

Maths Assignments

$$1) A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 2R_1$$

~~$$R_3 \rightarrow R_3 - 3R_1$$~~

~~$$(R_3 \rightarrow R_3 - 3R_1) \quad (SP-3)$$~~

$$R_4 \rightarrow R_4 - 4R_1$$

$$R_4 \rightarrow R_4 - R_3$$

↓

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Leftarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 1 & 0 & -3 & 2 \end{bmatrix}$$

$$R_4 \rightarrow R_4 - R_2$$

*

~~SEI0-5 = 85~~

No. of non zero rows = Rank(A)

$$\therefore \text{Rank}(A) = 3$$

~~Emittable~~

$$4) \begin{aligned} 3x - 0.1y - 0.2z &= 7.85 \\ 0.1x + 7y - 0.3z &= -19.3 \\ 0.3x - 0.2y + 10z &= 71.4 \end{aligned}$$

$$n = \frac{1}{3}[7.85 + 0.1(y + 0.2z)]$$

$$100-8 = 82$$

$$y = \frac{1}{7}[-19.3 - 0.1(n + 0.2z)]$$

$$z = \frac{1}{10}[71.4 - 0.3n + 0.2y]$$

Iteration 1:

$$z = y = 0$$

$$100-8 = 82$$

$$x_1 = \frac{7.85}{3} = 2.61$$

$$y_1 = \frac{1}{7}(-19.3 - 0.1(2.61)) = 2.79$$

$$z_1 = \frac{1}{10}[71.4 - 0.3(2.61) + 0.2(2.79)] = 7.1175$$

Iteration 2

$$n_2 = \frac{7.85 - 0.1(2.61) - 0.2(7.1408)}{2.18 - \mu - 0} = 3.$$

$$n_2 = \underline{\underline{2.9255}}$$

$$y_2 = \frac{19.3 - 0.1(2.92) - 0.3(7.1408)}{10}$$

$$\begin{bmatrix} 0 & y_2 & 3.0123 \\ S & S - O & \underline{\underline{0}} \\ S & S - \mu & 0 \\ S & S - \mu & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & S & 1 \\ S & S - \mu & 0 \\ S & S - \mu & 0 \\ S & S - \mu & 0 \end{bmatrix}$$

$$z_2 = \frac{71.4 - 0.3(2.9255) - 0.2(3.0123)}{10}$$

$$z_2 = 7.0132$$

~~(A) $\times 10^3$~~ = more precise form

Iteration 3

$$n_3 = \frac{7.85 - 0.1(2.9255) - 0.2(3.0123)}{2.18 - \mu} = 3$$

$$S.PI^3 = 5S.0 + S\Gamma + \kappa I.0$$

$$n_3 = \underline{\underline{3.0032}}$$

$$y_3 = \frac{19.3 - 0.1(3.0032) - 0.3(7.0132)}{10}$$

$$[5S.0 + \kappa I.0 - S.PI] \frac{1}{10} = P.$$

$$y_3 = \underline{\underline{3.001}}$$

$$[P + \kappa S.0 - \mu.1] \frac{1}{10} = S$$

$$z_3 = \frac{71.4 - 0.3(3.0032) - 0.2(3.0001)}{10}$$

$$z_3 = \underline{\underline{7.00}}$$

$$O = E = S$$

$$12-S = \frac{28.5}{S} = 1.5$$

$$n = 3.0032, y = 3.0001, z = 7.00$$

$$P.S = ((12-S)1.0 - S.PI) \frac{1}{10} = P$$

$$251.5 = [(PI)S.0 + (12-S)E.0 - H.1\Gamma] \frac{1}{10} = 15$$

JUNIOR SEMESTER

6) $A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix}$ ei T jo afmri oft :
 Difmorfizm $\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix}$ + compare sepsis fo

$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix}$ $R_2 \Rightarrow R_2 - 2R_1$ T fo X mod
 $R_3 \Rightarrow R_3 - 3R_1$
 $R_4 \Rightarrow R_4 - R_1$

afmri oft fo $\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix}$ T fo X mod

$\Rightarrow \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix}$ $R_3 \Rightarrow R_3 - 2R_2$
 free to me $\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (A)T

This corresponds to system:
 $\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
 $x + 3y + 2z = 0$
 $-7y - z = 0$
 $0 = 0 - 0$

Let

$y = t$

$x = -3t$

$z = -7t$

$\alpha = d = 0$

∴ System has infinitely many solutions.

∴ System is consistent and dependent.

2) T: $W \rightarrow P_2$: $t = \text{midterm}$

$$T \begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b)t + (b-c)n + (c-a)n^2$$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$T \begin{bmatrix} a & b \\ b & c \end{bmatrix} = (a-b) + (b-c)n + (c-a)n^2$$

$$= -a$$

Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$$T(A) = (a-b) + (b-c)n + (c-a)n^2$$

$$= a - bn + c(n^2 - n + 1)$$

∴ The image of T is the set of all polynomials of degree at most 2, denoted as P_2 .

Rank of T

The rank of T is the dimension of its image since P_2 has a dimension of 3 (coefficients for x^0, x^1 and x^2) the rank of T is 3

The Null space of symmetric matrix,

$$T(A) = 0.$$

This leads to the system of equations -

$$ad - bc = 0$$

$$ab - cd = 0$$

$$c - a = 0$$

$$\therefore a = b = c$$

∴ T is the set of symmetric matrices of the form

$\begin{bmatrix} t & t \\ t & t \end{bmatrix}$ where t is any scalar.

