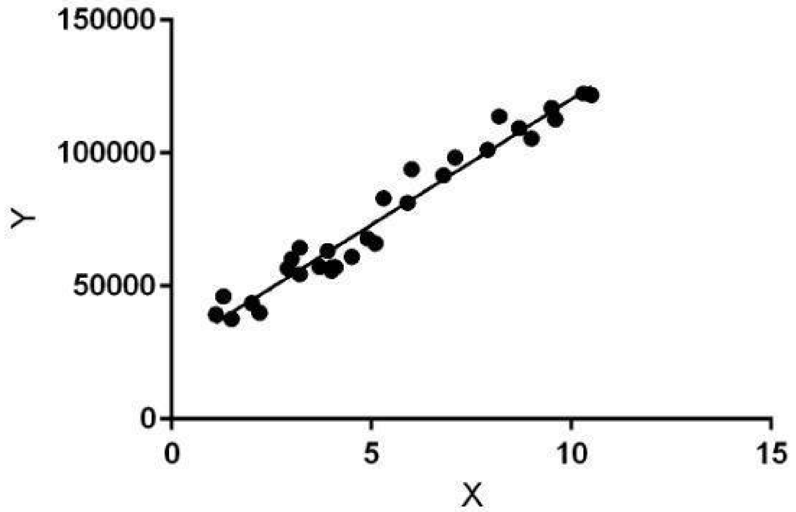
Experiment No. 1
Analyze the Boston Housing dataset and apply appropriate Regression Technique
Date of Performance: 24-07-2023
Date of Submission: 08-10-23

Aim: Analyze the Boston Housing dataset and apply appropriate Regression Technique.

Objective: Ability to perform various feature engineering tasks, apply linear regression on the given dataset and minimize the error.

Theory:

Linear Regression is a machine learning algorithm based on supervised learning. It performs a regression task. Regression models a target prediction value based on independent variables. It is mostly used for finding out the relationship between variables and forecasting. Different regression models differ based on – the kind of relationship between dependent and independent variables they are considering, and the number of independent variables getting used.



Linear regression performs the task to predict a dependent variable value (y) based on a given independent variable (x). So, this regression technique finds out a linear relationship between x (input) and y(output). Hence, the name is Linear Regression.

In the figure above, X (input) is the work experience and Y (output) is the salary of a person. The regression line is the best fit line for our model.

Dataset:

The Boston Housing Dataset

The Boston Housing Dataset is derived from information collected by the U.S. Census Service concerning housing in the area of Boston MA. The following describes the dataset columns:

CRIM - per capita crime rate by town

ZN - proportion of residential land zoned for lots over 25,000

sq.ft. INDUS - proportion of non-retail business acres per town.

CHAS - Charles River dummy variable (1 if tract bounds river; 0 otherwise)

NOX - nitric oxides concentration (parts per 10 million)

RM - average number of rooms per dwelling

AGE - proportion of owner-occupied units built prior to

1940 DIS - weighted distances to five Boston employment

centers RAD - index of accessibility to radial highways

TAX - full-value property-tax rate per

$10,000 PTRATIO - pupil-teacher ratio by

town

B - 1000(Bk - 0.63)^2 where Bk is the proportion of blacks by town

LSTAT - % lower status of the population

MEDV - Median value of owner-occupied homes in $1000's

ml-experiment01

October 9, 2023

[ ]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from statsmodels.graphics.gofplots import ProbPlot
import sklearn.datasets
from sklearn.model_selection import train_test_split
from statsmodels.formula.api import ols
import statsmodels.api as sm
from sklearn.linear_model import LinearRegression
from sklearn.preprocessing import MinMaxScaler
```

[ ]:
```
df = pd.read_csv('housing.csv')
```

[ ]:
```
print(df)
```

```
        CRIM    ZN  INDUS  CHAS    NOX     RM   AGE     DIS  RAD  TAX  \
0    0.00632  18.0   2.31     0  0.538  6.575  65.2  4.0900    1  296
1    0.02731   0.0   7.07     0  0.469  6.421  78.9  4.9671    2  242
2    0.02729   0.0   7.07     0  0.469  7.185  61.1  4.9671    2  242
3    0.03237   0.0   2.18     0  0.458  6.998  45.8  6.0622    3  222
4    0.06905   0.0   2.18     0  0.458  7.147  54.2  6.0622    3  222
..       …     …     …    …     …     …     …    …   …
501  0.06263   0.0  11.93     0  0.573  6.593  69.1  2.4786    1  273
502  0.04527   0.0  11.93     0  0.573  6.120  76.7  2.2875    1  273
503  0.06076   0.0  11.93     0  0.573  6.976  91.0  2.1675    1  273
504  0.10959   0.0  11.93     0  0.573  6.794  89.3  2.3889    1  273
505  0.04741   0.0  11.93     0  0.573  6.030  80.8  2.5050    1  273

     PTRATIO       B  LSTAT  MEDV
0       15.3  396.90   4.98  24.0
1       17.8  396.90   9.14  21.6
2       17.8  392.83   4.03  34.7
3       18.7  394.63   2.94  33.4
4       18.7  396.90   5.33  36.2
..       …       …    …     …
501     21.0  391.99   9.67  22.4
502     21.0  396.90   9.08  20.6
```

```
503      21.0  396.90   5.64  23.9
504      21.0  393.45   6.48  22.0
505      21.0  396.90   7.88  11.9

