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Maths Assignments

1) $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ $\Rightarrow$ $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$

~~$R_3 \to R_3 - 3R_3$~~

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 4R_1$

$R_4 \to R_4 - R_3$

$\Downarrow$

$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Leftarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$

$R_4 \to R_4 - R_2$

Non of non zero rows = Rank (A)

$\therefore$ Rank (A) = 3

4) $3x - 0.1y - 0.2z = 7.85$

$0.1x + 7y - 0.3z = -19.3$

$0.3x - 0.2y + 10z = 71.4$

$x = \frac{1}{3}[7.85 + 0.1y + 0.2z]$

$y = \frac{1}{7}[-19.3 - 0.1x + 0.3z]$

$z = \frac{1}{10}[71.4 - 0.3x + 0.2y]$

Iteration 1 :

$z = y = 0$

$x_1 = \frac{7.85}{3} = 2.61$

$y_1 = \frac{1}{7}(-19.3 - 0.1(2.61)) = 2.79$

$z_1 = \frac{1}{10}[71.4 - 0.3(2.61) + 0.2(2.79)] = 7.1175$

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Iteration 2

$$x_2 = \frac{7.86 - 0.1(2.61) - 0.2(7.1408)}{3}$$

$$x_2 = 2.9255$$

$$y_2 = \frac{19.3 - 0.1(2.92) - 0.3(7.1408)}{7}$$

$$y_2 = 3.0123$$

$$z_2 = \frac{71.4 - 0.3(2.9255) - 0.2(3.0123)}{10}$$

$$z_2 = 7.0132$$

Iteration 3

$$x_3 = \frac{7.85 - 0.1(2.9255) - 0.2(3.0123)}{3}$$

$$x_3 = 3.0032$$

$$y_3 = \frac{19.3 - 0.1(3.0032) - 0.3(7.0132)}{7}$$

$$y_3 = 3.001$$

$$z_3 = \frac{71.4 - 0.3(3.0032) - 0.2(3.0001)}{10}$$

$$z_3 = 7.00$$

$x = 3.0032$, $y = 3.0001$; $z = 7.00$

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5) $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix}$ $R_2 \Rightarrow R_2 - 2R_1$
$R_3 \Rightarrow R_3 - 3R_1$
$R_4 \Rightarrow R_4 - R_1$

$\Rightarrow \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \Rightarrow R_3 - 2R_2$

$\Rightarrow \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ This corresponds to system:

$x + 3y + 2z = 0$

$-7y - z = 0$

Let

$y = t$

$x = -3t$

$z = -7t$

$\therefore$ System has infinitely many solutions.

$\therefore$ System is consistent and dependent

2) $T : W \to P_2$.

$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$.

~~Let A~~ $= \left[\cancel{a} \; b \right.$

$T\left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = (a-b) + (b-c)x + (c-a)x^2$

$\cancel{= a}$

Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$T(A) = (a-b) + (b-c)x + (c-a)x^2$

$= a - bx + c(x^2 - x + 1)$

∴ The image of T is the set of all polynomials of degree at most 2, denoted as P_2.

Rank of T :

The rank of T is the dimension of its image since P_2 has a dimension of 3 (coefficients for x^0, x^1 and x^2) the rank of T is 3

The Null space of symmetric matrix,

$$T(A) = 0.$$

this leads to the system of equations.

$a - b = 0$

$b - c = 0$

$c - a = 0$

$\therefore\ a = b = c$

∴ T is the set of symmetric matrices of the form

$\begin{bmatrix} t & t \\ t & t \end{bmatrix}$ where t is any scalar.

