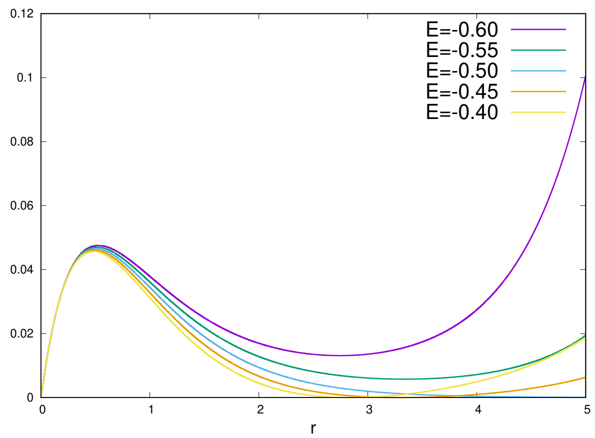
1. The radial Schrödinger equation for the central potential $V(r)$ is given by

$$\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}\right] R(r) + \frac{2\mu}{\hbar^2}\left[E + V(r) - \frac{l(l+1)\hbar^2}{2\mu r^2}\right] R(r) = 0.$$

Here, μ is the reduced mass of the system, l is the orbital-angular momentum quantum number, and $R(r)$ is the radial wave function. The above equation, in atomic units, for the ground state (l=0) of the hydrogen atom can be written as

$$\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr}\right] R + 2\left[E_0 + \frac{1}{r}\right] R = 0.$$

Write a Fortran program to solve the above equation using RK4 to find E_0 with following starting values: $R(r = 0.0005) = 0.000001, R'(r = 0.0005) = -1000.0$. The r grid will be from 0.0005 unit to 5 unit with 10000 points. The code will be for a range of E values, $-0.6 \leq E \leq -0.4$, with ΔE=0.01. For finding the correct value of E, plot $R(r)$ and the radial distribution function, $|rR(r)|^2$, against r and check their convergence with respect to E. At the E where both $R(r)$ and $|rR(r)|^2$ behave properly against r, that will be the answer. You will submit one zip file, q1.zip, containing the following files: the code, and 5 different output files containing the data for $R(r)$ and $|rR(r)|^2$ vs r. The figure below shows the plot of $|rR(r)|^2$ vs r for 5 different energies. You will show the results for E= -0.60, -0.55, -0.50, -0.45 and -0.40.



2. The energy of the simple harmonic oscillator is $E = \dfrac{p^2}{2m} + \dfrac{1}{2}kx^2$. Here m is the mass, k is the spring force constant, and $p = m\dot{x}$ is the momentum. Let us set $m = k$=1, so the angular frequency, ω, and period, T, are given by $\omega = \sqrt{\dfrac{k}{m}} = 1$, and $T = \dfrac{2\pi}{\omega} = 2\pi$. Hence, $2E = p^2 + x^2$ is a constant, and a plot of p vs x will be a circle of radius $\sqrt{2E}$. We will solve the Newton's equation of motion $\ddot{x} = -x$ which will be written as two first order differential equations

$$\dot{x} = p \text{ and } \dot{p} = -x. \tag{1}$$

Numerically integrate these equations using the Euler and RK4 methods.

- Use initial conditions, x =1 and p =0. Hence $2E$ =1. Use a time step h = 0.02T . Use 200 time steps in total. Plot $2E$ vs t/period, x vs p and x vs t/period.

In the figure below, results obtained using Euler's, RK2 and RK4 methods are shown. **But you need to show the results only for Euler's and RK4 methods.** You will submit a zip file, q2.zip, containing the code(s), and six output files, 3 for each method.