[506 rows x 14 columns]
```

[ ]:
```
df.head()
```

[ ]:
```
      CRIM    ZN  INDUS  CHAS    NOX     RM   AGE     DIS  RAD  TAX  PTRATIO  \
0  0.00632  18.0   2.31     0  0.538  6.575  65.2  4.0900    1  296     15.3
1  0.02731   0.0   7.07     0  0.469  6.421  78.9  4.9671    2  242     17.8
2  0.02729   0.0   7.07     0  0.469  7.185  61.1  4.9671    2  242     17.8
3  0.03237   0.0   2.18     0  0.458  6.998  45.8  6.0622    3  222     18.7
4  0.06905   0.0   2.18     0  0.458  7.147  54.2  6.0622    3  222     18.7

        B  LSTAT  MEDV
0  396.90   4.98  24.0
1  396.90   9.14  21.6
2  392.83   4.03  34.7
3  394.63   2.94  33.4
4  396.90   5.33  36.2
```

[ ]:
```
#The price of the house indicated by the variable MEDV is the target variable␣
↪and the rest are the independent variables based on which we will predict␣
↪house price.

# Info of dataframe
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 506 entries, 0 to 505
Data columns (total 14 columns):
 #   Column   Non-Null Count  Dtype
---  ------   --------------  -----
 0   CRIM     506 non-null    float64
 1   ZN       506 non-null    float64
 2   INDUS    506 non-null    float64
 3   CHAS     506 non-null    int64
 4   NOX      506 non-null    float64
 5   RM       506 non-null    float64
 6   AGE      506 non-null    float64
 7   DIS      506 non-null    float64
 8   RAD      506 non-null    int64
 9   TAX      506 non-null    int64
 10  PTRATIO  506 non-null    float64
 11  B        506 non-null    float64
 12  LSTAT    506 non-null    float64
 13  MEDV     506 non-null    float64
```

```
dtypes: float64(11), int64(3)
memory usage: 55.5 KB
```

[ ]:
```
# checking the number of rows and Columns in the data frame
df.shape
```

[ ]:
```
(506, 14)
```

[ ]:
```
# check for missing values
df.isnull().sum()
```

[ ]:
```
CRIM       0
ZN         0
INDUS      0
CHAS       0
NOX        0
RM         0
AGE        0
DIS        0
RAD        0
TAX        0
PTRATIO    0
B          0
LSTAT      0
MEDV       0
dtype: int64
```

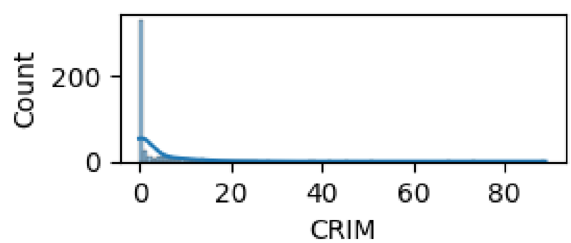
[ ]:
```
# statistical measures of the dataset
df.describe()
```

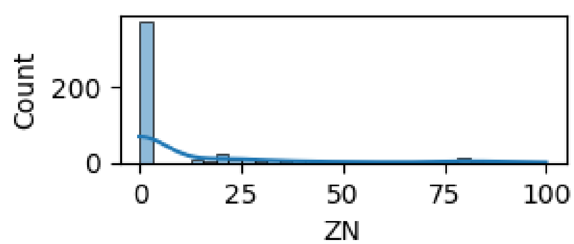
[ ]:
```
             CRIM          ZN       INDUS        CHAS         NOX          RM  \
count  506.000000  506.000000  506.000000  506.000000  506.000000  506.000000
mean     3.613524   11.363636   11.136779    0.069170    0.554695    6.284634
std      8.601545   23.322453    6.860353    0.253994    0.115878    0.702617
min      0.006320    0.000000    0.460000    0.000000    0.385000    3.561000
25%      0.082045    0.000000    5.190000    0.000000    0.449000    5.885500
50%      0.256510    0.000000    9.690000    0.000000    0.538000    6.208500
75%      3.677083   12.500000   18.100000    0.000000    0.624000    6.623500
max     88.976200  100.000000   27.740000    1.000000    0.871000    8.780000

              AGE         DIS         RAD         TAX     PTRATIO           B  \
count  506.000000  506.000000  506.000000  506.000000  506.000000  506.000000
mean    68.574901    3.795043    9.549407  408.237154   18.455534  356.674032
std     28.148861    2.105710    8.707259  168.537116    2.164946   91.294864
min      2.900000    1.129600    1.000000  187.000000   12.600000    0.320000
25%     45.025000    2.100175    4.000000  279.000000   17.400000  375.377500
50%     77.500000    3.207450    5.000000  330.000000   19.050000  391.440000
```

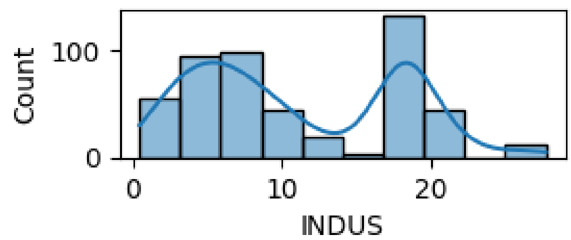
```
75%      94.075000    5.188425   24.000000  666.000000   20.200000  396.225000
max     100.000000   12.126500   24.000000  711.000000   22.000000  396.900000