∴ Dimension = 1 (using only t)

∴ Rank (T) is 3

The nullity of T is 1

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37) $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$$A^{-1} = \frac{1}{4-1}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2/3-\lambda & 1/3 \\ 1/3 & 2/3-\lambda \end{bmatrix} \Rightarrow \left(\frac{2}{3}-\lambda\right)^2 - \left(\frac{1}{3}\right)^2 = 0$$

$a^2 - b^2 = (a+b)(a-b)$

$\left(\frac{1}{3}-\lambda\right)(1-\lambda) = 0$

$\lambda = \frac{1}{3}, 1$

$\lambda = 1/3$

$$\Rightarrow \begin{bmatrix} 1/3 & 1/3 \\ 1/3 & 1/3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$x = -y \rightarrow$ Eigen vector $= \begin{bmatrix} k \\ -k \end{bmatrix} \Rightarrow k\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\lambda = 1$

$$\Rightarrow \begin{bmatrix} -1/3 & +1/3 \\ +1/3 & -1/3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$x = y \rightarrow$ Eigen vector $= \begin{bmatrix} k \\ k \end{bmatrix} \Rightarrow k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$Z = A + 4I$

$$\Rightarrow \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\cancel{Z-4}\ Z - \lambda I = \begin{bmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}$$

$(6-\lambda)^2 - (1)^2 \Rightarrow (6-\lambda-1)(6-\lambda+1)$

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$\Rightarrow (5-\lambda)(7-\lambda)=0$

$\lambda = 5, 7$

$\lambda = 5$

$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$x = y \rightarrow$ Eigen vector $= \begin{bmatrix} k \\ k \end{bmatrix} \rightarrow k \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda = 7$

$\Rightarrow \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$x = -y \rightarrow$ Eigen vector $= \begin{bmatrix} k \\ -k \end{bmatrix} \rightarrow k \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$

6> $T : P_2 \rightarrow P_2$ is linear transformation

$T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1)x^2$

$\rightarrow T(a + bx + c) = (a+1) + (b+1)x + (c+1)x^2$ is a linear transformation, we need to check two properties.

1> Additivity : $T(U+V) = T(U) + T(V)$

2> Homogenity of degree 1 :

$T(kU) = k\,T(U)$ for all in domain of T and all scalars k.

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$T(U+V) = T(a_1+b_1x+c_1) + (a_2+b_2x+c_2)$

$= T((a_1+a_2)+(b_1+b_2)x+(c_1+c_2))$

$\Rightarrow (a_2+a_1+1)+(b_1+b_2+1)x+(c_1+c_2+1)x^2$

$\Rightarrow (a_1+1)+(b_1+1)x+(c_1+1)x^2+(a_2+1)$

$+(b_2+1)x+(c_2+1)x^2$

$T(a_1+b_1x+c_1)+T(a_2+b_2x+c_2)$

So Function is additive.

$\therefore$ Homogenity of Degree : 1.

$T(kU) = T(k(a+bx+c))$

$= T(ka+kbx+kc)$

$\Rightarrow (ka+1)+(kb+1)x+(kc+1)x^2)$

$\Rightarrow kT(a+bx+c)$

So the function is homogenous of degree 1.

$\therefore$ It is indeed Linear transformation.

7) $S=\{(1,2,3),(3,1,0),(-2,1,3)\}$ is a basis.

It can be arranged as matrix.

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$$

$\therefore$ Row reduction to obtain echelon form.

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_3 \to R_3 + \frac{9}{5}R_2$

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∴ Third row of zeroes indicates that the vectors in S are linearly dependent.

For basis of the subspace spanned by S.

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \end{bmatrix}$$

(1,3,2) and (0,-5,5) these vectors form a basis for the subspace spanned by S.

∴ Dimension of subspace spanned by S = 2

∴ Set S is not a basis of R^3 because the row reduced form has a row of zeroes.

∴ the basis for the subspace spanned by S is $\{(1,3,-2), (0,-5,5)\}$.

