CSE 544.01 Probability and Statistics for Data Scientists

Assignment - 1

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1)

a) For a Series to be tied as 2 - 2 after first 4 games, both the teams (MIL) & (BKN) need to win 2 games each

→ Considering all possible outcomes of wins & loss in the 4 games Example (LLWW, LWLW, WLWL ... etc)

Total Outcomes = $2 \times 2 \times 2 \times 2 = 2^4$ (2 because there are 2 possible results)

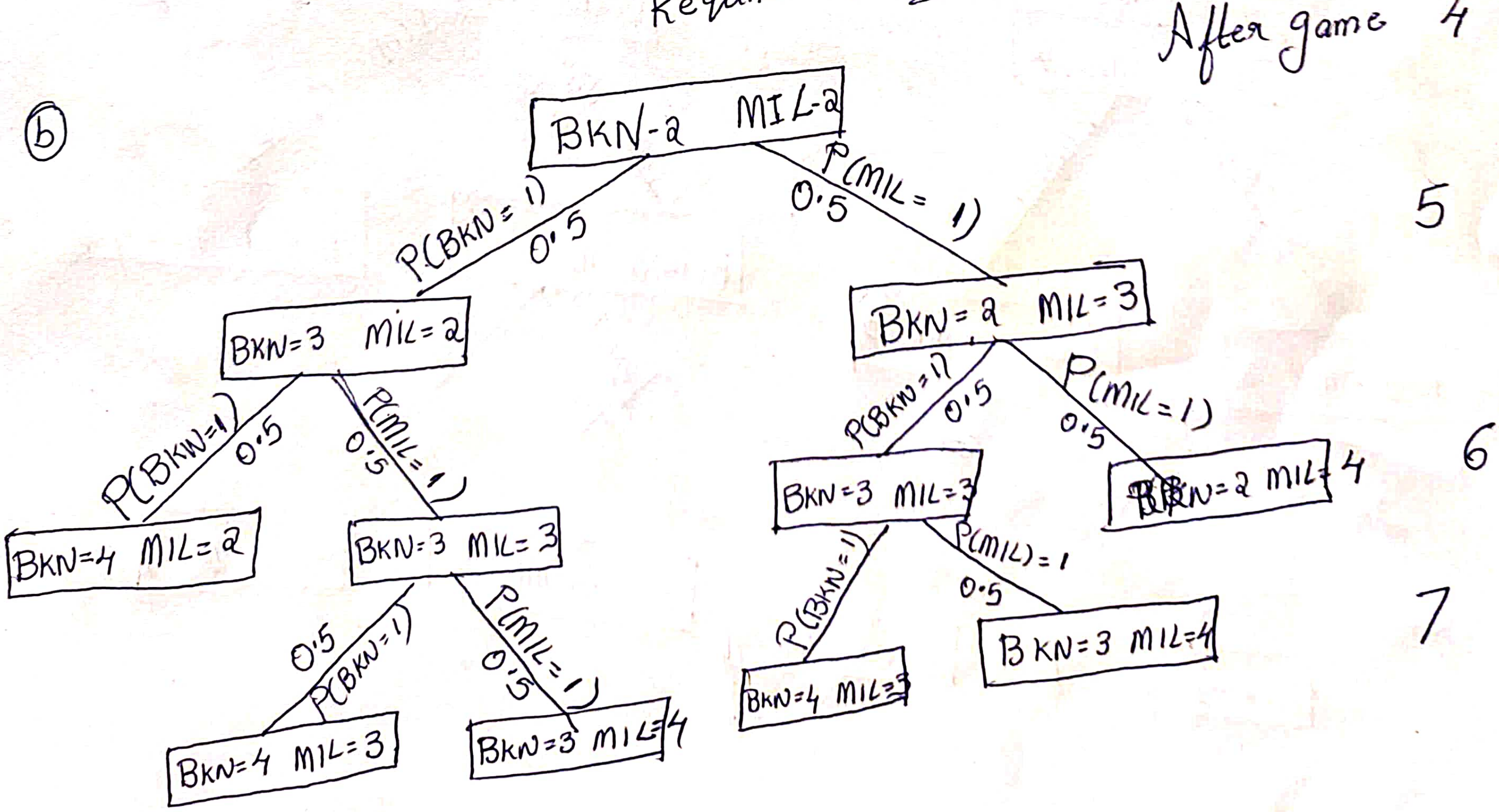
. The Possibility where each team wins 2 games out of 4 is obtained by choosing 2 out of 4

$$\Rightarrow 4C_2 \Rightarrow \frac{4!}{2!\,2!}$$

$$\therefore \text{Probability} = \frac{\frac{4!}{2!\,2!}}{2^4} = \frac{24}{4 \times 2^4} = \frac{6}{16} = \frac{3}{8}$$