            LSTAT        MEDV
count  506.000000  506.000000
mean    12.653063   22.532806
std      7.141062    9.197104
min      1.730000    5.000000
25%      6.950000   17.025000
50%     11.360000   21.200000
75%     16.955000   25.000000
max     37.970000   50.000000
```

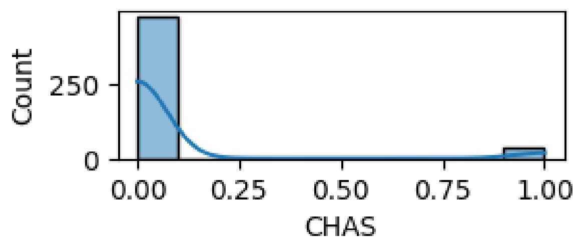
[ ]:
```
correlation = df.corr()
```

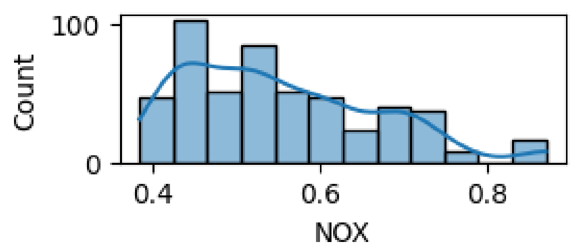
[ ]:
```
#plot all the columns to look at their distributions
for i in df.columns:
    plt.figure(figsize=(3, 1))
    sns.histplot(data=df, x=i, kde = True)
    plt.show()
```

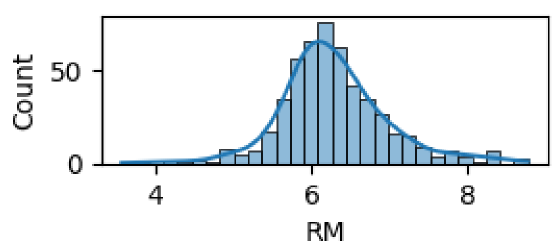


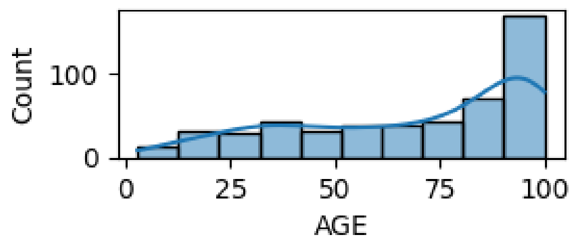


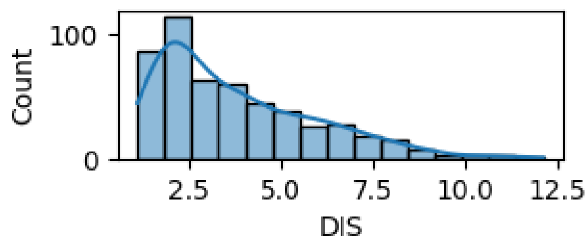

Count
100
0
0
10
20
INDUS

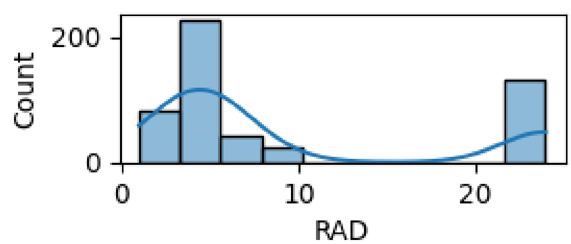

Count
250
0
0.00
0.25
0.50
0.75
1.00
CHAS

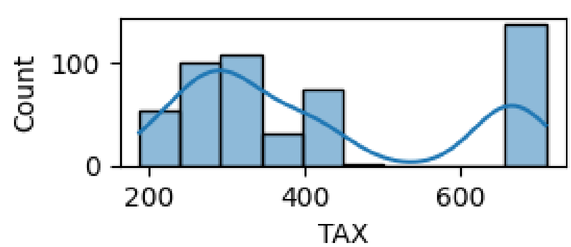

Count
100
0
0.4
0.6
0.8
NOX

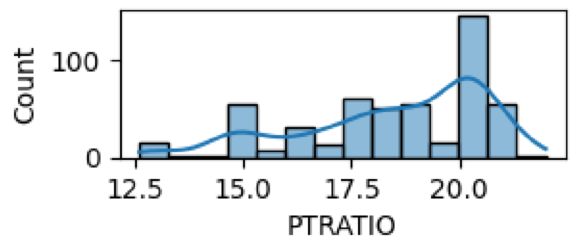

Count
50
0
4
6
8
RM

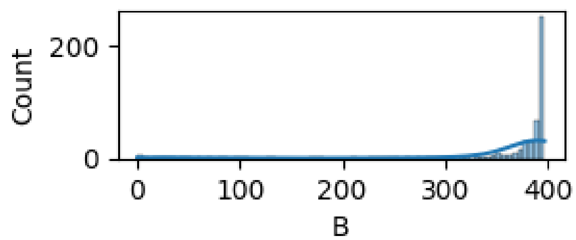

Count
100
0
0
25
50
75
100
AGE