∴ The dimension of the subspace is 2

8) $3x - 6y + 2z = 23$
$-4x + y - z = -15$
$x - 3y + 7z = 16$

with initial values

$x_0 = 1, y_0 = 1, z_0 = 1$

Iteration-1

$$x_1 = \frac{23 + 6y_0 - 2z_0}{3} = \underline{\underline{9}}$$

$$y_1 = \frac{-15 + 4x_0 + z_0}{1} = \underline{\underline{-9}}$$

$$z_1 = \frac{16 - x_0 - 3y_0}{7} = \underline{\underline{2}}$$

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Iteration 2

$$x_2 = \frac{23 + 6y_1 - 2z_1}{3} = \underline{\underline{5}}$$

$$y_2 = \frac{-15 + 4x_1 + z_1}{1} = \underline{\underline{-5}}$$

$$z_2 = \frac{16 - x_1 + 3y_1}{7} = \underline{\underline{3}}$$

Iteration 3,

$$x_3 = \frac{23 + 6y_2 - 2z_2}{3} = \underline{\underline{6}}$$

$$y_3 = \frac{-15 + 4x_2 + z_2}{1} = \underline{\underline{-6}}$$

$$z_3 = \frac{16 - x_2 + 3y_2}{7} = \underline{\underline{2}}$$

$\therefore x, y, z = (6, -6, 2)$

HW-1

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a)

1) $2x-3y+7z=5$

$3x+y-3z=13$

$2x+19y-47z=32$

$$A=\begin{bmatrix}2 & -3 & 7\\ 3 & 1 & -3\\ 2 & 19 & -47\end{bmatrix},\quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}\quad B=\begin{bmatrix}5\\ 13\\ 32\end{bmatrix}$$

~~$R_2 \to R_2 - \frac{3}{2}R_1$~~

$\downarrow$ $R_3 \to R_3 - R_1$

$$\begin{bmatrix}2 & -3 & 7\\ 3 & 1 & -3\\ 0 & 22 & -54\end{bmatrix}\quad\begin{bmatrix}5\\ 13\\ 27\end{bmatrix}$$

$R_2 \to R_2 - \frac{3}{2}R_1$

$\downarrow$

$$\begin{bmatrix}2 & -3 & 7\\ 0 & 11/2 & -27/2\\ 0 & 22 & -54\end{bmatrix}\quad\begin{bmatrix}5\\ 11/2\\ 27\end{bmatrix}$$

$$\begin{bmatrix}2 & -3 & 7\\ 0 & 11/2 & -27/2\\ 0 & 0 & 0\end{bmatrix}\quad\begin{bmatrix}5\\ 11/2\\ 5\end{bmatrix}$$

Rank (A) = 2

Rank (A : B) = 3

$P(A) \neq P(A:B)$ Inconsistent

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ii) $2x - y + 3z = 8$
$-x + 2y + z = 4$
$3x + y - 4z = 0$

$A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$

$A = \begin{bmatrix} -1 & 2 & 1 \\ 2 & -1 & 3 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 0 \end{bmatrix}$

$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 7 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$

$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 7 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$

$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & -38/5 \end{bmatrix} \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$

$\rho(A) = \rho(A:B)$ consistent.

iii) $4x - y = 12$
$-x + 5y - 2z = 0$
$-2x + 4z = -8$

$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -4 & 0 & 8 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix}$

~~$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & -1 & 8 \end{bmatrix}$ $B = \begin{bmatrix} 12 \\ 0 \\ -16 \end{bmatrix}$~~

$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -4 & 0 & 8 \end{bmatrix} \begin{bmatrix} 12 \\ 0 \\ -16 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & -1 & 8 \end{bmatrix} \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$

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$$A = \begin{bmatrix} 4 & -1 & 0 \\ -4 & 20 & -8 \\ 0 & -1 & 8 \end{bmatrix} \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & -1 & 8 \end{bmatrix} \begin{bmatrix} 12 \\ 12 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & 18 & 0 \end{bmatrix} \begin{bmatrix} 12 \\ 12 \\ 8 \end{bmatrix}$$

$\rho(A) = \rho(A:B)$

consistent.

b).

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = \mu$

Q) For what values of λ and μ

i) No solution

ii) Unique solution

iii) Infinite solution.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \lambda-3 \end{bmatrix}.$$

$$A:B = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

i) No solution $(\rho(A) \neq \rho(A:B))$

$\underline{\lambda = 3, \ \mu \neq 10}$

ii) Unique solution $(\rho(A) = \rho(A:B))$

$\underline{\lambda \neq 3, \ \mu \text{ any}}$

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iii) Infinite solution $(\rho(A) = \rho(A:B)) < n$ ~~(order)~~
(no of unknowns)

$\lambda = 3 ; \mu = 10$

$n \to x, y, z$ ie $\underline{\underline{3}}$.