Required Probability $= \frac{3}{8} = 0.375$

(b)



(c) The probability of MIL Winning the Series 4-3 is ⇒

Two ~~pa~~ paths

Path-1 Games 5, 6, 7 are won by BLK, MIL, MIL respectively

path-2 Games 5, 6, 7 are won by MIL, BKN, MIL respectively

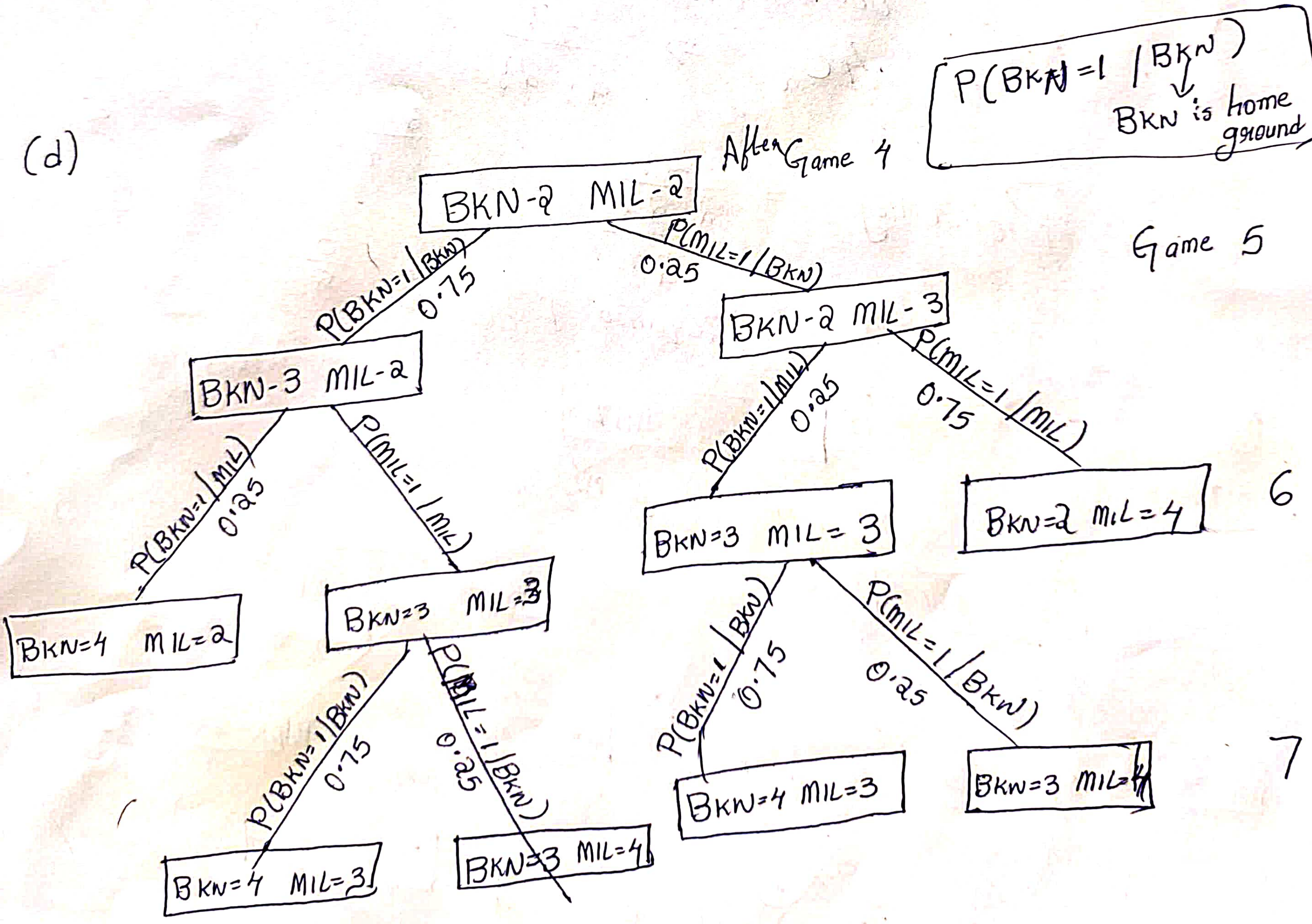
$$Pr = \text{Path 1} + \text{Path 2}$$

$$= (0.5)(0.5)(0.5) + (0.5)(0.5)(0.5)$$

$$= (0.5)^3 + (0.5)^3 = 0.125 + 0.125$$

$$= \underline{\underline{0.25}}$$

(d)



e)

Two paths

Path-1 Games 5,6,7 are won by BKN, MIL, MIL respectively

Path-2 Games 5,6,7 are won by MIL, BKN, MIL respectively.

$$Pr = \text{Path 1} + \text{Path 2}$$

$$= (0.75)(0.75)(0.25) + (0.25)^3$$

$$= 0.15625$$

1 - f

The outputs for the program associated with problem 1-f is as follows

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For N = 1000, the simulated value for part (a) is 0.385
For N = 1000, the simulated value for part (c) is 0.3978494623655914
For N = 1000, the simulated value for part (e) is 0.32894736842105265
For N = 10000, the simulated value for part (a) is 0.3669
For N = 10000, the simulated value for part (c) is 0.3780748663101604
For N = 10000, the simulated value for part (e) is 0.293640350877193
For N = 100000, the simulated value for part (a) is 0.37169
For N = 100000, the simulated value for part (c) is 0.3692267088941673
For N = 100000, the simulated value for part (e) is 0.29319007329751834
For N = 1000000, the simulated value for part (a) is 0.375264
For N = 1000000, the simulated value for part (c) is 0.37368661245426327
For N = 1000000, the simulated value for part (e) is 0.2971918165233255
For N = 10000000, the simulated value for part (a) is 0.3750895
For N = 10000000, the simulated value for part (c) is 0.3751737556112332
For N = 10000000, the simulated value for part (e) is 0.296871667806731
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We can observe that as the N value increases, the output is becoming closer to the actual value we got using the formula of probability.