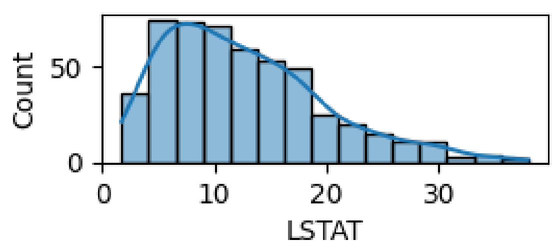

Count
100
0
2.5
5.0
7.5
10.0
12.5
DIS

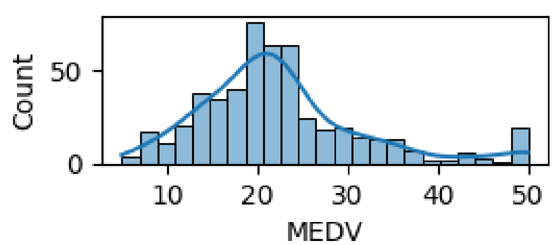

Count
200
0
0
10
20
RAD


Count
100
0
200
400
600
TAX





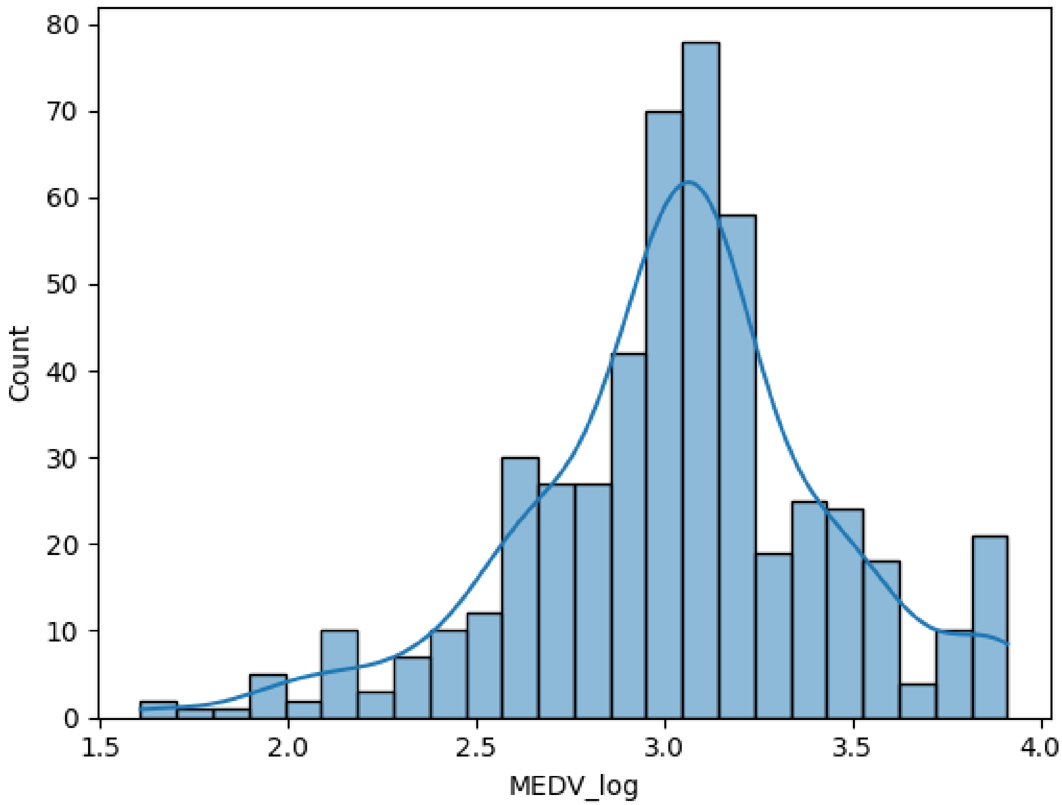




[ ]:
```
#The dependent variable MEDV seems to be slightly right skewed, apply a log␣
↪transformation on the 'MEDV' column and check the distribution of the␣
↪transformed column.
df['MEDV_log'] = np.log(df['MEDV'])
```

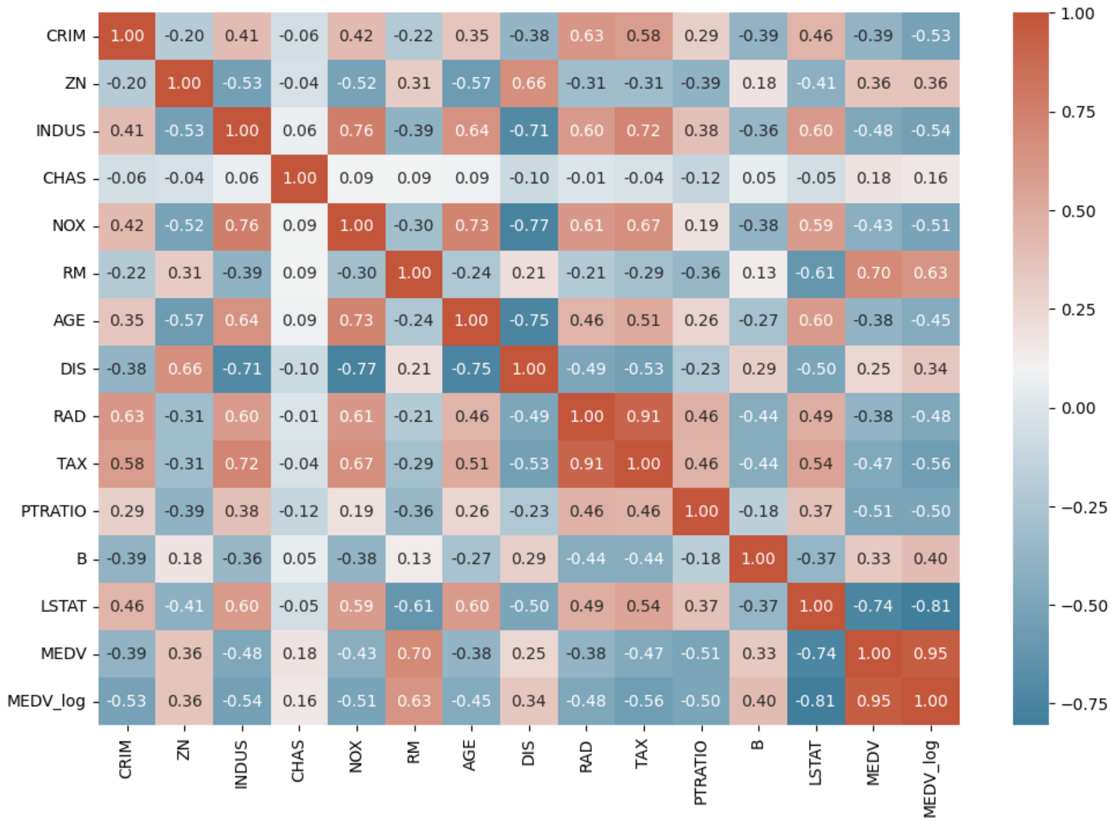
[ ]:
```
sns.histplot(data=df, x='MEDV_log', kde = True)
```

[ ]: <Axes: xlabel='MEDV_log', ylabel='Count'>



The log-transformed variable (MEDV_log) appears to have a nearly normal distribution without skew, and hence we can proceeed.

[ ]:
```
plt.figure(figsize=(12,8))
cmap = sns.diverging_palette(230, 20, as_cmap=True)
sns.heatmap(df.corr(),annot=True,fmt='.2f',cmap=cmap )
plt.show()
```



[ ]:
```python
# separate the dependent and indepedent variable
Y = df['MEDV_log']
X = df.drop(columns = {'MEDV', 'MEDV_log'})