∴ Dimension = 1 (using only t)

∴ Rank (T) is 1

\therefore The nullity of T is $1 - 1 = 0$

$$d - e = 0 \Rightarrow d = e$$

$$d - e + e - e + d - e = \begin{bmatrix} d & e \\ e & e \end{bmatrix} = T$$

$$\begin{bmatrix} d & e \\ e & e \end{bmatrix} = T$$

$$3) A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\lambda_1 = 1, \lambda_2 = 3$$

$$A^{-1} = \frac{1}{4-1} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2/3 - \lambda & 1/3 \\ 1/3 & 2/3 - \lambda \end{bmatrix} \Rightarrow \left(\frac{2}{3} - \lambda \right)^2 - \left(\frac{1}{3} \right)^2 = 0$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} \lambda \\ \lambda \end{bmatrix} = \text{solution implies } \lambda = 1, \lambda = \frac{1}{3}$$

$$\lambda = \frac{1}{3}, 1$$

$$|A - \lambda I| = \begin{bmatrix} 2/3 - \lambda & 1/3 \\ 1/3 & 2/3 - \lambda \end{bmatrix} = 0$$

$$\lambda = 1/3$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2/3 - 1/3 & 1/3 \\ 1/3 & 2/3 - 1/3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \Rightarrow x = y$$

$$\underline{x = y} \rightarrow \text{Eigenvector} = k \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x = 1 \rightarrow x(x+1) + y(1+y) + (1+y) = (x+y+1)^2$$

$$\Rightarrow \begin{bmatrix} x(x+1) + y(1+y) + (1+y) \\ x \\ y \end{bmatrix} = 0$$

or $\begin{bmatrix} x(x+1) + y(1+y) + (1+y) \\ x \\ y \end{bmatrix} = 0$ result in
writing out steps

$$\underline{x = y} \rightarrow \text{Eigenvector} = k \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

1 step for component

$$Z = A + 4I$$

method of finding $(Z)^{-1} \cdot x = (Ax)^{-1}$

$$\Rightarrow \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$$Z - 4I = \begin{bmatrix} 6-4 & 0 \\ 0 & 6-4 \end{bmatrix}$$

$$(6-4)^2 - (0)^2 \Rightarrow (6-4)(6-4)$$

$$\Rightarrow (5-\lambda)(7-\lambda) = 0$$

$$\lambda = 5, 7$$

$$\lambda = 5 \quad \left[\begin{array}{cc} 1-s & s \\ s & 1-s \end{array} \right] - \left[\begin{array}{cc} 1-s & s \\ s & 1-s \end{array} \right] \frac{1}{1-s} = A$$

$$0 = \left(\frac{1-s}{s} \right) - \left[\begin{array}{cc} 1-s & s \\ -1 & 1-s \end{array} \right] \left[\begin{array}{c} x \\ y \end{array} \right] = 0 \quad \left[\begin{array}{cc} 1-s & s \\ -1 & 1-s \end{array} \right] \left[\begin{array}{c} x \\ y \end{array} \right] = I\lambda - A$$

$$(d-s)(d+s) = s^2 d - s^2 0 \quad n=y \rightarrow \text{Eigen vector} = \left[\begin{array}{c} k \\ k \end{array} \right] \rightarrow k \left[\begin{array}{c} 1 \\ 1 \end{array} \right]$$

$$\lambda = 7 \quad \left[\begin{array}{cc} 1-s & s \\ s & 1-s \end{array} \right] = A$$

$$\Rightarrow \left[\begin{array}{cc} -1 & -1 \\ -1 & -1 \end{array} \right] \left[\begin{array}{c} x \\ y \end{array} \right] = 0$$

$$n = -y \rightarrow \text{Eigen vector} = \left[\begin{array}{c} k \\ -k \end{array} \right] \rightarrow k \left[\begin{array}{c} 1 \\ -1 \end{array} \right].$$

6) $T: P_2 \rightarrow P_2$ is a linear transformation.

$$T(a + bn + cn^2) = (a+1) + (b+1)n + (c+1)n^2$$

$$\Rightarrow T(a + bn + c) = (a+1) + (b+1)n + (c+1)n^2$$

is a linear transformation, we need to check two properties.

1) Additivity: $T(U+V) = T(U) + T(V)$

2) Homogeneity of degree 1:

$$T(kU) = k T(U) \text{ for all } U \in \text{domain}$$

T and all scalars k .

$$\left[\begin{array}{cc} 1 & 1-s \\ s & 1-s \end{array} \right] = I\lambda - s A - s$$

$$\begin{aligned}
 T(u+v) &= T(a_1+b_1x+c_1) + T(a_2+b_2x+c_2) \\
 &= T((a_1+a_2)+(b_1+b_2)x+(c_1+c_2)) \\
 &\Rightarrow (a_2+a_1+1) + (b_1+b_2+1)x + (c_1+c_2+1)x^2 \\
 &\Rightarrow (a_1+1) + (b_1+1)x + (c_1+1)x^2 + (a_2+1) \\
 &\quad + (b_2+1)x + (c_2+1)x^2 \quad (\text{S.E.V}) \\
 T(a_1+b_1x+c_1) + T(a_2+b_2x+c_2)
 \end{aligned}$$

So Function is additive.

\therefore Homogeneity of Degree : 1.

$$T(ku) = T(k(a+bx+c))$$

$$= T(ka+kbx+kc)$$

$$\Rightarrow (ka+1) + (kb+1)x + (kc+1)x^2$$

$$\Rightarrow k T(a+b x + c)$$

So the function is homogeneous of degree 1.

\therefore It is indeed Linear Transformation.

$\Rightarrow S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ is a basis.

It can be arranged as matrix.

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$$

\therefore Row reduction to obtain echelon form.

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$R_3 \rightarrow R_3 - 3R_1$$

$$R_3 \rightarrow R_3 + \frac{9}{5}R_2$$

∴ Third row of zeroes indicates that the vectors in S are linearly dependent.
for basis of the subspace spanned by S .

$$(1, 3, 2) + \lambda(0, -5, 5) = (1 + \lambda, 3 + 5\lambda, 2 + 5\lambda)$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -5 & 5 \end{bmatrix} + \lambda(1, 5, 1) = (1 + \lambda, 3 + 5\lambda, 2 + 5\lambda)$$

$(1, 3, 2)$ and $(0, -5, 5)$ These vectors form a basis for the subspace spanned by S .