2)

Let E_n be the event in which the n^{th} phone will be undiscarded.
$\Rightarrow E_1$ is the event in which the first phone is undiscarded, E_2 is the event in which the second phone is undiscarded and so on.
Therefore, for atleast one phone to be undiscarded after the given process, the required event is $E_1 \cup E_2 \cup ..... \cup E_n$
Therefore the probability that atleast one phone is undiscarded is Pr($E_1 \cup E_2 \cup ..... \cup E_n$)

According the Principle of Inclusion-Exclusion (PIE), Pr($E_1 \cup E_2 \cup ..... \cup E_n$) = $\sum_i Pr(E_i) - \sum_{i<j} Pr(E_i \cap E_j) + \sum_{i<j<k} Pr(E_i \cap E_j \cap E_k) - ...... + (-1)^{n+1} \sum_{i<j<<...n} Pr(E_i \cap E_j \cap .... \cap E_n)$
$\Rightarrow Pr(E_1 \cup E_2 \cup ..... \cup E_n\ )$ = Sum of Probabilities of one phone being undiscarded - Sum of probabilities of two phones being undiscarded + Sum of probabilities of three phones being undiscarded and so on.
$\Rightarrow Pr(E_1 \cup E_2 \cup ..... \cup E_n\ )$ = Probability of any one phone being undiscarded - Probability of any two phones being undiscarded + Probability of any three phones being undiscaded and so on.
In all these cases, we care about the atleast number of mentioned phones being undiscarded, the remaining phones can be in any order.
As there are n phones, the number of possibilites in which the phones can be arranged is n!.
For atleast one phone to be undiscarded, atleast one phone should be in its original position. By original position we mean , the position that is equal to the number of the phone.
That means that one phone is in fixed position and the remaining n-1 phones can be in any order and this can be done for n phones.
The probability of the mentioned event is $\frac{(n-1)!*\binom{n}{1}}{n!} = 1$

In the similar way, the probability that any two phones are undiscarded (the second term in the sequence) is $\frac{(n-2)!*\binom{n}{2}}{n!} = \frac{1}{2!}$