# add the intercept term
X = sm.add_constant(X)
```

[ ]:
```python
# splitting the data in 70:30 ratio of train to test data
X_train, X_test, y_train, y_test = train_test_split(X, Y, test_size=0.30 , random_state=1)
```

[ ]:
```python
#Check for Multicollinearity
#use the Variance Inflation Factor (VIF), to check if there is multicollinearity in the data.
#Features having a VIF score > 5 will be dropped/treated till all the features have a VIF score < 5

from statsmodels.stats.outliers_influence import variance_inflation_factor

# function to check VIF
def checking_vif(train):
```

```
    vif = pd.DataFrame()
    vif["feature"] = train.columns

    # calculating VIF for each feature
    vif["VIF"] = [
        variance_inflation_factor(train.values, i) for i in range(len(train.
 ↪columns))
    ]
    return vif


print(checking_vif(X_train))
```

```
    feature         VIF
0     const  585.099960
1      CRIM    1.993439
2        ZN    2.743911
3     INDUS    4.004462
4      CHAS    1.078490
5       NOX    4.430555
6        RM    1.879494
7       AGE    3.155351
8       DIS    4.361514
9       RAD    8.369185
10      TAX   10.194047
11  PTRATIO    1.948555
12        B    1.385213
13    LSTAT    2.926462
```

There are two variables with a high VIF - RAD and TAX. Remove TAX as it has the highest VIF values and check the multicollinearity again.

[ ]:
```
# create the model after dropping TAX
X_train = X_train.drop(['TAX'],1)

# check for VIF
print(checking_vif(X_train))
```

```
    feature         VIF
0     const  581.372515
1      CRIM    1.992236
2        ZN    2.483521
3     INDUS    3.277778
4      CHAS    1.052841
5       NOX    4.397232
6        RM    1.876243
7       AGE    3.154114
8       DIS    4.339453
```

```
9       RAD    2.978247
10  PTRATIO    1.914523
11        B    1.384927
12    LSTAT    2.924524
```

```
<ipython-input-17-31a12e8753ff>:2: FutureWarning: In a future version of pandas
all arguments of DataFrame.drop except for the argument 'labels' will be
keyword-only.
  X_train = X_train.drop(['TAX'],1)
```

[ ]:
```
#create the linear regression model using statsmodels OLS and print the model
 ↪summary.
model1 = sm.OLS(y_train, X_train).fit()

# get the model summary
model1.summary()
```

[ ]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:               MEDV_log   R-squared:                       0.771
Model:                            OLS   Adj. R-squared:                  0.763
Method:                 Least Squares   F-statistic:                     95.56
Date:                Tue, 01 Aug 2023   Prob (F-statistic):          2.97e-101
Time:                        01:45:20   Log-Likelihood:                 78.262
No. Observations:                 354   AIC:                            -130.5
Df Residuals:                     341   BIC:                            -80.22
Df Model:                          12
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          4.4999      0.253     17.767      0.000       4.002       4.998
CRIM          -0.0122      0.002     -7.005      0.000      -0.016      -0.009
ZN             0.0010      0.001      1.417      0.157      -0.000       0.002
INDUS         -0.0002      0.003     -0.066      0.947      -0.006       0.005
CHAS           0.1164      0.039      3.008      0.003       0.040       0.193
NOX           -1.0297      0.187     -5.509      0.000      -1.397      -0.662
RM             0.0569      0.021      2.734      0.007       0.016       0.098
AGE            0.0003      0.001      0.390      0.697      -0.001       0.002
DIS           -0.0496      0.010     -4.841      0.000      -0.070      -0.029
RAD            0.0080      0.002      3.885      0.000       0.004       0.012
PTRATIO       -0.0458      0.007     -6.762      0.000      -0.059      -0.033
B              0.0002      0.000      1.796      0.073   -2.35e-05       0.001
LSTAT         -0.0291      0.002    -11.772      0.000      -0.034      -0.024
==============================================================================
Omnibus:                       33.707   Durbin-Watson:                   1.924
```

```
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              100.726
Skew:                           0.387   Prob(JB):                     1.34e-22
Kurtosis:                       5.496   Cond. No.                     1.01e+04
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 1.01e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
"""
```

