## Problem Statement for Optimization

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Let us consider a two way relay (TWR) network as shown in Fig. 1, where bidirectional information exchange takes place between two source nodes  $S_a$  and  $S_b$  via a relay node R. We also consider that  $S_a$  and  $S_b$  can exchange information via direct link available between them. The transceiver used at the  $S_a$ ,  $S_b$ , and R have hardware impairments. An amplify-and-forward protocol used at the relay R for information processing. The relay node suffers from non-linearity which caused due to high power amplifier. The relay node R harvests energy from the radio-frequency signals, which is received from both the source nodes  $S_a$  and  $S_b$ . A hybrid receiver is used at the relay node for energy harvesting (EH) and information processing (IP). The hybrid receiver combines both the basic receivers time-switching (TS) and power splitting (PS). The time-switching factor  $\alpha$  is an important parameter which decides the fraction of the time period dedicated for EH and IT at the relay node to achieve the optimum performance. Outage probability (OP) is an important performance metric which determines the link failure probability. For the considered network OP at  $S_i$  is given as

$$P_{out,i}(\gamma_{th}) = \left[1 - \sum_{p=0}^{m_j - 1} \sum_{t=0}^{p} \sum_{q=0}^{t} {p \choose t} {t \choose q} \left(\frac{m_i}{\Omega_i}\right)^{m_i} \frac{(C_2 \zeta_i)^q (C_4)^{p-t} (C_3)^{t-q}}{p! \Gamma(m_i)} \left(\frac{m_j \gamma_{th}}{\Omega_j \zeta_j (1 - C_1 \gamma_{th})}\right)^p \right] \times e^{-\frac{m_j C_3 \gamma_{th}}{\Omega_j \zeta_j (1 - C_1 \gamma_{th})}} 2 \left(\frac{m_j \Omega_i C_4 \gamma_{th}}{m_i \Omega_j \zeta_j (1 - C_1 \gamma_{th}) + m_j \Omega_i C_2 \zeta_i \gamma_{th}}\right)^{\frac{m_i + t + q - p}{2}} \times \mathcal{K}_{m_i + t + q - p} \left(2 \sqrt{\left(\frac{m_i}{\Omega_i} + \frac{m_j C_2 \zeta_i \gamma_{th}}{\Omega_j \zeta_j (1 - C_1 \gamma_{th})}\right) \frac{m_j C_4 \gamma_{th}}{\Omega_j \zeta_j (1 - C_1 \gamma_{th})}}\right) \frac{1}{\Gamma(m_d)} \gamma \left(m_d, \frac{m_d \gamma_{th}}{\Omega_d \zeta_i}\right),$$

$$(1)$$

where  $\{m_i, m_j, m_d\}$  and  $\{\Omega_i, \Omega_j, \Omega_d\}$  denotes fading severity and average power between the links  $S_a - R$ ,  $S_b - R$ , and  $S_a - S_b$ , respectively. The constant terms  $C_1 = k^2 + \frac{1+k^2}{\gamma_{pa}} + \frac{k^2(1+k^2)}{K_0^2}$ ,  $C_2 = \frac{1+k^2}{\gamma_{pa}} + \frac{k^2(1+k^2)}{K_0^2}$ ,  $C_3 = \frac{2\sigma_r^2}{\sigma_i^2}$ ,  $C_4 = \frac{1+k^2}{K_0^2\Xi}$ ,  $\Xi = \frac{\eta(3\alpha+(1-\alpha)\beta)}{1-\alpha}$ ,  $\gamma_{pa} = \frac{P_rK_0^2}{\sigma_{ND}^2}$ ,  $K_0 = 1 - \exp(\frac{-A_{sat}^2}{P_r}) + \frac{A_{sat}\sqrt{\pi}}{2\sqrt{P_r}} \operatorname{erfc}(\frac{A_{sat}}{\sqrt{P_r}})$ ,  $\sigma_{ND}^2 = P_r(1 - \exp(\frac{-A_{sat}^2}{P_r}) - |K_0|^2)$ ,  $\zeta_i = \frac{P_i}{\sigma_i^2}$ ,  $\zeta_j = \frac{P_j}{\sigma_i^2}$ , and  $\gamma_{th} = 2^{\frac{3r_{th}}{1-\alpha}} - 1$ . The  $\mathcal{K}_v(\cdot)$  denotes  $v^{th}$  order modified Bessel function of second kind and  $r_{th}$  denotes threshold datarate. Energy efficiency is an important performance metric for the design of low-powered system. For the considered network energy efficiency can be given as

$$\eta^{EE} = \frac{(1-\alpha)[(1-P_{out,a}(\gamma_{th}))r_{th} + (1-P_{out,b}(\gamma_{th}))r_{th}]}{(1+2\alpha)(P_a + P_b)}.$$
 (2)

where  $P_{out,a}(\gamma_{th})$  and  $P_{out,b}(\gamma_{th})$  denote the OPs at  $S_a$  and  $S_b$ , respectively.

The typical value of the system parameters are as follows:  $\{m_i, m_j, m_d\} = \{1, 1, 1\}, \{\Omega_i, \Omega_j, \Omega_d\} = \{8, 8, 1\}, A_{sat} = 1, \kappa = 0.1, P_i = P_j = 0.5 \text{ Watt}, r_{th} = 1 \text{ bps/Hz}, \eta = 0.7, \beta = 0.2, \text{ and } 0 < \alpha \leq 1.$ 

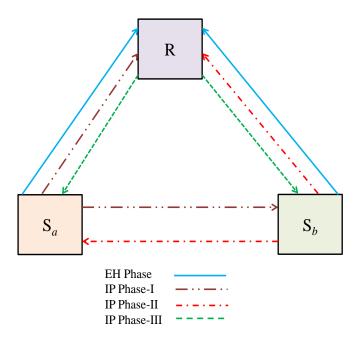


Figure 1: System model.

Problem statement: Find the optimal value of the time-switching factor  $\alpha$  to design an energy efficient system.

Note: For more details please follow the reference paper [R1] D. Kumar, P. K. Singya and V. Bhatia, "Performance analysis of hybrid two-way relay network with NLPA and hardware impairments," IEEE Wireless Commun. Net. Conf. (WCNC), Apr. 2021, pp. 1-6.