Therefore the probability that atleast one phone remains undiscarded = $Pr(E_1 \cup E_2 \cup ..... \cup E_n)$
$= 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + ... + (-1)^{n+1}\frac{1}{n!}$

3)

Let O be the event in which the ring is the one ring.
Let A be the event in which the person has an above average age.
Let W be the event in which the writing appeared on the wring.
a)
The given information is as follows
Pr(O) = 1/10000
Pr(A/O) = 0.92
Pr(A/$\tilde{O}$) = 0.3
Probability of the ring being one ring given that the person has above average is Pr(O/A).
According to Bayes Theorem

Pr(O/A) = $\frac{Pr(A/O)Pr(O)}{Pr(A)}$
To calculate Pr(A), we can use Total Probability Theorem.
Pr(A) = Pr(A/O) * Pr(O) + $Pr(A/\tilde{O})Pr(\tilde{O})$
Pr(A) = 0.92* $\frac{1}{10^4} + 0.3 * \frac{9999}{10000}$
$\Rightarrow Pr(A) = 0.300062$

$\Rightarrow Pr(O/A) = \frac{0.92*\frac{1}{10^4}}{0.300062}$

Therefore the probability that the ring with Bilbo is one ring is 0.00031.

b)
Given that

Pr(W/O) = 0.95
$Pr(W/\tilde{O}) = 0.1$
Let WA be the event where both the person has above average lifespan and the writing appears on the ring.
Also given that both are independent events.
We need to find out ,Pr(O/WA).
According to Bayes Theorem,

Pr(O/WA) = $\frac{Pr(WA/O)*Pr(O)}{Pr(WA)}$
As W and A are independent events, Pr(WA/O) = Pr(W/O) * Pr(A/O).
Similar to the above problem , we can find out Pr(W) using total probability theorem.
Pr(W) = Pr(W/O)* Pr(O) + $Pr(W/\tilde{O}) * Pr(\tilde{O})$
⇒ Pr(W) = 0.95* 0.0001 + 0.1*0.9999 = 0.100085

Pr(O/WA) = $\frac{0.95*0.92*0.0001}{0.100085*0.300062}$

Therefore the probability of the ring being one ring is 0.0029.

4) $E[X] = \sum_{x=0}^{\infty} Pr[X > x]$

Consider R.H.S

$$\sum_{x=0}^{\infty} Pr[X > x]$$

$$\Rightarrow \sum_{x=0}^{\infty} \sum_{i=x+1}^{\infty} Pr[X = i]$$

consider the limits

$0 \leq x < \infty$

$x < i < \infty$

Combining the limits

$0 \leq x < i < \infty$

The summations can be interchanged as follows

$$\sum_{i=x+1}^{\infty} \sum_{x=0}^{i-1} Pr[X = i]$$

As x varies from 0 to $i-1$, the lower limit of the outer summation starts at 1.

$$\sum_{i=1}^{\infty} \sum_{x=0}^{i-1} Pr[X = i]$$

$$= \sum_{i=1}^{\infty} Pr[X = i] \sum_{x=0}^{i-1} 1$$

$$= \sum_{i=1}^{\infty} Pr[X=i] \cdot i$$ (As there are i terms in the inner summation)

$$= \sum_{i=1}^{\infty} i \, Pr[X=i]$$

$$= \sum_{i=1}^{\infty} i \cdot Pr[X=i] + 0 \cdot Pr[X=0]$$

$$= \sum_{i=0}^{\infty} i \cdot Pr[X=i]$$

$$= E[X]$$

$$= L.H.S$$

Hence Proved.

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6) Given

$$P_X(i) = \frac{e^{-\lambda} \lambda^i}{i!}, \quad i \geq 0$$

a) The summation of the given P.M.F should be 1.