Independent variables (ZN, AGE, and INDUS) have a high p-value and low t, which implies a minimum significance. Drop insignificant variables from the above model and create the regression model again

[ ]:
```python
# create the model after dropping TAX
Y = df['MEDV_log']

X = df.drop(columns = {'MEDV', 'MEDV_log', 'ZN', 'AGE', 'INDUS', 'TAX'})
X = sm.add_constant(X)

#splitting the data in 70:30 ratio of train to test data
X_train, X_test, y_train, y_test = train_test_split(X, Y, test_size=0.30 ,␣
↪random_state=1)

# create the model
model2 = sm.OLS(y_train, X_train).fit()

# get the model summary
model2.summary()
```

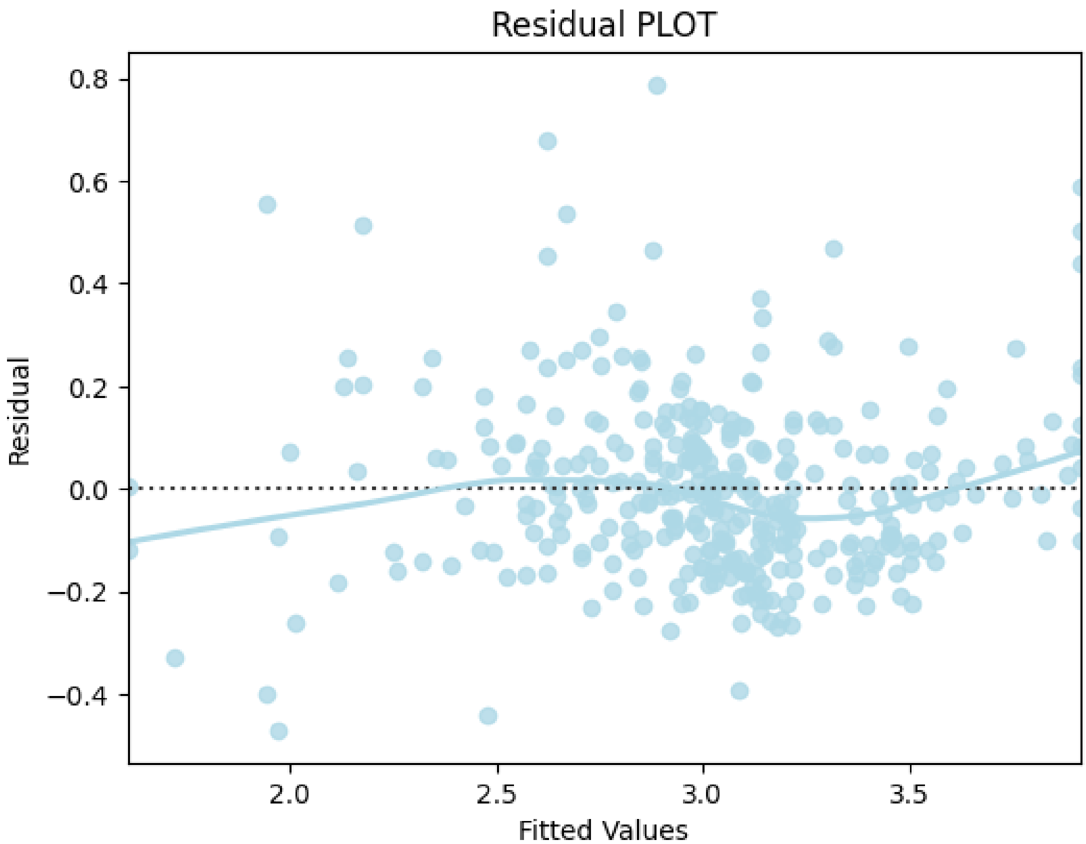
[ ]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:               MEDV_log   R-squared:                       0.769
Model:                            OLS   Adj. R-squared:                  0.763
Method:                 Least Squares   F-statistic:                     127.5
Date:                Tue, 01 Aug 2023   Prob (F-statistic):          6.21e-104
Time:                        01:45:24   Log-Likelihood:                 77.190
No. Observations:                 354   AIC:                            -134.4
Df Residuals:                     344   BIC:                            -95.69
Df Model:                           9
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
```