∴ Dimension of subspace spanned by $S = 2$

∴ Set S is not a basis of \mathbb{R}^3 because the row reduced form has a row of zeroes.

∴ the basis for the subspace spanned by S is $\{(1, 3, 2), (0, -5, 5)\}$.

∴ The dimension of the subspace is 2

8) $3x - 6y + 2z = 23$
 $-4x + y - z = -15$

$$x - 3y + 7z = 16$$

with initial values

$$x_0 = 1, y_0 = 1, z_0 = 1$$

Iteration - 1

$$x_1 = \frac{23 + 6y_0 - 2z_0}{3} = \underline{\underline{9}}$$

$$y_1 = \frac{-15 + 4x_0 + z_0}{-4} = \underline{\underline{-9}}$$

$$z_1 = \frac{16 - x_0 - 3y_0}{7} = \underline{\underline{2}}$$

Iteration 2

$$\begin{aligned} d &= \bar{x} + \bar{y} - \bar{z} \\ \bar{c} &= \bar{x} + \bar{y} + \bar{z} \end{aligned}$$

$$n_2 = \frac{2\bar{x} + 6\bar{y}_1 - 2\bar{z}_1}{3} = \underline{\underline{-5}}$$

$$\begin{bmatrix} \bar{x}_2 \\ \bar{y}_2 \\ \bar{z}_2 \end{bmatrix} = \underline{\underline{-15}} + \frac{4x_1 + z_1}{1} = \begin{bmatrix} \bar{x} - \bar{y} - \bar{z} \\ \bar{y} - \bar{z} \\ \bar{z} \end{bmatrix} \quad \bar{\Gamma} = \underline{\underline{-15}}$$

$$z_2 = \frac{16 - n_1 + 3y_1}{7} = \underline{\underline{\frac{6}{3}}}$$

Iteration 3

$$8x_3 = \frac{2\bar{x} + 6\bar{y}_2 - 2\bar{z}_2}{3} = \underline{\underline{6}}$$

$$y_3 = \underline{\underline{-15 + 4x_2 + z_2}} = \underline{\underline{-16}}$$

$$z_3 = \frac{16 - n_2 + 3y_2}{7} = \underline{\underline{2}}$$

$$\therefore \underline{\underline{x_1, y_1, z_1}} = (6, -6, 2)$$

$$S = (4) \text{ rad/s}$$

$$\theta = (\pi/4) \text{ radian}$$

$$\text{Initial phase: } \theta = (\pi/4) \text{ radian}$$

Q)

$$1) \quad 2x - 3y + 7z = 5$$

$$3x + y - 3z = 13$$

$$2x + 19y - 47z = 32$$

Solve it

$$A = \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - \frac{3}{2}R_1 \quad \underline{18x + 12y - 21z = 27}$$

$$\downarrow \quad R_3 \rightarrow R_3 - R_1$$

$$\left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 0 & 22 & -54 & 32 \end{array} \right] \xrightarrow{\text{Row Echelon Form}} \left[\begin{array}{ccc|c} 1 & -\frac{3}{2} & \frac{7}{2} & \frac{5}{2} \\ 0 & 1 & -3 & \frac{13}{2} \\ 0 & 0 & -\frac{54}{2} & \frac{32}{2} \end{array} \right] \xrightarrow{\text{Simplify}}$$

$$R_2 \rightarrow R_2 - \frac{3}{2}R_1 \quad \underline{5x + 2y + 21z = 27}$$

$$\left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 0 & 1 & -3 & \frac{13}{2} \\ 0 & 0 & -27 & 27 \end{array} \right] \xrightarrow{\text{Simplify}} \left[\begin{array}{ccc|c} 1 & -\frac{3}{2} & \frac{7}{2} & \frac{5}{2} \\ 0 & 1 & -3 & \frac{13}{2} \\ 0 & 0 & 1 & 1 \end{array} \right] \xrightarrow{\text{Simplify}}$$

$$\left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 0 & 1 & -3 & \frac{13}{2} \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\text{Simplify}} \left[\begin{array}{ccc|c} 1 & -\frac{3}{2} & \frac{7}{2} & \frac{5}{2} \\ 0 & 1 & -3 & \frac{13}{2} \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\text{Rank}(A) = 2$$

$$\text{Rank}(A:B) = 3$$

$\text{P}(A) \neq \text{P}(A:B)$ inconsistent

$$\text{ii) } \begin{aligned} 2x - y + 3z &= 8 \\ -x + 2y + z &= 4 \\ 3x + y - 4z &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{array} \right] \xrightarrow{\begin{array}{l} R_1 \leftrightarrow R_2 \\ R_1 + R_1 \\ R_3 - 3R_1 \end{array}} \left[\begin{array}{ccc|c} 0 & 1 & 4 & 8 \\ 2 & 0 & 4 & 4 \\ 0 & 1 & 0 & 0 \end{array} \right] = A$$

$$A = \left[\begin{array}{ccc} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{array} \right], X = \left[\begin{array}{c} x \\ y \\ z \end{array} \right], B = \left[\begin{array}{c} 8 \\ 4 \\ 0 \end{array} \right], \left[\begin{array}{ccc|c} 0 & 1 & 4 & 8 \\ 2 & 0 & 4 & 4 \\ 0 & 1 & 0 & 0 \end{array} \right] = A$$

$$A = \left[\begin{array}{ccc} -1 & 2 & 1 \\ 2 & -1 & 3 \\ 3 & 1 & -4 \end{array} \right], \left[\begin{array}{c} 4 \\ 8 \\ 0 \end{array} \right], \left[\begin{array}{ccc|c} 0 & 1 & 4 & 8 \\ 2 & 0 & 4 & 4 \\ 0 & 1 & 0 & 0 \end{array} \right] = A$$