~~Sum => $\int_0^{\infty} e^{-\lambda} \lambda^i$~~

$$\text{Sum} \Rightarrow \sum_{i=0}^{\infty} \frac{e^{-\lambda} \lambda^i}{i!}$$

$$= e^{-\lambda} \sum_{i=0}^{\infty} \frac{\lambda^i}{i!}$$

$$= e^{-\lambda}\left(0 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots \frac{\lambda^k}{k!} + \cdots\right)$$

This is the expansion of e^{λ}

$$= e^{-\lambda} \cdot e^{\lambda}$$

$$= 1$$

$\therefore$ The P.M.F adds up to 1.

b) $$E[X] = \sum_{i=0}^{\infty} i \cdot Pr[X=i]$$

$$= \sum_{i=0}^{\infty} i \cdot \frac{e^{-\lambda}\lambda^i}{i!}$$

$$= e^{-\lambda} \sum_{i=0}^{\infty} \frac{i \cdot \lambda^i}{i!}$$

$$= e^{-\lambda} \sum_{i=\cancel{0}}^{\infty} \frac{\lambda^i}{(i-1)!}$$ (For $i=0$, the term becomes 0)

$$= e^{-\lambda} \cdot \frac{\lambda}{\lambda} \sum_{i=\cancel{0}}^{\infty} \frac{\lambda^i}{(i-1)!}$$

$$= e^{-\lambda} \cdot \lambda \sum_{i=1}^{\infty} \frac{\lambda^{i-1}}{(i-1)!}$$

$$= e^{-\lambda} \cdot \lambda \left(\frac{1}{1} + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots\right)$$

$$= e^{-\lambda} \cdot \lambda \cdot e^{\lambda}$$

$$= \lambda$$

$\therefore E[X] = \lambda$

8)

Given $f_x(x) = \alpha x^{-\alpha-1}$, $x \geq 1$

a) $\int_1^\infty \alpha x^{-\alpha-1} dx$

$= \alpha \int_1^\infty x^{-\alpha-1} dx$

$= \alpha \left[\frac{x^{-\alpha-1+1}}{-\alpha-1+1} \right]_1^\infty$

$= \frac{\alpha}{(-\alpha)} [x^{-\alpha}]_1^\infty$

$= - \left[\frac{1}{x^\alpha} \right]_1^\infty$

As $x \geq 1$ and $\alpha > 1$

$\frac{1}{x^\alpha}$ becomes 0 as $x \to \infty$

$= - [0 - 1]$

$= -(-1)$

$= 1$

$\therefore \int_1^\infty f_x(x) dx = 1$

b) $E[x] = \int_1^{\infty} x \cdot f_x(x) \, dx$

$= \int_1^{\infty} x \cdot \alpha \cdot x^{-\alpha-1} \, dx$

$= \alpha \int_1^{\infty} x^{-\alpha} \, dx$

$= \alpha \left[\frac{x^{-\alpha+1}}{-\alpha+1} \right]_1^{\infty}$

$= \frac{\alpha}{1-\alpha} \left[x^{1-\alpha} \right]_1^{\infty}$

As $1-\alpha < 0$, since $1 < \alpha < 2$

Again $x^{1-\alpha}$ tends to 0 as $x \to \infty$

$\Rightarrow \frac{\alpha}{1-\alpha} [0 - 1]$

$= \frac{\alpha}{\alpha - 1}$

$\therefore E[X] = \frac{\alpha}{\alpha - 1}$

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c)

Variance $= E[x^2] - (E[x])^2$

$= \int_1^{\infty} x^2 f_x(x) \, dx - \left(\frac{\alpha}{\alpha - 1} \right)^2$

Let us first calculate $E[x^2]$

$$E[x^2] = \int_1^\infty x^2 f_x(x)\,dx$$

$$= \int_1^\infty x^2 \cdot \alpha \quad x^{-\alpha-1}\,dx$$

$$= \alpha \int_1^\infty x^{-\alpha+1}\,dx$$

$$= \alpha \left[\frac{x^{2-\alpha}}{2-\alpha} \right]_1^\infty$$

$$= \frac{\alpha}{2-\alpha} \left[x^{2-\alpha} \right]_1^\infty$$

As $1 < \alpha < 2 \Rightarrow 2-\alpha > 0$

Therefore As $x \to \infty$ ~~$x^{2-\alpha}$~~ $x^{2-\alpha} \to \infty$

$\Rightarrow E[x^2] = \infty$

$\therefore$ $Var[x] = \infty$

5.) a.) Given, Indicator Random Variable with event E
we know for Indicator Random Variable,