```
==============================================================================
const          4.5147      0.252     17.925      0.000       4.019       5.010
CRIM          -0.0119      0.002     -6.909      0.000      -0.015      -0.009
CHAS           0.1165      0.039      3.016      0.003       0.041       0.192
NOX           -1.0234      0.168     -6.086      0.000      -1.354      -0.693
RM             0.0622      0.020      3.089      0.002       0.023       0.102
DIS           -0.0434      0.008     -5.488      0.000      -0.059      -0.028
RAD            0.0083      0.002      4.092      0.000       0.004       0.012
PTRATIO       -0.0490      0.006     -7.936      0.000      -0.061      -0.037
B              0.0002      0.000      1.824      0.069   -1.95e-05       0.001
LSTAT         -0.0287      0.002    -12.577      0.000      -0.033      -0.024
==============================================================================
Omnibus:                       35.608   Durbin-Watson:                   1.927
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              104.246
Skew:                           0.425   Prob(JB):                     2.31e-23
Kurtosis:                       5.519   Cond. No.                     9.76e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 9.76e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
"""
```

[ ]:
```
residuals = model2.resid
residuals.mean()
```

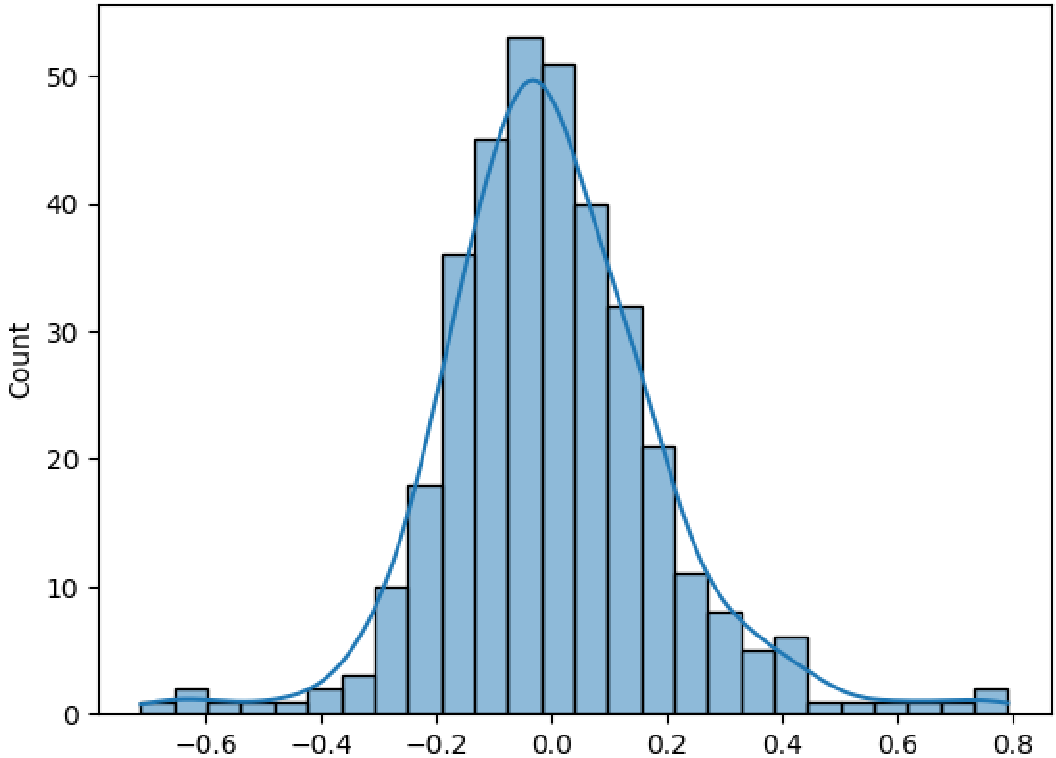
[ ]: 8.154180406851432e-17

[ ]:
```
# predicted values
fitted = model2.fittedvalues

#sns.set_style("whitegrid")
sns.residplot(x = y_train, y = residuals , color="lightblue", lowess=True)
plt.xlabel("Fitted Values")
plt.ylabel("Residual")
plt.title("Residual PLOT")
plt.show()
```

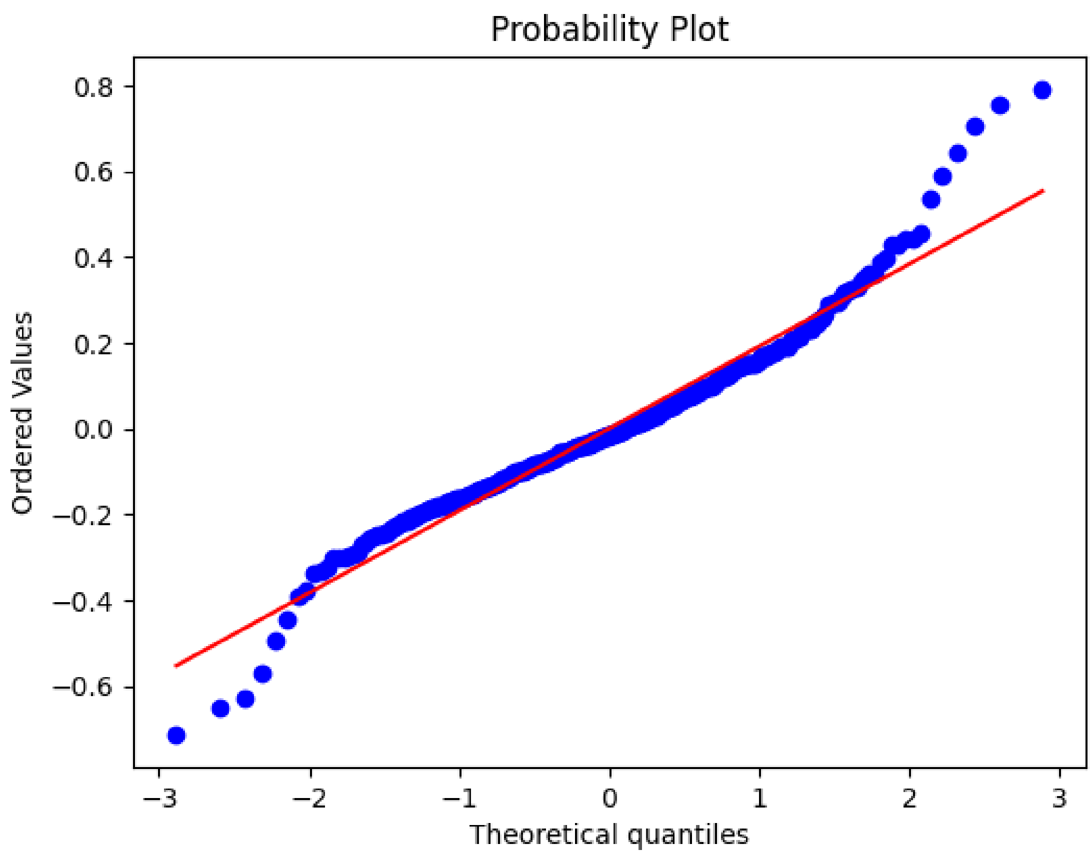


[ ]:
```
# Plot histogram of residuals
sns.histplot(residuals, kde=True)
```

[ ]: <Axes: ylabel='Count'>



[ ]:
```python
# Plot q-q plot of residuals
import pylab
import scipy.stats as stats

stats.probplot(residuals, dist="norm", plot=pylab)
plt.show()
```



[ ]:
```
# RMSE
def rmse(predictions, targets):
    return np.sqrt(((targets - predictions) ** 2).mean())