CQ). $x + y + z = 1$
$x + 2y + 4z = \lambda$.
$x + 4y + 10z = \lambda^2$

For what value of λ have a solution & Solve for each value of λ.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{bmatrix} \qquad A:B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{bmatrix}$$

$\rho(A) = 2$

$\rho(A:B) = 2$.

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-1-3\lambda+3 \end{bmatrix}$$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda^2 - 2\lambda - \lambda + 2 = 0$

$\lambda(\lambda-1) - 2(\lambda-1) = 0$

$(\lambda-1)(\lambda-2) = 0$

$\underline{\lambda = 1, 2}$

$\rho(A) = \rho(A:B) \neq n$.

3 unknow

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case-1

$\lambda = 1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x + y + z = 1$

$y + 3z = 0$

$z = k$ where $x = 1 - y - z$

$y = -3k$ where $= 1 + 3k - k = \underline{1 + 2k}$

case-2

$\lambda = 2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x + y + z = 1$

$y + 3z = 1$

$y = 1 - 3k \; ; \; z = k$

$x = 1 - 1 + 3k - k$

$\underline{x = 2k}$

d Q) Find solution!

$x + 3y - 2z = 0$

$2x - y + 4z = 0$

$x - 11y + 14z = 0$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & -14 & 16 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = \rho(A:B) \neq n$

infinite solutions.

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e) For what values of λ, the equations:

$$3x + y - \lambda z = 0$$
$$4x - 2y - 3z = 0$$
$$2\lambda x + 4y + \lambda z = 0$$

may possess non trivial solution and solve in each case

A) $A = \begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} = \begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda+3 & 5 & 0 \end{bmatrix}$

$R_3 = R_3 + R_1$

$\begin{bmatrix} 12 & 4 & -4\lambda \\ 12 & -6 & -9 \\ 2\lambda+3 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 12 & 4 & -4\lambda \\ 0 & -10 & -9+4\lambda \\ 2\lambda+3 & 5 & 0 \end{bmatrix}$

$R_1 = 4R_1$ | $R_2 = 3R_2$ $\qquad R_2 = R_2 - R_1$.

$= \begin{bmatrix} 12 & 4 & -4\lambda & 0 \\ 0 & -10 & -9+4\lambda & 0 \\ 2\lambda+3 & 5 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 12 & -6 & -9 & 0 \\ 0 & -10 & -9+4\lambda & 0 \\ 2\lambda+3 & 5 & 0 & 0 \end{bmatrix}$

$R_1 = R_1 + R_2$

$\Rightarrow 12x - 6y - 9z = 0$

$-10y + (-9+4\lambda)z = 0$

$(2\lambda+3)x + 5y = 0$

$\dfrac{-(2\lambda+3)x}{5} = y$

$12x - 6y - 9z = 0$

$12x + \dfrac{6(2\lambda+3)x}{5} - 9\left(\dfrac{10y}{(-9+4\lambda)}\right) = 0$

$12x + \dfrac{6(2\lambda+3)}{5}x + 9\left(\dfrac{\cancel{10}\,2}{(-9+4\lambda)}\right)\left(\dfrac{2\lambda+3}{\cancel{5}\,1}\right)x = 0.$

$12 + \dfrac{6(2\lambda+3)}{5} + \dfrac{18(2\lambda+3)}{4\lambda-9} = 0.$

$12(4\lambda-9)5 + \underbrace{6(2\lambda+3)}_{(12\lambda+18)}(4\lambda-9) + 18(5)(2\lambda+3) = 0$

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$x(\lambda-1)-9(x-1)$ $(\lambda-9$

$\Rightarrow 240\lambda - 840 + 180\lambda + 270 + 48\lambda^2 - 108\lambda + 72\lambda - 162 = 0$