$$E[I_E] = \sum_{x=0} x \cdot P_x^{(x)}$$

$$= 1 \cdot P_x^{(1)} + 0 \cdot P_x^{(0)}$$

$$\boxed{E[I_E] = Pr(x)}$$

5.) b.) For, Indicator Rv $Var(I_E)$

$$Var(I_E) = E(x^2) - (E(x))^2$$

$$= E(x^2) - Pr(x)^2 \quad [\because \text{from above } E[I_E] = Pr(x)]$$

$\Downarrow$

let us Expand $E(x^2)$

$$E(x^2) = \sum_{x=0}^{1} x^2 P(x) \quad \left[\because E[x^i] = \sum_{\forall x} P(x) x^i\right]$$

$$= P(0) \cdot (0)^2 + P(1)(1)^2 \quad \left[\because \text{Pmf } (f) \begin{cases} Pr(x) & \text{if } x=1 \\ 1-Pr(x) & \text{if } x=0 \\ 0 & \text{otherwise} \end{cases}\right]$$

$$= (1-Pr(x))(0) + Pr(x)(1)$$

$$= Pr(x)$$

$$\therefore Var(I_x) = Pr(x) - Pr(x)^2 \Rightarrow \boxed{Pr(x)(1-Pr(x))}$$

5) c.) Given $X \sim$ Geometric with $P<1$

Pmf $P_X(i) = (1-P)^{i-1} \cdot (P)$ (i flips to get first success)

$$E[X] = \sum_{x=1}^{\infty} i \cdot P(x)$$

$$= \sum_{i=\cancel{0}}^{\infty} i\,(1-P)^{i-1} \cdot P = P\sum_{i=1}^{\infty} i\,(1-P)^{i-1} = \frac{P}{1-P}\sum_{i=1}^{\infty} i\,(1-P)^{i} \quad — ①$$

Assume $x = 1-P$

~~Assume~~ $x = \cancel{1-P}$

$$\Rightarrow \frac{P}{1-P}\sum_{i=1}^{\infty} i\,x^{i}$$

Given from the question $\sum_{i=0}^{\infty} x^{i} = \frac{1}{1-x}$ $(\forall x<1)$

Differentiate on both sides

$$\sum_{i=0}^{\infty} i\,x^{i-1} = \frac{(-1)(-1)}{(1-x)^2}$$

$$\sum_{i=0}^{\infty} \frac{i \cdot x^{i}}{x} = \frac{1}{(1-x)^2}$$

$$\sum_{i=0}^{\infty} i \cdot x^{i} = \frac{x}{(1-x)^2} \quad — ②$$

Since ① & ② are similar

From ② $\sum_{i=0}^{\infty} i \cdot x^{i} = \frac{x}{(1-x)^2}$

$$\sum_{i=1}^{\infty} i x^i = \frac{x}{(1-x)^2}$$ [Since for $x = 0$
$(0) x^0 = 0$]

$$\therefore \quad E(X) = \frac{P}{1-P} \sum_{i=1}^{\infty} i x^i$$

$$= \frac{P}{1-P} \left(\frac{x}{(1-x)^2} \right)$$ [Substitute back $x = 1-P$]

$$= \frac{P}{\cancel{1-P}} \frac{\cancel{(1-P)}}{(1-1+P)^2} = \frac{P}{P^2} = \frac{1}{P}$$

$$\therefore \quad E(X) = \frac{1}{P}$$

5(d) Compute $Var(x)$

$$Var(x) = E(X^2) - (E(X))^2$$

$$E(x) = \frac{1}{P} \longrightarrow \text{From Q5(c)}$$

$$\left[E(X^i) = \sum_x P(x) \cdot x^i\right]$$

$$E(X^2) = \sum_{i=1}^{\infty} i^2 (1-P)^{i-1} P$$

$$= P \sum_{i=1}^{\infty} i^2 \frac{(1-P)^i}{1-P}$$

$$= \frac{P}{1-P} \sum_{i=1}^{\infty} i^2 (1-P)^i$$

Let us consider $x = 1-P$

$$= \text{Compute} \sum_{n=1}^{\infty} n^2 x^n$$

We have

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

Apply differentiation on both sides

$$\left[\frac{d x^n}{dx} = n x^{n-1}\right]$$

$$\Rightarrow \frac{d}{dx} \sum_{n=0}^{\infty} x^n = \frac{d}{dx}\left(\frac{1}{(1-x)}\right)$$