# MAPE
def mape(predictions, targets):
    return np.mean(np.abs((targets - predictions)) / targets) * 100


# MAE
def mae(predictions, targets):
    return np.mean(np.abs((targets - predictions)))


# Model Performance on test and train data
def model_pref(olsmodel, x_train, x_test):

    # Insample Prediction
    y_pred_train = olsmodel.predict(x_train)
```

```
    y_observed_train = y_train

    # Prediction on test data
    y_pred_test = olsmodel.predict(x_test)
    y_observed_test = y_test

    print(
        pd.DataFrame(
            {
                "Data": ["Train", "Test"],
                "RMSE": [
                    rmse(y_pred_train, y_observed_train),
                    rmse(y_pred_test, y_observed_test),
                ],
                "MAE": [
                    mae(y_pred_train, y_observed_train),
                    mae(y_pred_test, y_observed_test),
                ],
                "MAPE": [
                    mape(y_pred_train, y_observed_train),
                    mape(y_pred_test, y_observed_test),
                ],
            }
        )
    )

# Checking model performance
model_pref(model2, X_train, X_test)
```

```
    Data      RMSE       MAE      MAPE
0  Train  0.194565  0.141729  4.919107
1   Test  0.191732  0.146199  5.069304
```

The errors have increased slightly on the test data. This suggested further investigation to improve the performance on general data.

[ ]:
```
# import the required function

from sklearn.model_selection import cross_val_score

# build the regression model and
linearregression = LinearRegression()

cv_Score11 = cross_val_score(linearregression, X_train, y_train, cv = 10)
cv_Score12 = cross_val_score(linearregression, X_train, y_train, cv = 10,␣
 ↪scoring = 'neg_mean_squared_error')
```

```
print("RSquared: %0.3f (+/- %0.3f)" % (cv_Score11.mean(), cv_Score11.std() * 2))
print("Mean Squared Error: %0.3f (+/- %0.3f)" % (-1*cv_Score12.mean(),␣
↪cv_Score12.std() * 2))
```

```
RSquared: 0.726 (+/- 0.251)
Mean Squared Error: 0.041 (+/- 0.024)
```

Get model Coefficients in a pandas dataframe with column 'Feature' having all the features and column 'Coefs' with all the corresponding Coefs. Write the regression equation.

[ ]:
```
coef = pd.Series(index = X_train.columns, data = model2.params.values)

coef_df = pd.DataFrame(data = {'Coefs': model2.params.values }, index = ␣
↪X_train.columns)
coef_df
```

[ ]:
```
            Coefs
const    4.514720
CRIM    -0.011919
CHAS     0.116497
NOX     -1.023431
RM       0.062203
DIS     -0.043391
RAD      0.008288
PTRATIO -0.049038
B        0.000249
LSTAT   -0.028659
```

[ ]:
```
#Write the equation of the fit
Equation = "log (Price) ="
print(Equation, end='\t')
for i in range(len(coef)):
    print('(', coef[i], ') * ', coef.index[i], '+', end = ' ')
```

```
log (Price) =   ( 4.514720483568433 ) *  const + ( -0.011918775173037938 ) *
CRIM + ( 0.11649715902151694 ) *  CHAS + ( -1.0234312247045108 ) *  NOX + (
0.062202691330254856 ) *  RM + ( -0.04339113889561087 ) *  DIS + (
0.008287691091705261 ) *  RAD + ( -0.04903790360575723 ) *  PTRATIO + (
0.00024900512380057695 ) *  B + ( -0.0286587316944412 ) *  LSTAT +
```

Conclusion:
Certain features have been selected as influential factors in predicting house prices:

- CRIM: Higher crime rates may be associated with lower property values.
- CHAS: Proximity to the Charles River can potentially increase house prices.
- NOX: Areas with higher air pollution (nitric oxides concentration) may have lower property values.
- RM: A higher number of rooms in a dwelling tends to lead to higher house prices.
- DIS: Shorter distances to employment centers can result in higher demand and potentially higher prices.
- RAD: Improved accessibility to radial highways may affect housing demand and, consequently, prices.
- PTRATIO: A lower pupil-teacher ratio is often viewed as desirable, indicating better educational resources in the area.
- B: The proportion of Black residents can influence housing prices due to historical segregation patterns.
- LSTAT: The percentage of lower-status population can reflect the economic condition of the area and affect property values.

These selected features are meaningful and relevant for estimating house prices.
The model's performance is assessed using Mean Squared Error (MSE), which is found to be 0.041 (+/- 0.024). This indicates that, on average, the squared difference between the predicted and actual house prices is 0.041. The relatively low MSE suggests that the model is performing well and providing accurate predictions for house prices.
Furthermore, the low standard deviation of MSE (+/- 0.024) indicates consistent model performance across different folds. Overall, the Mean Squared Error of 0.041 suggests that the linear regression model is effective in capturing the relationships within the dataset and making accurate predictions for house prices.