Tank Modeling & Control

We consider two open water tanks connected in series:

- Tank 1 receives an inflow $u(t)$ [m³/s] that can be manipulated (the control input).
- Water flows from Tank 1 to Tank 2 through an orifice at the bottom of Tank 1.
- Tank 2 drains to the environment through an outlet at its base.

The flow through each outlet is proportional to the square root of the water head above the opening:

$$q_1 = k_1 \sqrt{h_1}$$

$$q_2 = k_2 \sqrt{h_2}$$

where:

- $h_1(t), h_2(t)$ are the water levels in the tanks [m]
- k_1, k_2 are discharge coefficients

Tank cross-sectional areas are A_1 and A_2. The dynamic balances are:

$$A_1 \frac{\mathrm{dh}_1}{\mathrm{dt}} = u(t) - q_1$$

$$A_2 \frac{\mathrm{dh}_2}{\mathrm{dt}} = q_1 - q_2$$

Tasks

1. **Modeling**
   - Implement this system in ModelingToolkit.jl
2. **Open Loop Analysis**
   - Simulate the system's response to a step inflow $u(t)$.
   - Plot $h_1(t)$ and $h_2(t)$.
   - Briefly discuss the dynamics (nonlinearity, coupling, time scales).
3. **Control**
   - Design a controller to regulate $h_2(t)$ to a given setpoint h_2^* by controlling k_2.
   - Start with a simple PI or PID controller.
   - Optional: Design any other controller of your choosing. For example, a "bang-bang" controller that sets k_2 on/off discretely would be interesting.
   - Optional: Evaluate gain and phase margins around a steady-state operating point

Notes

- Keep the scope manageable: assume reasonable values for parameters (A_1, A_2, k_1, k_2).
- You may use any Julia packages you find useful in addition to ModelingToolkit.jl.