Assignment 11

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Outline

Question

Show that if
$\hat{E}\{s(t+\lambda)|s(t), s(t-\tau)\} = \hat{E}\{s(t+\lambda)|s(t)\}$ then $R_s(\tau) = Ie^{-\alpha|\tau|}$.

Solution

Since

$$\hat{E}\left\{s(t+\lambda)|s(t)\right\} = a\, s(t) \tag{1}$$

$$a = R(\lambda)/R(0) \tag{2}$$

it follows from the assumption that

$$s(t+\lambda) - a\, s(t) \perp s(t-\tau) \tag{3}$$

Hence

$$R(\lambda+\tau) = \frac{R(\lambda)}{R(0)} R(\tau) \tag{4}$$

The only continuous function satisfying the above is an exponential. This is easily shown if we assume that $R(\lambda)$ is differentiable for $\lambda > 0$. Differentiating (4) with respect to λ and setting $\lambda = 0^+$,we obtain

$$R'(\tau) + \alpha R(\tau) = 0 \tag{5}$$

$$\alpha = \frac{-R'(0^+)}{R(0)}, \tau > 0 \tag{6}$$

Equation (5) yields $R(\tau) = Ie^{-\alpha\tau}$ for $\tau > 0$.