$$= \sum_{n=0}^{\infty} n x^{n-1} = \frac{-1 \, (-1)}{(1-x)^2}$$

$$\sum_{n=0}^{\infty} n x^n = \frac{x}{(1-x)^2}$$

Differentiate on both sides

$$\left[\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\right]$$

$$= \sum_{n=0}^{\infty} n\left(n x^{n-1}\right) = \frac{(1-x)^2(1) - x\,2(1-x)(-1)}{(1-x)^4}$$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{n^2 x^n}{x} = \cancel{(1-x)}\left[\frac{(1-x) + 2x}{(1-x)^{\cancel{4}\,3}}\right]$$

$$\sum_{n=0}^{\infty} n^2 x^n = \frac{x(1-x) + 2x^2}{(1-x)^3}$$

$$\therefore \sum_{n=1}^{\infty} n^2 x^n = \frac{x(1-x) + 2x^2}{(1-x)^3}$$ Substitute $x = (1-P)$

$$\therefore \sum_{i=1}^{\infty} i^2 (1-P)^i = \frac{(1-P)(1-(1-P)) + 2(1-P)^2}{(1-(1-P))^3}$$

$$= \frac{(1-P)(P) + 2(1-P)^2}{P^3} =$$

$$E(X^2) = \frac{P}{1-p}\sum_{i=1}^{\infty} i^2 (1-P)^i$$

$$E(X^2) = \frac{\cancel{p}^{\,1}}{(1-\cancel{p})}\left[\cancel{(1-p)}\left(\frac{p+2(1-p)}{p^{\cancel{3}\,2}}\right)\right]$$

$$= \frac{p+2-2p}{p^2}$$

$$Var(x) = E(x^2) - (E(x))^2$$

$$= \frac{p+2-2p}{p^2} - \left(\frac{1}{p}\right)^2$$

$$= \frac{2-p-1}{p^2} = \frac{1-p}{p^2}$$

$$\boxed{Var(x) = \frac{1-p}{p^2}}$$

7)

Given X is distributed as Geometric distribution

$\Rightarrow Pr(i) = (1-p)^i p$

$Pr(X \geq a+b \mid X \geq a) = Pr(X \geq b)$

Consider L.H.S

$Pr(X \geq a+b \mid X \geq a)$

According to Bayes Theorom

$$\Rightarrow \frac{Pr(X \geq a \mid X \geq a+b) \cdot P(X \geq a+b)}{Pr(X \geq a)}$$

The term $Pr(X \geq a \mid X \geq a+b)$ is always 1

Because given a, b are positive, so if x is greater than or equal to $a+b$, then it is definitely greater than a.

$$\Rightarrow \frac{Pr(X \geq a+b)}{Pr(X \geq a)}$$

$$= \frac{\sum_{i=a+b}^{\infty} Pr(X = i)}{\sum_{i=a}^{\infty} Pr(X = i)}$$

$$= \frac{\sum_{i=a+b}^{\infty} p \cdot (1-p)^{i-1}}{\sum_{i=a}^{\infty} p \cdot (1-p)^{i-1}}$$

$$= \frac{p \sum_{i=a+b}^{\infty} (1-p)^{i-1}}{p \cdot \sum_{i=a}^{\infty} (1-p)^{i-1}}$$

$$= \frac{p \cdot \left[(1-p)^{a+b-1} + (1-p)^{a+b} + ----\right]}{p \left[(1-p)^{a-1} + (1-p)^{a} + ---\right]}$$

$$= \frac{p \cancel{(1-p)^{a-1}} \left[(1-p)^{b} + (1-p)^{b+1} + --\right]}{p \cancel{(1-p)^{a-1}} \left[(1-p)^{0} + (1-p)^{1} + --\right]}$$

$$= \frac{p \left[\sum_{i=b+1}^{\infty} (1-p)^{i-1}\right]}{p \left[\sum_{i=1}^{\infty} (1-p)^{i-1}\right]}$$

$$= \frac{\sum_{i=b+1}^{\infty} p \cdot (1-p)^{i-1}}{\sum_{i=1}^{\infty} p \cdot (1-p)^{i-1}}$$

$$= \frac{Pr(X \geq b+1)}{Pr(X \geq 1)}$$

As ~~X~~ is a positive integer x is always ≥ 1

and $X \geq b+1 \Rightarrow X > b$.

$$= \frac{Pr(X > b)}{1}$$

$= Pr(X > b)$

$= R.H.S$

Hence Proved.

