1. $CDF(x) = x^2$.

$D = \{0.245, 0.424, 0.436, 0.592, 0.648, 0.685, 0.774, 0.842, 0.959, 0.995\}$

x	$F_Y(x)$	$\hat{F}_x^-$	$\hat{F}_x^+$	$\lvert\hat{F}_x^- - F_Y(x)\rvert$	$\lvert\hat{F}_x^+ - F_Y(x)\rvert$
0.245	0.06	0.0	0.1	0.06	0.04
0.424	0.18	0.1	0.2	~~0.08~~ 0.08	0.02
0.436	0.19	0.2	0.3	0.01	0.11
0.592	0.35	0.3	0.4	0.05	0.05
0.648	0.42	0.4	0.5	0.02	0.08
0.685	0.47	0.5	0.6	0.03	$\boxed{0.13}$
0.774	0.60	0.6	0.7	0	0.10
0.842	0.71	0.7	0.8	0.01	0.09
0.959	0.92	0.8	0.9	0.12	0.02
0.995	0.99	0.9	1.0	0.01	0.01

$0.13 < c = 0.25$ accept.

2. $X = \{3, 2\}$ $Y = \{2, 5\}$.

$H_0 : X \equiv Y$,

$T_{obs} = |2.5 - 3.5| = 1$.

Calculate the difference of means for the 24 possible permutations.

No.	Permutation	$T_i = \lvert X - Y \rvert$
1	32, 25	1
2	32, 52	1
3	32, 25	1
4	32, 52	1
5	35, 22	2
6	35, 22	2
7	22, 35	2
8	22, 53	2
9	23, 25	1
10	23, 52	1
11	25, 23	1
12	25, 32	1
13	22, 35	2
14	22, 53	2
15	23, 25	1
16	23, 52	1
17	25, 23	1
18	25, 32	1
19	52, 23	1
20	52, 32	1
21	52, 23	1
22	52, 32	1

23	53, 22	2
24	53, 22	2

$$P_{val} = \frac{1}{N!} \sum_{i=1}^{N!} I(T_i > T_{obs})$$

$$P_{val} = \frac{1}{24} \times 8 = 0.33. > 0.05.$$

Reject.

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3. From the given table, we get

$$P(\text{Player 1 wins}) = \frac{\text{Total Player 1 wins}}{\text{Total judged matches}} = \frac{146}{199}$$

	Judge A	Judge B	Judge C	
Player 1 wins	72	50	24	
Draw	8	5	3	
Player 1 loses	20	8	9	
Total	100	63	36	199

$$P(\text{Judge A}) = \frac{\text{Total for Judge A}}{\text{Grand total}}$$

$$= \frac{100}{199}.$$

If the outcomes are independent of judges,

Expected freq. of Player 1 win and Judge A

$= (\text{Grand total}) \times P(\text{Player 1 wins}) \times P(\text{Judge A})$

$\neq \cancel{1/199}$

Similarly, we populate the table below with the expected frequencies.

	Judge A	Judge B	Judge C	Total
P1 win	73.36	46.22	26.41	146
Draw	8.04	5.07	2.89	16
P1 lose	18.59	11.71	6.69	37
Total	100	63	36	199

Evaluating the χ^2 test:

Observed	Expected	$(E-O)^2/E$
72	73.36	0.025
50	46.22	0.309
24	26.41	0.220
8	8.04	0.00
5	5.07	0.001
3	2.89	~~0.107~~ 0.004
20	18.59	~~1.175~~ 0.107
8	11.71	1.175
9	6.69	0.798
$\Sigma O = 199$	$\Sigma E = 199$	$\Sigma = 2.639$

We now have a Q-statistic of 2.639

df (Degree of freedom) $= (3-1) \times (3-1) = 4$.

P-value $= P_r(\chi^2_4 > 2.639) = 1 - 0.38 = 0.62$.