$$A = \left[\begin{array}{ccc} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 7 & -1 \end{array} \right], \left[\begin{array}{c} 4 \\ 16 \\ 12 \end{array} \right]$$

to solve for x from $R_1 \rightarrow R_1 - R_2$, $x = 5 + y + z$

$$A = \left[\begin{array}{ccc} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 7 & -1 \end{array} \right], \left[\begin{array}{c} 4 \\ 16 \\ 12 \end{array} \right], 01 = 5x + yz + 12$$

$$\text{and also } 01 = 5x + yz + 12$$

$$A = \left[\begin{array}{ccc} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & -3815 \end{array} \right], \left[\begin{array}{c} 4 \\ 16 \\ 12 \end{array} \right], \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 3 & 5 & 12 \\ 0 & 0 & 1 & 1 \end{array} \right] = A$$

$$S(A) = S(A; B) \text{ consistent} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 3 & 5 & 12 \\ 0 & 0 & 1 & 1 \end{array} \right] =$$

$$\text{iii) } \begin{aligned} 4x - y &= 12 \\ -x + 5y - 2z &= 0 \\ -2x + 4z &= -8 \end{aligned}$$

$$A = \left[\begin{array}{ccc|c} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 4 & -8 \end{array} \right], X = \left[\begin{array}{c} x \\ y \\ z \end{array} \right], B = \left[\begin{array}{c} 12 \\ 0 \\ -8 \end{array} \right]$$

$$A = \left[\begin{array}{ccc|c} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ 0 & -10 & 8 & -16 \end{array} \right], B = \left[\begin{array}{c} 12 \\ 0 \\ -16 \end{array} \right] \xrightarrow{\text{add } R_2 + R_3} \left[\begin{array}{ccc|c} 4 & -1 & 0 & 12 \\ 0 & 4 & -2 & -16 \\ 0 & -10 & 8 & -16 \end{array} \right] = A$$

$$A = \left[\begin{array}{ccc} 4 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & 4 & -2 \end{array} \right], \left[\begin{array}{c} 12 \\ 0 \\ -16 \end{array} \right] \xrightarrow{\text{row } R_3 \rightarrow R_3 + R_2} \left[\begin{array}{ccc} 4 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & 0 & -4 \end{array} \right], \left[\begin{array}{c} 12 \\ 0 \\ -4 \end{array} \right] = A$$

VACETV JEEVA

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -4 & 20 & -8 \\ 0 & -1 & 8 \end{bmatrix} \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

$$S = 5S + B - RS \quad \text{Lii}$$

$$H = S + B^T + R -$$

$$O = SH - B + RB$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & -1 & 8 \end{bmatrix} \begin{bmatrix} 12 \\ 12 \\ -4 \end{bmatrix} = X$$

$$\begin{bmatrix} S & H & O \\ I & S & H \\ R & B & E \end{bmatrix} = A$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & 18 & 0 \end{bmatrix} \begin{bmatrix} 12 \\ 12 \\ 0 \end{bmatrix} \begin{bmatrix} S & H & O \\ I & S & H \\ R & B & E \end{bmatrix} = A$$

$$P(A) = P(A \geq B)$$

consistent.

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} S & H & O \\ I & S & H \\ R & B & E \end{bmatrix} = A$$

$$x + y + z = 6 \quad \Rightarrow \text{For what values of}$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu \quad \begin{cases} \text{i} \triangleright \text{No solution} \\ \text{ii} \triangleright \text{Unique solution} \\ \text{iii} \triangleright \text{Infinite solution.} \end{cases}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix} \quad \begin{bmatrix} S & H & O \\ I & S & H \\ R & B & E \end{bmatrix} = A$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \lambda - 3 \end{bmatrix} \quad (A: B) \Rightarrow (A) Q$$

$$S = B - 2AH \quad \text{Lii}$$

$$A: B = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 10 \\ 0 & 0 & \lambda - 3 & \mu \end{bmatrix} \quad \begin{bmatrix} S & H & O \\ I & S & H \\ R & B & E \end{bmatrix} = A$$

$$\triangleright \text{No solution } (P(A) \neq P(A: B)) \quad \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 10 \\ 0 & 0 & \lambda - 3 & \mu \end{bmatrix} \quad \begin{bmatrix} S & H & O \\ I & S & H \\ R & B & E \end{bmatrix} = A$$

$$\lambda = 3, \mu \neq 10$$

$$\text{ii} \triangleright \text{Unique solution } (P(A) = P(A: B)) \quad \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 10 \\ 0 & 0 & \lambda - 3 & \mu \end{bmatrix} \quad \begin{bmatrix} S & H & O \\ I & S & H \\ R & B & E \end{bmatrix} = A$$

$$\text{iii} \triangleright \text{Infinite solution } (\lambda \neq 3, \mu \neq 10) \quad \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 10 \\ 0 & 0 & \lambda - 3 & \mu \end{bmatrix} \quad \begin{bmatrix} S & H & O \\ I & S & H \\ R & B & E \end{bmatrix} = A$$

iii) infinite solution ($P(A) = P(A:B)$) $\leftarrow n$ (order)

(no of unknowns)

$$\lambda = 3; \mu = 10$$

$n \rightarrow x, y, z$ ie 3.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 8 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$I = S + P + N$$

$$O = S - P$$

CQ) $x+y+z=1$ For what value of λ
 $x+2y+4z=\lambda$ have a solution &
 $x+4y+10z=\lambda^2$ solve for each value of λ .