$48\lambda^2 + 384\lambda - 432 = 0$

$\lambda^2 + 8\lambda - 9 = 0$

$\lambda^2 + 9\lambda - \lambda - 9 = 0$

$\lambda(\lambda-1) + 9(\lambda-1) = 0$

$\underline{\lambda = 1}$

$-x = y$, $z = -2y$

$12x - 6y - 9z = 0$

$12(-y) - 6y - 9(-2y) = 0$

$18y - 18y = 0$

$y = 0$ Trivial solution

$\underline{\lambda = -9}$

$y = \left(-\frac{(2\lambda+3)x}{5}\right)$ $z = \frac{10y}{(-9+4\lambda)}$

$= -\frac{(-18+3)}{5} x = 3x$ $= \frac{10y}{-45} = \frac{2y}{-9}$

$12x - 6y - 9z = 0.$

$12\left(\frac{4}{3}\right) - 6y - 9\left(\frac{2y}{-9}\right) = 0$

$4y - 6y + 2y = 0$

$\underline{\underline{y = 0}}$ Trivial solution.

HW - 2

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1) $[1,0,0]$, $[1,1,0]$, $[1,1,1]$

$[c_1 + c_2 + c_3 \quad c_2 + c_3 \quad c_3] = [0 \; 0 \; 0]$

$c_3 = 0$, $c_2 = 0$, $c_1 = 0$

unique solution, Linearly Independent.

2) $[7 \; -3 \; 11 \; -6]$, $[-56 \; 24 \; -88 \; 48]$

$[7c_1 - 56c_2 \quad -3c_1 + 24c_2 \quad 11c_1 - 88c_2 \quad -6c_1 + 48c_3]$

$= [0 \; 0 \; 0 \; 0]$

$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 8k \\ k \end{bmatrix} = k\begin{bmatrix} 8 \\ 1 \end{bmatrix}$

$7c_1 = 56c_2$ Linear ~~Deti~~ Dependent.

$c_1 = 8c_2$

$\underline{c_1 = 8k}$

3) $[-1 \; 5 \; 0]$, $[16 \; 8 \; -3]$, $[-64 \; 56 \; 9]$

$[-c_1 + 16c_2 - 64c_3, 5c_1 + 8c_2 + 56c_3, -3c_2 + 9c_3]$

$= [0 \; 0 \; 0]$

$-3c_2 + 9c_3 = 0$ $\qquad -c_1 + 16c_2 - 64c_3 = 0$

$9c_3 = 3c_2$ $\qquad 16c_2 = 64c_3 + c_1$

$c_2 = 3c_3$ $\qquad 16(3c_3) = 64c_3 + c_1$

$= \underline{3k}$ $\qquad \underline{c_1 = 0}$

vector $= \begin{bmatrix} 0 \\ 3k \\ k \end{bmatrix} = k\begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$ Linear dependent.

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4> $[1\ -1\ 1], [1\ 1\ -1], [-1\ 1\ 1], [0\ 1\ 0]$

$[c_1+c_2-c_3 \quad -c_1+c_2+c_3+c_4 \quad c_1-c_2+c_3]$

$c_1+c_2-c_3=0$

$c_1+c_2=c_3$

$c_1+c_1=0$

$2c_1=0$

$\underline{\underline{c_1=0}}$

$-c_1+c_2+c_3+c_4=0$

$c_1=c_2+c_3+c_4$

$c_4=-2k$

$c_1-c_2+c_3=0$

$c_2=c_1+c_3$

$c_2=c_3=k$

$$\text{vector} = k\begin{bmatrix}0\\1\\1\\2\end{bmatrix}$$

Linearly dependent

5> $[2\ -4], [1\ 9], [3\ 5]$

$[2c_1+c_2+3c_3, \ -4c_1+9c_2+5c_3] = [0\ 0]$

$2c_1+c_2+3c_3=0$

$4c_1+2c_2+6c_3=0$

$9c_2+5c_3+2c_2+6c_3=0$

$11c_2+11c_3=0$

$\cancel{c_2=c}$ $c_2=-c_3=-k$

$4c_1=9c_2+5c_3$

$4c_1=9(k)-5k$

$4c_1=4k$

$\underline{\underline{c_1=k}}$

$$\text{vector} = \begin{bmatrix}k\\k\\-k\end{bmatrix} = k\begin{bmatrix}1\\1\\-1\end{bmatrix}$$

