Assignment 10

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Outline

Question

The random variable x is uniformly distributed in the interval $\theta - 2 < x < \theta + 2$. We observe 100 samples x_i and find that their average equals $\bar{x} = 30$. Find the 0.95 confidence interval of θ.

Solution

In this problem, x is uniform with $E\{x\} = \theta$ and $\sigma^2 = \frac{4}{3}$.
We can use , however, the normal approximation for $\bar{x}$ because n =100.
Here, $\gamma = 0.95$
We know,

$$\gamma = 1 - \delta \tag{1}$$

$$\implies \delta = 1 - \gamma = 1 - 0.95 = 0.05 \tag{2}$$

Also,

$$u = 1 - \frac{\delta}{2} = 1 - \frac{0.05}{2} = 0.975 \tag{3}$$

Standard normal percentile z_u for $u = 1 - \frac{\delta}{2}$ is $z_{0.975}$ which is equal to 1.967.
We obtain the interval

$$\bar{x} \pm z_{0.975}\frac{\sigma}{\sqrt{n}} = 30 \pm 1.967\frac{\frac{2}{\sqrt{3}}}{\sqrt{100}} \tag{4}$$

$$= 30 \pm 1.967\frac{1}{5\sqrt{3}} \tag{5}$$

$$= 30 \pm 0.227 \tag{6}$$