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix} S = A$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{bmatrix} A:B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix} S = A + P + N$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix} N = S + N = P$$

$$P(A) = 2$$

$$P(A:B) = 2.$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda - 1 \\ 0 & 3 & 9 & \lambda^2 - 1 \end{bmatrix}$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\lambda^2 - 2\lambda - \lambda + 2 = 0$$

$$\lambda(\lambda - 1) - 2(\lambda - 1) = 0$$

$$(\lambda - 1)(\lambda - 2) = 0$$

$$\begin{cases} \lambda = 1, 2 \end{cases} \Rightarrow \begin{cases} \lambda = 1 \\ \lambda = 2 \end{cases} \Rightarrow P(A) = P(A:B) \neq n.$$

A: (1, 2) : (1, 2)

3 unknowns

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{bmatrix} \in \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} = A$$

case-1

$\begin{pmatrix} \text{Grobner} \\ x=1 \end{pmatrix} \Rightarrow ((x+y+z) - (1))$ without stimable L.H.S
 (approximate now for 01)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$01 = N_1, \epsilon = x$$

$$x+y+z = 1$$

$$y+3z=0$$

$$\begin{aligned} z = k &\quad \text{take } x = 1 - y - z \\ x = 1 - y - z &\quad I = S + B + N \\ y = -3k &\quad S = H + \underline{B} + N \\ &\quad \cancel{B} \rightarrow 1 + 3k - k = 1 + 2k \\ &\quad S_k = S01 + \underline{B}P + N \end{aligned}$$

Case-2

$$x=2$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = A \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} = X, \begin{bmatrix} 1 & 1 & 1 \\ H & S & N \\ 0 & H & 1 \end{bmatrix} = B$$

$$x+y+z = 1 \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} = S: A \quad \begin{bmatrix} P & 1 & 1 \\ S & 1 & 0 \\ P & S & 0 \end{bmatrix} = B$$

$$y = 1 - 3k ; z = k$$

$$x = 1 - 1 + 3k - k \quad \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 1 - 3k & P & S \end{bmatrix} = B$$

$$x = 2k \quad \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = B$$

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = S \quad S = (A)^0$$

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = S \quad S = (\emptyset : A)^0$$

d Q.

Find solution?

$$x + 3y - 2z = 0$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 4 \\ 3 & 11 & 14 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 4 \\ 3 & 11 & 14 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P(A) = P(A : B) \neq 0$$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 0 & 7 & 8 \\ 0 & -14 & 16 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 7 & 8 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{infinite solutions.}$$

e) For what values of λ , the equations:

$$3x + 4 - \lambda z = 0$$

$$4x - 2y - 3z = 0$$

$$3\lambda x + 4y + x - z = 0$$

may possess non-trivial solution and solve
in each case

$$\text{a)} A = \begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} = \begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda+3 & 5 & 0 \end{bmatrix}$$

$$R_3 = R_3 + R_1 \quad O = SP - Pd - RSI$$

$$\begin{bmatrix} 12 & 4 & -4\lambda \\ 12 & -6 & -9 \\ 2\lambda+3 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 12 & 4 & -4\lambda \\ 0 & -10 & -9+4\lambda \\ 2\lambda+3 & 5 & 0 \end{bmatrix}$$

$$R_1 = R_1 - R_2 / 12 \quad O = P$$

$$R_2 = R_2 - R_1.$$

$$= \begin{bmatrix} 12 & 4 & -4\lambda & 0 \\ 0 & -10 & -9+4\lambda & 0 \\ 2\lambda+3 & 5 & 0 & 0 \end{bmatrix} \quad P = 1$$

$$R_1 = R_1 + R_2$$

$$\frac{P^3}{P} - \frac{P^3}{2\lambda+3} = P^3 - \frac{(8+8\lambda)}{2\lambda+3} =$$

$$P^3 = SP - \frac{(8+8\lambda)}{2\lambda+3} \Rightarrow 12x - 6y - 9z = 0$$

$$O = SP - \frac{-10y + (-9+4\lambda)z}{6} = 0$$

$$O = \left(\frac{P^3}{P}\right)P - Pd - \left(\frac{P}{6}\right)S = (2\lambda+3)x + 5y = 0$$

$$12x - 6y - 9z = 0 \quad O = PS + Pd - \frac{(2\lambda+3)x}{5} = 0$$

$$12x + \frac{6(2\lambda+3)}{5}x - 9 \left(\frac{10y}{(-9+4\lambda)} \right) = 0$$

$$12x + \frac{6(2\lambda+3)}{5}x + 9 \left(\frac{10y}{(-9+4\lambda)} \right) \left(\frac{2\lambda+3}{5} \right) x = 0$$

$$12 + \frac{6(2\lambda+3)}{5} + \frac{18(2\lambda+3)}{4\lambda-9} = 0.$$

$$12(4\lambda-9)5 + 6(2\lambda+3)(4\lambda-9) + 18(5)(2\lambda+3) = 0$$

(cancel 18)

$$\lambda(\lambda-1)(\lambda-9)(\lambda-1) \quad \lambda(\lambda-1)-9(\lambda-1) \quad (\lambda-9) \quad \text{AKHIL YEDDOO}$$

$$\rightarrow 240\lambda - 640 + 180\lambda + 270 + 48\lambda^2 - 108\lambda + 72\lambda - 162 = 0$$

$$48\lambda^2 + 384\lambda - 432 = 0$$

$$\lambda^2 + 8\lambda - 9 = 0$$

$$\lambda^2 + 9\lambda - \lambda - 9 = 0$$

$$\lambda(\lambda-1) + 9(\lambda-1) = 0$$

$$\lambda = 1$$

$$-n = \begin{bmatrix} \lambda-1 & \varepsilon \\ \varepsilon & \lambda-1-\varepsilon \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \lambda-1 & \varepsilon \\ \varepsilon & \lambda-1-\varepsilon \\ \lambda & \lambda-1-\varepsilon \end{bmatrix} = A$$