Linearly dependent

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6> $[3\ -2\ 0\ 4], [5\ 0\ 0\ 1], [-6\ 1\ 0\ 1], [2\ 0\ 0\ 3]$

$[3c_1 + 5c_2 - 6c_3 + 2c_4 \quad -2c_1 + c_3 \quad 0 \quad 4c_1 + c_2 + c_3 + 3c_4]$

$= [0\ \ 0\ \ 0\ \ 0]$

$-2c_1 + c_3$

$2c_1 = c_3$

$c_1 = k.$

$c_3 = 2c_1$

$= 2k$

$4c_1 + c_2 + c_3 + 3c_4 = 0$

$20c_1 + 5c_2 + 5c_3 + 15c_4 = 0$

$3c_1 + 5c_2 - 6c_3 + 2c_4 = 0$

$-\quad\ -\quad +\quad -$

$17c_1 + 11c_3 - 13c_4 = 0$

$17k + 11(2k) = 13c_4$

$(17 + 22)k = 13c_4$

$\frac{\cancel{39}k}{\cancel{13}} = c_4 = 3k$

$4c_1 + c_2 + c_3 + 3c_4$

$4k + c_2 + 2k + 3(3k) = 0.$

$c_2 + 15k = 0$

$\underline{c_2 = -15k}$

$$\text{vector} = \begin{bmatrix} k \\ -15k \\ 2k \\ 3k \end{bmatrix} = k \begin{bmatrix} 1 \\ -15 \\ 2 \\ 3 \end{bmatrix},$$

Linearly dependent.

8> $[6\ 0\ 3\ 1\ 4\ 2], [0\ -1\ 2\ 7\ 0\ 5], [12\ 3\ 0\ -19\ 8\ -11]$

$[6c_1 + 12c_3 \quad -1c_2 + 3c_3 \quad 3c_1 + 2c_2 \quad c_1 + 7c_2 - 19c_3 \quad 4c_1 + 8c_3 \quad 5c_2 - 11c_3] = [0\ 0\ 0\ 0\ 0\ 0]$

$c_1 + 2c_3 = 0$

$c_2 + 3c_3 = 0$

$c_2 = 3c_3$

$\underline{c_2 = 0}$

$c_1 + 2c_3 = 0$

$c_1 + 7c_2 - 19c_3 = 0$

$\underline{c_1 = 0}$

$3c_1 + 2c_2 = 0$

$5c_2 - 11c_3 = 0$

$15c_3 - 11c_3 = 0$

$4c_3 = 0$

$\therefore$ Linearly Independent $\boxed{c_3 = 0}$

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7) $[3\ 4\ 7], [2\ 0\ 3], [8\ 2\ 3], [5\ 5\ 6]$

$[3c_1+2c_2+8c_3+5c_4 \quad 4c_1+2c_3+5c_4 \quad 7c_1+3c_2+3c_3+6c_4]$

$= [0\ 0\ 0]$

$3c_1+2c_2+8c_3+5c_4=0$
$4c_1+2c_3+5c_4=0$

$-c_1+2c_2+6c_3=0$

$c_1=2c_2+6c_3$

$5c_1=10c_2+30c_3$

x2 $7c_1+3c_2+3c_3+6c_4=0$
$\to 14c_1+6c_2+6c_3+12c_4=0$
$9c_1+6c_2+24c_3+15c_4=0$

