

③ Independent Tests to Save Your Boxing King

(a)

Null Hypothesis H_0 : Outcome of the match should be independent of the judge.

H_1 : Outcome of the match is dependent on the judge.

	Judge A	Judge B	Judge C	Total
Player 1 Wins	72	50	24	146
Draw	8	5	3	16
Loses	20	8	9	37
Total	100	63	36	199

From the above table, Grand total = 199

$$P(\text{Player 1 Wins}) = \frac{\text{Total Wins}}{\text{Grand Total}} = \frac{146}{199} = 0.73$$

$$P(\text{Judge A}) = \frac{\text{Total A}}{\text{Grand Total}} = \frac{100}{199} = 0.5025$$

Expected frequency of win player 1 & Judge A is = Grand Total * P(win) * P(Judge A)

Similarly, $\Rightarrow E_{ij} = \frac{T_{4j} \times T_{i4}}{T_{44}}$

\downarrow row
 \downarrow column

Similarly, we populate the below table with expected frequencies:

	Judge A	Judge B	Judge C
Wins	73.36	46.22	26.41
Draw	8.04	5.065	2.89
Player 1 Loses	18.54	11.71	6.693

$$Q_{obs} = \sum_r \sum_c \frac{(E_{rc} - O_{rc})^2}{E_{rc}}$$

Observed	Expected	$\frac{(E - O)^2}{E}$
72	73.36	0.0252
50	46.22	0.309
24	26.41	0.2199
8	8.04	0.00099
5	5.065	0.00083
3	2.89	0.0041
20	18.54	0.1069
8	11.71	1.175
9	6.693	0.7951

$$Q_{obs} = 2.63702$$

$$df(\text{Degree of Freedom}) = (3-1) * (3-1) = 4$$

$$P\text{-value} = P(\chi^2_4 > Q_{obs}) = 1 - P(\chi^2_4 < 2.63702) = 0.620283$$

As p-value (0.620283) > 0.05, we fail to reject H_0 .
We Accept H_0 .

3)b) Pearson Correlation Coefficient

$$r_{X,Y} = \frac{\sum_{i=1}^n (x_i - \bar{X})(y_i - \bar{Y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{X})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{Y})^2}}$$

Let

$x_i \rightarrow$ number of player wins for Judge A for 10 days

$y_i \rightarrow$ number of player wins for Judge B

$z_i \rightarrow$ number of player wins for Judge C.

$$\bar{X} = \sum_{i=1}^n \frac{x_i}{n} = 42.6 \quad \bar{Y} = 43 \quad \bar{Z} = 40.2$$

Correlation between Judges A, B

$$\hat{r}_{A,B} = \frac{\sum_{i=1}^{10} (x_i - 42.6)(y_i - 43)}{\sqrt{\sum_{i=1}^{10} (x_i - 42.6)^2} \sqrt{\sum_{i=1}^{10} (y_i - 43)^2}}$$

$$= 0.5726 (> 0.5) - \text{Positive linear Correlation}$$

Correlation between Judges B, C

$$\hat{r}_{B,C} = \frac{\sum_{i=1}^{10} (y_i - 43)(z_i - 40.2)}{\sqrt{\sum_{i=1}^{10} (y_i - 43)^2} \sqrt{\sum_{i=1}^{10} (z_i - 40.2)^2}}$$

$$= -0.0837$$

$$= |-0.0837| \leq 0.5 \Rightarrow$$

No linear Calculation

Correlation between Judges A, C

$$\hat{r}_{A,C} = \frac{\sum_{i=1}^{10} (x_i - 42.6)(z_i - 40.2)}{\sqrt{\sum_{i=1}^{10} (x_i - 42.6)^2} \sqrt{\sum_{i=1}^{10} (z_i - 40.2)^2}}$$

$$= 0.0913$$

$$\hat{s}_{A,C} \Rightarrow \hat{s}_{X,Z} = 0.0913$$

$$|\hat{s}_{X,Z}| \leq 0.5 \rightarrow \text{No linear Correlation}$$

As the probability of winning each game is same; the results for each Judges should be correlated.

We observe from the results ~~from~~ Judge C ^{that it is rare} not linearly correlated with Judge A & Judge B.

From this we can infer that Judge C is not doing their job.

4 a) Step 1: Define Null hypothesis

$$H_0: \mu \leq \mu_0$$

$$H_1: \mu > \mu_0 \quad \text{⊗} \rightarrow$$

Given $X = [21.5, 24.5, 18.5, 17.2, 14.5, 23.2, 22.1, 20.5, 19.4, 18.1, 24.1, 18.5]$

$$H_0: \bar{X} \leq 20$$

$$H_1: \bar{X} > 20$$

$n=12$ Since this is One Sample T Test, the degrees

$$\text{of freedom} = n-1 = 12-1 = 11$$

$\alpha = 0.05$ to meet 95% Confidence interval

Step 2 Calculate Test statistic

$$T = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

$$\bar{X} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{20.2 + 21.5 + 24.5 + 18.5 + 17.2 + 14.5 + 23.2 + 22.1 + 20.5 + 19.4 + 18.1 + 24.1 + 18.5}{12}$$
$$= 20.175$$

$$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{1.768 + 18.74 + 2.78 + 8.82 + 32.14 + 9.18 + 3.72 + 0.108 + 0.59 + 4.28 + 15.44 + 2.78}{11}}$$
$$= 3.0211$$

$$T = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{(20.175 - 20)}{\frac{3.0211}{\sqrt{12}}} = 0.2006$$

Step-3 ~~$t_{n-1,0}$~~ If $T > t_{n-1, \alpha}$ reject H_0

$$t_{n-1, \alpha} = t_{11, 0.05} = 1.796$$

$$T \not> t_{n-1, \alpha}$$

$$0.2006 \not> 1.796$$

\therefore We accept the null hypothesis