$$12n - 6y - 9z = 0$$

$$\begin{bmatrix} 12(-4) - 6y - 9(-2y) = 0 \\ 4P + P - 0 - 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4H - H - S \\ P - \delta - S \\ 0 - 2 - S + \delta \end{bmatrix}$$

$$18 - 5S = 5S \quad y = 0 \quad \text{Trivial solution}$$

$$\lambda = -9$$

$$\begin{bmatrix} 0 & P - \delta - S \\ 0 & KHP - \delta I - y \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 4H - H - S \\ 0 & 4P - \delta I - S \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 10y \\ 0 & 10y \\ 0 & 10y \end{bmatrix} = \frac{10y}{(-9+4\lambda)} = \frac{10y}{-45} = \frac{2y}{-9}$$

$$0 - 2(4P + P - 12n) - 6y - 9z = 0.$$

$$0 = P\delta + \kappa(S + \delta)$$

$$12\left(\frac{4}{3}\right) - 6y - 9\left(\frac{2y}{-9}\right) = 0$$

$$\frac{4}{3} - \frac{2}{3} = 4y - 6y + 2y = 0 \quad 0 = S\delta + \kappa\delta + \kappa\delta$$

$$0 = \frac{4}{3} - \frac{2}{3} = \frac{2}{3} \quad \frac{2}{3} = \frac{\kappa(\delta + \kappa\delta)}{\delta} + \kappa\delta \quad \text{Trivial solution.}$$

$$0 = \frac{2}{3} - \frac{2}{3} = 0 \quad P + \frac{\kappa(\delta + \kappa\delta)}{\delta} + \kappa\delta$$

HW-2

$$[0, 1, 0], [1, 1, 1], [1, 1, 1] \quad \text{Linearly Independent}$$

1) $[1, 0, 0], [1, 1, 0], [1, 1, 1]$

$$\begin{bmatrix} c_1 + c_2 + c_3 & c_2 + c_3 & c_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$c_3 = 0, c_2 = 0, c_1 = 0$$

Unique solution, Linearly Independent.

2) $[7 -3 11 -6], [-56 24 -88 48]$

$$\begin{bmatrix} 7c_1 - 56c_2 & -3c_1 + 24c_2 & 11c_1 - 88c_2 & -6c_1 + 48c_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 8k \\ k \end{bmatrix}, \quad [k \ 0 \ 0] \quad \text{Linearly Independent}$$

$$\begin{bmatrix} 0 & 0 \end{bmatrix} = c_1 \begin{bmatrix} 8 & 1 \end{bmatrix} \quad \text{Linearly Dependent}$$

$$c_1 = 8k$$

$$c_1 = 8k \quad 0 = 8k + 0$$

$$0 = 8k + 0 \quad 0 = 8k$$

3) $[-1 5 0], [16 8 -3], [-64 56 9]$

$$\begin{bmatrix} -c_1 + 16c_2 - 64c_3, 5c_1 + 8c_2 + 56c_3, -3c_2 + 9c_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$-3c_2 + 9c_3 = 0 \quad -c_1 + 16c_2 - 64c_3 = 0$$

$$9c_3 = 3c_2$$

$$c_2 = 3c_3$$

$$\div 3k$$

$$16c_2 = 64c_3 + c_1$$

$$16(3c_3) = 64c_3 + c_1$$

$$\text{Vector} = \begin{bmatrix} 0 \\ 3k \\ k \end{bmatrix} = k \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix} \quad \text{Linearly dependent}$$

Final answer: Linearly independent

JHNA

AKHIL VEDDA

4) $\begin{bmatrix} 1 & -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$

$$\begin{bmatrix} c_1 + c_2 - c_3 & -c_1 + c_2 + c_3 + c_4 & c_1 - c_2 + c_3 \end{bmatrix}$$

$$\begin{bmatrix} c_1 + c_2 - c_3 = 0 & -c_1 + c_2 + c_3 + c_4 = 0 \\ c_1 + c_2 = c_3 & c_1 = c_2 + c_3 + c_4 \\ c_1 + c_1 = 0 & c_4 = -2k \\ c_1 = 0 & c_1 - c_2 + c_3 = 0 \end{bmatrix}$$

$$\begin{bmatrix} 3H - 8E - 4G & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 11E - 13G & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} c_1 = 0 & c_2 = c_1 + c_3 \\ c_2 = c_3 = k & \end{bmatrix}$$

$$\text{Vector} = k \begin{bmatrix} 0 \\ H \\ 1 \\ 2 \end{bmatrix}$$

Linearly dependent.

5) $\begin{bmatrix} 2 & -4 \end{bmatrix}, \begin{bmatrix} 1 & 9 \end{bmatrix}, \begin{bmatrix} 3 & 5 \end{bmatrix}, \begin{bmatrix} 5 & 5 \end{bmatrix}$

$$\begin{bmatrix} 2c_1 + c_2 + 3c_3 = -4c_1 + 9c_2 + 8c_3 = 0 \\ 0 = 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$2c_1 + c_2 + 3c_3 = 0$$

$$4c_1 + 2c_2 + 6c_3 = 0$$

$$11c_2 + 11c_3 = 0$$

$$c_2 = k, c_2 = -c_3 = -k$$

$$4c_1 = 9c_2 + 5c_3 \Rightarrow \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} =$$

$$4c_1 = 9(k) - 5k \quad 0 = 30P + 85E - 50G$$

$$4c_1 = 4k \quad 30E = 30P$$

$$4c_1 = 4k \Rightarrow c_1 = k \quad 30E = 30$$

$$\text{Vector} = \begin{bmatrix} k \\ 9k \\ -5k \\ 5k \\ k \end{bmatrix} = k \begin{bmatrix} 1 \\ 9 \\ -5 \\ 5 \\ 1 \end{bmatrix}$$