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b)

Given $Pr(Y \geq a+b \mid Y \geq a) = Pr(Y > b)$

Let us assume Y is distributed as Geometric Distribution and use induction method to prove it.

For $a = 1$

$L.H.S = Pr(Y \geq b+1 \mid Y \geq 1)$

$$= \frac{Pr(Y \geq 1 \mid Y \geq b+1)\, Pr(Y \geq b+1)}{Pr(Y \geq 1)}$$ (According to Baye's Theorem)

As Y follows Geometric Distribution

Y is always greater than 1.

$$\Rightarrow L.H.S = \frac{(1)\ Pr(Y \geq b+1)}{(1)}$$

$$= Pr(Y \geq b+1)$$

As b is a positive integer

$$Y \geq b+1 \Rightarrow Y > b$$

$$= Pr(Y > b)$$

$$= R.H.S$$

$\therefore$ It is true for $a = 1$

Assume it is true for $a = t$, where t is a positive constant.

$$\Rightarrow Pr(Y \geq b+t \mid Y \geq t) = Pr(Y > b)$$

$$\Rightarrow \frac{Pr(Y \geq t \mid Y \geq b+t)\, Pr(Y \geq b+t)}{Pr(Y \geq t)} = Pr(Y > b)$$

$$= \frac{(1) \sum_{i=b+t}^{\infty} Pr(Y = i)}{\sum_{i=t}^{\infty} Pr(Y = i)} = \sum_{i=b+1}^{\infty} Pr(Y = i)$$

$$\Rightarrow \frac{\sum_{i=b+t}^{\infty} (1-p)^{i-1} p}{\sum_{i=t}^{\infty} (1-p)^{i-1} p} = \sum_{i=b+1}^{\infty} (1-p)^{i} p$$

$$\Rightarrow \sum_{i=b+t}^{\infty} (1-p)^{i-1} \cancel{p} = \sum_{i=b+1}^{\infty} (1-p)^{i-1} p \cdot \sum_{i=t}^{\infty} (1-p)^{i-1} \cancel{p} \quad - (1)$$

Let the above equation be eq (1).

Prove it is true for $a = t+1$

$$L.H.S = \frac{\sum_{i=b+t+1}^{\infty} (1-p)^{i-1} p}{\sum_{i=t+1}^{\infty} (1-p)^{i-1} p}$$

$$R.H.S = \sum_{i=b+1}^{\infty} (1-p)^{i-1} p.$$

Let $L.H.S = K \cdot (R.H.S)$, where K is some real constant — (2)

$$\Rightarrow \frac{\sum_{i=b+t+1}^{\infty} (1-p)^{i-1} p}{\sum_{i=t+1}^{\infty} (1-p)^{i-1} p} = K \cdot \sum_{i=b+1}^{\infty} (1-p)^{i-1} p.$$

$$\Rightarrow \frac{p\left[(1-p)^{b+t} + (1-p)^{b+t+1} + \cdots\right]}{p\left[(1-p)^{t} + (1-p)^{t+1} + \cdots\right]} = K \cdot p \cdot \left[(1-p)^{b} + (1-p)^{b+1} + \cdots\right]$$

Using S_∞ formula for G.P

$$\Rightarrow \frac{\dfrac{p(1-p)^{b+t}}{1-(1-p)}}{\dfrac{p(1-p)^{t}}{1-(1-p)}} = \frac{K \cdot p \cdot (1-p)^{b}}{1-(1-p)}$$

$$\Rightarrow \frac{\dfrac{\cancel{p}(1-p)^{b+t}}{\cancel{p}}}{\dfrac{\cancel{p}(1-p)^{t}}{\cancel{p}}} = \frac{K \cdot \cancel{p} \cdot (1-p)^{b}}{\cancel{p}}$$

$$\Rightarrow \frac{(1-p)^{b+t}}{(1-p)^{t}} = K \cdot (1-p)^{b}$$

$$\Rightarrow \cancel{(1-p)^{b}} = K \cancel{(1-p)^{b}}$$

$$\therefore K = 1$$

From equation (2)

$$L.H.S = K\,(R.H.S)$$

$\therefore$ As $K = 1$

$L.H.S = R.H.S$

Hence proved that the random variable Y is distributed as Geometric Distribution.