Since, $0.62 > 0.05$, we fail to reject H_0.

$$\rho_{x,y} = \frac{\sum_{i=1}^{n} (x_i - \bar{x}).(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^{n} (y_i - \bar{y})^2}}$$

Judge A: Mean = 42.6
Judge B: Mean = 43.0
Judge C: Mean = 40.2

$\rho_{A,B} = \frac{277}{483.72} = 0.573$ (Positive corr)

$\rho_{B,C} = \frac{-30}{358.62} = -0.084$ (No corr.)

$\rho_{A,C} = \frac{42.8}{468.85} = 0.09$ (No corr)

As Probability of Player 1 winning each game is the same, the results for each of the judges should be correlated.

We infer that since the results from Judge C are not linearly correlated with those of Judge A and B, Judge C is not doing their job right.

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4. a) $x = [21.5, 24.5, 18.5, 17.2, 14.5, 23.2, 22.1, 20.5, 19.4, 18.1, 24.1, 18.5]$

$H_0: \bar{x} = 20$

$H_1: \bar{x} > 20$.

$n = 12$; degree of freedom $= n-1 = 11$.

$\alpha = 0.05$.

Sample mean $(\bar{x}) = 20.175$

Sample standard deviation $(\sigma) = 3.02$

$$T\text{-statistic} = \frac{\bar{x} - \mu}{se} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$T = 0.2006$

T-critical (For one tailed T-test) = 1.796

Since $T < T_{critical}$, we do not reject the null hypothesis.

b) Sample mean ($\bar{x}$) = 22.
$n = 12$.
Population mean (μ) = 20.
Population Std. dev. (σ) = 3.
$\alpha = 0.05$.

Since standard deviation is known, we will use the z-test.

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$z = \frac{22 - 20}{3/\sqrt{12}} = 2.31.$$

$P(z > 2.31) = 0.01$.

Since, $p < 0.05$, we reject the null hypothesis.

5. First we know $\bar{X} \sim N\left(\mu_1, \frac{\sigma_1^2}{n}\right)$ and $\bar{Y} \sim N\left(\mu_2 \frac{\sigma_2^2}{m}\right)$. Since X and Y are independent, $\bar{X} - \bar{Y} \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{m}\right)$.

$$T = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}}.$$

a) Type-I error is shown by

$$P(T < -\delta \mid H_0) = P\left(\frac{\bar{X} - \bar{Y}}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}} < -\delta \,\Big|\, \mu_1 > \mu_2\right)$$

$$= P\left(\frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}} < -\delta - \frac{\mu_1 - \mu_2}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}}\right)$$

$$= \Phi\left(-\delta - \frac{\mu_1 - \mu_2}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}}\right)$$

since $\frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}} \sim N(0,1)$.

Type-2 error is,

$$P(T > -\delta \mid H_1) = P\left(\frac{\bar{x}-\bar{y}}{\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}} > -\delta \,\Big|\, \mu_1 \le \mu_2\right)$$

$$= 1 - P\left(\frac{\bar{x}-\bar{y}}{\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}} \le -\delta \,\Big|\, \mu_1 \le \mu_2\right)$$

$$= 1 - P\left(\frac{\bar{x}-\bar{y}-(\mu_1-\mu_2)}{\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}} \le -\delta - \frac{\mu_1-\mu_2}{\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}}\right)$$

$$= 1 - \Phi\left(-\delta - \frac{\mu_1-\mu_2}{\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}}\right)$$

b) Let $t_{obs} = \dfrac{\bar{x}-\bar{y}}{\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}}$.

$$\text{P-value} = P(T < t_{obs})$$

$$= P\left(\frac{\bar{x}-\bar{y}}{\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}} < t_{obs}\right)$$

$$= P\left(\frac{\bar{x}-\bar{y}-(\mu_1-\mu_2)}{\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}} < t_{obs} - \frac{\mu_1-\mu_2}{\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}}\right)$$

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$$= \Phi\left(\frac{\bar{x}-\bar{y}-(\mu_1-\mu_2)}{\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}}\right)$$

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