$5c_1-18c_3-3c_4=0$

$5c_1=18c_3+3c_4$

$10c_2+30c_3=18c_3+3c_4$

$12c_3=3c_4-10c_2$

x4 $4c_1+2c_3+5c_4=0$
$\to 36c_1+18c_3+45c_4=0$
$5c_1-18c_3-3c_4=0$

$41c_1+42c_4=0$

$41c_1=-42c_4$

$c_1=\frac{-42c_4}{41} \Rightarrow \frac{-42}{41}k$

$18c_3=5c_1-3c_4$

$c_3=5\left(\frac{-42}{41}\right)k-3k\left(\frac{1}{18}\right)$

$=\frac{(-210(18)-6(41))k}{41\times18}=\frac{(1260+216)k}{216}$

$=\frac{1806}{216}k$

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$$\frac{-42}{41}k = 2c_2 + 6\left(\frac{251}{36}\right)k$$

$$2c_2 = \frac{-42k}{41} - \frac{251}{6}k$$

$$c_2 = -\left(\frac{(252+10291)}{2 \times 246}\right)k = \frac{10543k}{492}$$

$$\therefore \text{vector} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -42/41 \\ 10543/492 \\ 1 \\ 1566/216 \end{bmatrix}$$

$\therefore$ Linearly dependent.

2)
$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$$A - \lambda I \Rightarrow \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix}$$

$$\Rightarrow (-2-\lambda)(-\lambda(1-\lambda) + 6(-2))$$

$$\Rightarrow -2(-\lambda(2) + 6(-1))$$

$$\Rightarrow -3(-4+1-\lambda)$$

$$= (-2-\lambda)(-\lambda + \lambda^2 - 12) + 2(\lambda+6) + 3(3+\lambda)$$

$$\Rightarrow 2\lambda - 2\lambda^2 + 24 + \lambda^2 - \lambda^3 + 12\lambda + 2\lambda + 12 + 9 + 3\lambda$$

$$\Rightarrow 19\lambda - \lambda^2 - \lambda^3 + 45$$

$$\lambda^3 + \lambda^2 - 19 - 45 = 0$$

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2) $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix}$

$(4-\lambda)(1-\lambda)(1-\lambda) + 1(2(1-\lambda))$

$= (1-\lambda)((4-\lambda)(1-\lambda)+2)$

$\Rightarrow (4-\lambda-4\lambda+\lambda^2+2)$

$\Rightarrow (\lambda^2-5\lambda+6)$

$(1-\lambda)(\lambda-2)(\lambda-3)$

$\lambda = 1, 2, 3$

$\underline{\lambda = -1}$

$\begin{bmatrix} 5 & 0 & 1 \\ -2 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$5x+z=0$

$-2x+2y=0$

$-2x+2z=0$

$10x+2z=0$

$-12x=0$

$x=0$

$y=0$ vector $= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$z=0$

$\lambda = 3.$

$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$x+z=0$

$-2x-2y=0$

$-2x-2z=0$

$x=-z=-k$

$y=-x$

$z=k$

$y=k$

vector $= k\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

$\underline{\lambda = 2}$

$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$2x+z=0$ $\quad x=k$

$-2x-y=0$ $\quad y=-2k$

$-2x-z=0$ $\quad z=2k$

$\therefore$ vector $= K\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$

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3) $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}$

$(5-\lambda)(-\lambda)(3-\lambda) = 0$

$\lambda = 0, 3, 5$

$\lambda = 0$

$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\cancel{2x = 0}$, $\cancel{-3y = 0}$, $-x = 0$ $\quad 5x = 0$, $x = 0$, $-x + 3z = 0 \Rightarrow z = 0$

~~vector = $k\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$~~

vector $= k\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\lambda = 3$

$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $2x = 0$, $-3y = 0$, $-x = 0$

vecta $= k\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\lambda = 5$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $-5y = 0$

$x - 2z = 0$

$x = 2z = -2k$

vector $= k\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

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4) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 0-\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & & 0-2-\lambda \end{bmatrix}$

$\lambda(3-\lambda)(2+\lambda)=0$

$\lambda = 0, 3, -2$

$\lambda=0$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$3y+4z=0 \qquad z=0,\ y=0$

$-2z=0$

vector $= k\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\lambda=3$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$4z=0$

$-5z=0$

vector $= k\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$\lambda=-2$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$5y+4z=0$

$5y=-4z=-4k$

$y=\frac{-4}{5}k$

$\therefore$ vector $= \begin{bmatrix} 0 \\ -4/5k \\ k \end{bmatrix} = k\begin{bmatrix} 0 \\ -4/5 \\ 1 \end{bmatrix}$