Linearly dependent

$$6) \begin{bmatrix} 3 & 8 \\ 3 & -2 \end{bmatrix}, \begin{bmatrix} 3 & 8 & 4 \end{bmatrix}, \begin{bmatrix} 5 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 5 & 10 & 8 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3c_1 + 5c_2 - 6c_3 + 2c_4 & -2c_1 + c_3 & 0 & 0 & 0 & 4c_1 + c_2 + c_3 + 3c_4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

$$-2c_1 + c_3$$

$$2c_1 = c_3$$

$$c_1 = k.$$

$$c_3 = 2c_1$$

$$= 2k$$

$$0 = \mu 2k + 8k + 8k + 13k$$

$$0 = \mu 2k + 8k + 13k$$

$$4c_1 + c_2 + c_3 + 3c_4 = 0$$

$$20c_1 + 5c_2 + 5c_3 + 15c_4 = 0$$

$$\underline{3c_1 + 5c_2 - 6c_3 + 2c_4 = 0}$$

$$17c_1 + 11c_2 + 13c_4 = 0$$

$$0 = \mu 2k + 18k + 11(2k) = 13c_4$$

$$4c_1 + c_2 + c_3 + 3c_4 = 0 \quad (17 + 22)k = 13c_4$$

$$4k + c_2 + 2k + 3(3k) = 0$$

$$0 = \mu 2k + 8k + 3(3k) \quad \frac{13}{13}k = c_4 = 3k$$

$$c_2 + 15k = 0$$

$$\underline{c_2 = -15k}$$

$$0 = \mu 2k + 8k + 13k$$

$$\text{vector} = \begin{bmatrix} k \\ 5k \\ 2k \\ 3k \end{bmatrix} = k \begin{bmatrix} 1 \\ 5 \\ 2 \\ 3 \end{bmatrix}$$

~~Linearly dependent~~

$$6) \begin{bmatrix} 6 & 0 & 3 & 1 & 4 & 2 \end{bmatrix}, \begin{bmatrix} 0 & -1 & 2 & 7 & 0 & 5 \end{bmatrix},$$

$$0 = \mu 2k + 8k + 13k$$

$$0 = \mu 2k + 8k + 13k$$

$$\begin{bmatrix} 12 & 3 & 0 & -19 \\ 8 & -11 \end{bmatrix}$$

$$\begin{bmatrix} 6c_1 + 12c_3 & -1c_2 + 3c_3 & 3c_1 + 2c_2 & c_1 + 7c_2 - 19c_3 \\ 4c_1 + 8c_3 & 5c_2 - 11c_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$c_1 + 2c_3 = 0$$

$$c_2 + 3c_3 = 0$$

$$c_2 = 3c_3$$

$$\underline{c_2 = 0}$$

$$c_1 + 2c_3 = 0 - \cancel{x} \left(\frac{5k}{15} \right) 3c_1 + 2c_2 = 0$$

$$c_1 + 7c_2 - 19c_3 = 0 \quad 15c_3 - 11c_3 = 0$$

$$\cancel{c_1 = 0}$$

$$12k + 11k = 4c_3 = 0$$

~~Linearly independent~~ $\boxed{c_3 = 0}$

$$\Rightarrow [3 \ 4 \ 7], [2 \ 0 \ 3], [8 \ 2 \ 3], [5 \ 6 \ 6]$$

$$[3c_1 + 2c_2 + 8c_3 + 5c_4 \quad 4c_1 + 2c_3 + 5c_4 \quad 7c_1 + 3c_2 + 3c_3] \\ [0 \ 0 \ 0 \ 0] \rightarrow [0 \ 0 \ 0 \ 0]$$

$$3c_1 + 2c_2 + 8c_3 + 5c_4 = 0 \quad [0 \ 0 \ 0 \ 0] \\ -4c_1 + 2c_3 + 5c_4 = 0$$

$$0 = \mu - c_1 + 2c_2 + 6c_3 = 0$$

$$0 = \mu - 8c_1 + 8c_2 + 8c_3 = 0$$

$$0 = \mu - 8c_1 + 8c_2 + 8c_3 = 0$$

$$5c_1 = 10c_2 + 30c_3$$

$$(x2) \quad [7c_1 + 3c_2 + 3c_3 + 6c_4 = 0] \quad NS = \\ [4c_1 + 6c_2 + 6c_3 + 12c_4 = 0] \quad NS = \\ 9c_1 + 6c_2 + 24c_3 + 15c_4 = 0 \quad NS = \\ 5c_1 = 18c_3 + 3c_4 \quad NS = \\ 10c_2 + 30c_3 = 18c_3 + 3c_4 \quad NS = \\ 12c_3 = 3c_4 - 10c_2 \quad NS =$$

$$(x4) \quad [4c_1 + 2c_3 + 5c_4 = 0] \quad NS = \\ [36c_1 + 18c_3 + 45c_4 = 0] \quad NS =$$

$$6c_1 + 18c_3 - 3c_4 = 0 \quad [0 \ 0 \ 0 \ 0] \quad NS = \\ 41c_1 + 42c_3 = 0$$

$$[0 \ 0 \ 0 \ 0] \quad 41c_1 = -42c_3$$

$$c_1 = \frac{-42c_3}{41} \Rightarrow \frac{42}{41}K \quad NS =$$

$$18c_3 = 5c_1 - 3c_4 \quad NS =$$

$$c_3 = 5 \left(\frac{-42}{41} \right) K - 3K \left(\frac{1}{18} \right)$$

$$= \frac{(-310)(8) - 6(41))K}{41 \times 16} = \frac{(1260 + 216)K}{216}$$

$$= \frac{1506}{216} K$$

$$\frac{-42}{41} k = 2c_2 + b^1 \left(\frac{251}{36} \right) k \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & \frac{1}{36} \end{bmatrix} = A \quad \text{LHS}$$

$$2c_2 = -\frac{42k}{41} - \frac{251}{36} k \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & k-1 & \frac{1}{36} \end{bmatrix} = \text{RHS}$$

$$c_2 = \frac{(252 + 10291)0}{(2 \times 246)} k = \frac{10543k}{492}$$

$$\therefore \text{vector} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -42(41)(k-1) \\ 10543/492 \\ (1+2+k(k-1)-k(k-1)) \\ 1566/216 \end{bmatrix}$$

(Ex: linearly dependent.)

