

   Subsets : $\{1, 2, 3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\},$
   $\{3, 4\}, \{1\}, \{2\}, \{3\}, \{4\}, \phi$

Power Set: Set of subsets is called power set. P(A) denotes power set of the set A.

E.g.: $A = \{\phi\}$

Subsets : $\{\phi\}, \phi$

Power set of A, P(A) = $\{\{\phi\}, \phi\}$

Subset as intervals of R

Let a and b be any two real numbers. If $a < b$, then

i. $\{x : x \in R, a < x < b\}$ is known as open interval a,b. It is denoted as (a,b).

Graph: 

ii. $\{x : x \in R, a \le x \le b\}$ is known as closed interval a,b. It is denoted as [a,b].

Graph: 

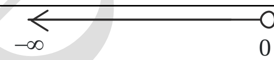
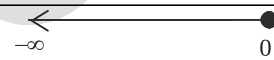
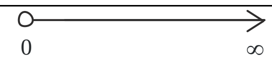
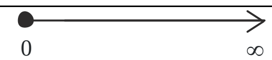
iii. $\{x : x \in R, a \le x < b\}$ is known as semi-closed interval a,b. It is denoted as [a,b).

Graph: 

iv. $\{x : x \in R, a < x \le b\}$ is known as semi-open interval a,b. It is denoted as (a,b].

Graph: 

Infinite intervals

Set builder form	Roster form	Graph
$\{x : x \in R, -\infty < x < \infty\}$	$(-\infty, \infty)$	
$\{x : x \in R, -\infty < x < 0\}$	$(-\infty, 0)$	
$\{x : x \in R, -\infty < x \le 0\}$	$(-\infty, 0]$	
$\{x : x \in R, 0 < x < \infty\}$	$(0, \infty)$	
$\{x : x \in R, 0 \le x < \infty\}$	$[0, \infty)$	

Symmetric Difference of Sets: If A and B are any two sets, then $A \Delta B = (A - B) \cup (B - A)$

Laws of Algebra for operations on sets:

1. $A \cup A = A$ $A \cap A = A$ [Idempotent Laws]
2. $A \cup \phi = A$ $A \cup U = U$ [Identity Laws]
   $A \cap \phi = \phi$ $A \cap U = A$
3. $A \cup (B \cup C) = (A \cup B) \cup C$
   $A \cap (B \cap C) = (A \cap B) \cap C$ [Associative Laws]
4. $A \cup B = B \cup A$
   $A \cap B = B \cap A$ [Commutative Laws]
5. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [Distributive Law]
6. De Morgan's Law

   $(A \cup B)' = A' \cap B'$ $(A \cap B)' = A' \cup B'$
7. $A \cup A' = U$ $A \cap A' = \phi$
   $\phi' = U$ $U' = \phi$ [Complement Laws]

8. (A') ' = A [Involution Law or Double complement law]

9. $A \cap B' = A - B$

10. If $A = B$ then $A \cup B = A \cap B$

11. If $B \subset A$, $(A - B) \cup B = A$

12. For any two finite sets A and B, not disjoint
   - $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
   - $n(A \cup B) = n(A-B) + n(B-A) + n(A \cap B)$
   - $n(A) = n(A-B) + n(A \cap B)$
   - $n(B) = n(B-A) + n(A \cap B)$

13. If A and B are two disjoint sets then $n(A \cup B) = n(A) + n(B)$

14. If A, B and C are any three disjoint sets, then $n(A \cup B \cup C) = n(A) + n(B) + n(C)$

15. If A, B and C are any three finite sets, not disjoint, then
   $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

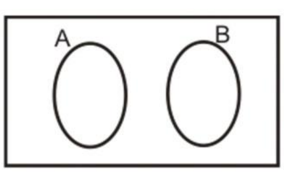
Extra Formulae:

1. $n(A \Delta B) = n(A) + n(B) - 2 \times n(A \cap B)$.
2. $n(A - B) = n(A) - n(A \cap B)$
3. $n(A - B) = n(A \cup B) - n(B)$
4. $n(B - A) = n(B) - n(A \cap B)$
5. $n(B - A) = n(A \cup B) - n(A)$
6. $n(A^c) = n(U) - n(A)$
7. $n(A^c \cap B^c) = n(A \cup B)^c = n(U) - n(A \cup B)$
8. $n(A^c \cup B^c) = n(A \cap B)^c = n(U) - n(A \cap B)$
9. $n(A\ only) = n(A \cap B^c \cap C^c) = n(A) - [n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)]$

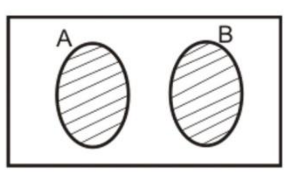
Venn diagram

It is a pictorial representation of sets. It consists of two closed figures – a rectangle for universal set and circles or oval shaped circles for sets. It was introduced by two mathematicians John Venn and Euler. Hence it is known as Venn-Euler diagram or simply Venn diagram. The following are some examples:

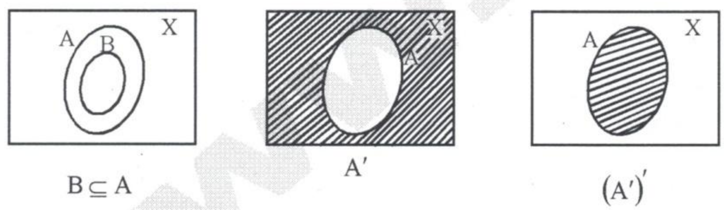
If A and B are disjoint sets,



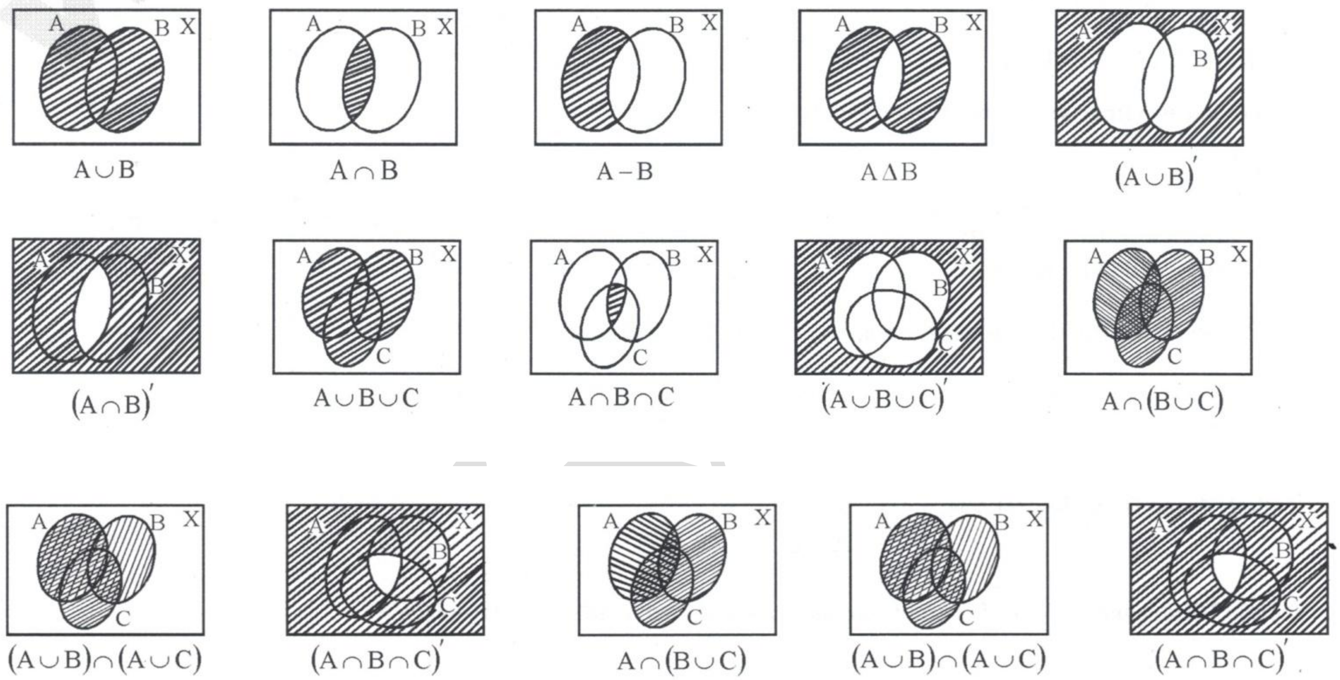
$A \cap B$



$A \cup B$



If A and B and A, B and C are not disjoint sets



Multiple choice questions: Do it yourself

1. The number of proper subsets of the set {1,2,3} is

   a) 8 b) 7 c) 6 d) 5

2. A,B and C are non-empty sets, then $(A-B)\cup(B-A)$ equals

   a) $(A\cup B)-B$ b) $A-(A\cap B)$ c) $(A\cup B)-(A\cap B)$ d) $(A\cap B)\cup(A\cup B)$

3. In a class of 100 students, 55 students have passed in Mathematics and 67 students have passed in Physics. Then the number of students who have passed in Physics only is

   a) 22 b) 33 c) 10 d) 45

4. Out of 800 boys in a school, 224 played cricket, 240 played hockey and 336 played basket ball. Of the total, 64 played both basketball and hockey; 80 played cricket and basketball and 40 played cricket and hockey; 24 played all the three games. The number of boys who did not play any game is

a) 128 b) 216 c) 240 d) 160

5. The number of non-empty subsets of the set $\{1,2,3,4\}$ is

a) 15 b) 14 c) 16 d) 17

6. The set of intelligent students in the class is

a) a null set b) a singleton set c) a finite set d) not a well defined collection