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -2 & 0 & 6 \end{bmatrix} \quad \text{Solve } A - \lambda I = 0 \quad \text{LHS}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad A - \lambda I \Rightarrow \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -2 & 0 & 6-\lambda \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{RHS}$$

$$0 = S \Rightarrow (-2-\lambda)(-1-\lambda) + 6(-2) \quad 0 = S + \lambda I$$

$$0 = P - \lambda S \Rightarrow -2(-\lambda(2) + 6(-1)) \quad 0 = P + \lambda S$$

$$0 = Q - \lambda S \Rightarrow -3(-4+1-\lambda) \quad 0 = Q + \lambda S$$

$$= (-2-\lambda)(-\lambda + \lambda^2 - 12) + 2(\lambda + 6) + 3(3+\lambda)$$

$$\Rightarrow -2\lambda^3 - 2\lambda^2 + 24 + \lambda^2 - \lambda^3 + 12\lambda + 2\lambda + 12 + 9 + 3\lambda$$

$$\Rightarrow 19\lambda^3 - \lambda^2 - \lambda^3 + 45 \quad 0 = S$$

$$\lambda^3 + \lambda^2 - 19 - 45 = 0 \quad \text{LHS}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$2) A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix}$$

$$(4-\lambda)(1-\lambda)(1-\lambda) + 1(2(1-\lambda))$$

$$= (1-\lambda)((4-\lambda)(1-\lambda)+2)$$

$$\Rightarrow (4-\lambda-4\lambda+\lambda^2+2)$$

$$\Rightarrow (\lambda^2-5\lambda+6)$$

$$(1-\lambda)(\lambda-2)(\lambda-3)$$

$$\lambda = 1, 2, 3$$

$$\underline{\lambda = -1}$$

$$\begin{bmatrix} 5 & 0 & 1 \\ -2 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} n \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & 1 \\ -2 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix} = A$$

$$\lambda = 3.$$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} n \\ y \\ z \end{bmatrix}$$

$$5n+2=0$$

$$-2n+2y=0$$

$$-2n+2z=0$$

$$10n+2z=0$$

$$(5-1)n + (2-1)y - (1-1)z = 0$$

$$-2n-2y=0$$

$$-2n-2z=0$$

$$n = -z = -k$$

$$(-12n=0) \quad (2+1)s + (3-2k+k-1)y + (2-k)z = 0$$

$$z = k$$

$$y = k$$

$$\text{vector} = k \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$\underline{\lambda = 2}$$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} n \\ y \\ z \end{bmatrix}$$

$$2n+z=0$$

$$-2n-y=0$$

$$-2n-z=0$$

$$n=k$$

$$y=-2k$$

$$z=3k$$

$$\therefore \text{vector} = k \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

$$3) A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{AKHIL YEDDU}$$

$$A - \lambda I = \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix} \rightarrow \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix} = 0$$

$$(5-\lambda)(-\lambda)(3-\lambda) = 0$$

$$\underline{\lambda = 0, 3, 5.}$$

$$\underline{\lambda = 0}$$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$0 = 5x \Rightarrow x = 0$
 $0 = 0 \Rightarrow 0 = 0$
 $-x + 3z = 0 \Rightarrow z = 0$
 $\cancel{-3y = 0}$
 $\cancel{-x = 0}$
 $\cancel{-x + 3z = 0} \Rightarrow \cancel{z = 0}$
 $\cancel{x = 0}$
 $\cancel{y = 0}$
 $\cancel{0 = 0}$
 $\text{vector} = k \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\underline{\lambda = 3}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x = 0 \Rightarrow x = 0$
 $-3y = 0 \Rightarrow y = 0$
 $-x + 3z = 0 \Rightarrow z = 0$
 $\cancel{2x = 0}$
 $\cancel{-3y = 0}$
 $\cancel{-x + 3z = 0}$
 $\text{vector} = k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$$\underline{\lambda = 5}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-5y = 0 \Rightarrow y = 0$
 $x - 2z = 0 \Rightarrow x = 2z$
 $x = 2z \Rightarrow z = -2k$
 $\text{vector} = k \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$
 $0 = 5x + b^2$

$$x^2 + y^2 + z^2 = 0$$

$$x^2 + y^2 = b^2$$

$$\left(\frac{x}{b}, \frac{y}{b} \right) = \left[\begin{array}{c} x \\ y \end{array} \right] \cdot \text{mod } \mathbb{R}^2$$

$$4) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 0-\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0-2-\lambda & \end{bmatrix}$$

$$\lambda(3-\lambda)(2+\lambda) = 0$$

$$\lambda = 0, 3, -2$$

$$\lambda = 0$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$3y + 4z = 0 \quad z = 0, y = 0$$

$$0+0-2z = 0 \quad 0 = 0$$

$$\text{vector} = k \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{rank } A = 2$$

$$\lambda = 3 - 0$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$4z = 0 \quad z = 0$$

$$-5z = 0 \quad z = 0$$

$$\text{vector} = k \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda = -2$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$5y + 4z = 0$$

$$5y = -4z = -4k$$

$$y = \frac{-4}{5} k$$

$$\therefore \text{vector} = \begin{bmatrix} 0 \\ -4/5k \\ k \end{bmatrix} = k \begin{bmatrix} 0 \\ -4/5 \\ 1 \end{bmatrix}$$