7. If A and B are given two sets, then $A \cap (A \cap B)^c$ is

a) A b) B c) ϕ d) $A \cap B^c$

8. Which of the following is an empty set?

a) $\{x : x \in R, x^2 - 1 = 0\}$ b) $\{x : x \in R, x^2 + 1 = 0\}$

c) $\{x : x \in R, x^2 - 9 = 0\}$ d) None of these

9. For two sets $A \cup B = A$ iff

a) $B \subseteq A$ b) $A \subseteq B$ c) $A \neq B$ d) $A = B$

10. Let $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$, then $n(A^c \cap B^c) =$

a) 400 b) 600 c) 300 d) 200

11. In rule method, the null set is represented by

a) { } b) ϕ c) $\{x : x = x\}$ d) $\{x : x \neq x\}$

12. Which set is the subset of all the given sets?

a) $\{1,2,3, \ldots\}$ b) $\{1\}$ c) $\{0\}$ d) { }

13. The smallest A such that $A \cup \{1,2\} = \{1,2,3,5,9\}$ is?

a) $\{2,3,5\}$ b) $\{3,5,9\}$ c) $\{1,2,5,9\}$ d) None of these

14. If A and B are sets $A \cap (B - A)$ is

a) ϕ b) A c) B d) None of these

15. Let $A = \{(x, y) : y = e^x, x \in R\}$; $B = \{(x, y) : y = e^{-x}, x \in R\}$, then

a) $A \cap B = \phi$ b) $A \cap B \neq \phi$ c) $A \cup B = R^2$ d) None of these

16. If $A \subset B$ *and* $A \neq B$, then A is known as ....

a) subset of B b) Proper subset of B c) Super set of B d) none of these

17. Sets A and B have 3 and 6 elements respectively. What can be the maximum number of elements in $A \cup B$?

a) 3 b) 6 c) 9 d) 18

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SET ASSIGNMENT 1

1. Describe the following sets by roster method:

   i. $\{x : 25x^2 + 30x + 7 = 0, x \in Q\}$
   ii. $\{x : 7x + 9 < 55, x \in N\}$
   iii. $\{x : |x| = 4, x^2 + 16 = 0, x \in N\}$
   iv. $\{x : x^2 + x + 1 = 0, x \in C\}$
   v. $\{x : |x| < 3, x \in Z\}$

2. Describe the following sets by property method:

   i. $\{3,5,9,17,33\}$
   ii. $\left\{\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}\right\}$
   iii. $\{5,7,11,13,17\}$

3. Which of the following sets are null sets:

   i. $A = \{x : x < 1 \, and \, x > 3\}$
   ii. $B = \{x : x^2 = 9 \, and \, 3x = 7\}$
   iii. $C = \{x : x^2 - 1 = 0, x \in R\}$
   iv. $D = \{x : x \, is \, an \, even \, prime \, number\}$

4. Which of the following are singleton sets:

   i. $A = \{x : 3x - 2 = 0, x \in Q\}$
   ii. $B = \{x : x^3 - 1 = 0, x \in R\}$
   iii. $C = \{x : 30x - 59 = 0, x \in N\}$
   iv. $D = \{x : |x| = 1, x \in Z\}$

5. Write all the proper subsets of the set $\{-1,3,4\}$.
6. If $A = \{3,6,8,15,19\}$ and $B = \{1,2,6,8,14,15\}$, then verify that $A \Delta B = (A \cup B) - (A \cap B)$.
7. If $A = \{x : x^3 - 1 = 0\}, B = \{x : x^2 + x + 1 = 0\}$ *find* $A \cap B$ *when* x is a (i) real number (ii) complex number.
8. Write the following intervals in set builder form:

   a) $[-2,3)$ b) $(0, 5]$ c) $(-1, 3)$ d) $[-2, 3]$

9. Write the following intervals in the roster form:

   a) $\{x : x \in R, -2 < x < 0\}$ b) $\{x : x \in R, \ 2 \le x < 4\}$ c) $\{x : x \in R, \ 2 < x \le 4\}$
   d) $\{x : x \in R, \ -3 \le x \le 5\}$

10. If $U = \{1,2,3,4,5,6,7,8,9\}$; $A = \{1,2,3\}$; $B = \{2,3,5\}$; $C = \{3,5,7\}$, then prove that:

    i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

11. Using the above sets, find the following questions:

    i) $A - (B \cup C)'$ ii) $A \cap (B \cap C)'$

12. If A={1,2,3,4,5} , then the number of proper subsets of A is ……… a) 120 b) 30 c) 31 d) 32.
13. If A and B are two given sets, then $A \cap (A \cap B)^c$ is equal to

    a) A b) B c) $(A \cap B)^c$ d) ϕ

14. If A ={1,2,3,4} and B={3,4,5,6} , then shows that $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$.
15. Draw the Venn diagrams for the following sets:

    a) $A - (B \cup C)'$ b) $A \cup (B \cap C)$ c) $A \cap (B \cup C)$ d) $(A \cup B) - (A \cap B)$.