Assignment 12

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June 16, 2022

Outline

Question

Let $P_n(z)$ represent the Levinson polynomial of the first kind.
(a) If one of the roots of $P_n(z)$ lie on the unit circle, then show that all other roots of $P_n(z)$ are simple and lie on the unit circle.
(b) If the reflection coefficient $s_k \neq 0$, then show that $P_k(z)$ and $P_{k+1}(z)$ have no roots in common.

Solution

(a) Let $z = e^{j\theta_1}$ represent one of the roots of the Levinson Polynomial $P_n(z)$ that lie on the unit circle.In that case

$$P_n\left(e^{j\theta_1}\right) = 0 \tag{1}$$

and substituting this into the recursion equation..

$$\sqrt{1 - s_n^2}P_n(z) = P_{n-1}(z) - zs_n\tilde{P}_{n-1}(z) \tag{2}$$

We get

$$|s_n| = \left|\frac{P_{n-1}\left(e^{j\theta_1}\right)}{\tilde{P}_{n-1}\left(e^{j\theta_1}\right)}\right| = 1 \tag{3}$$

so that

$$s_n = e^{j\alpha} \tag{4}$$

Let

$$P_{n-1}(e^{j\theta}) = R(\theta)e^{j\psi(\theta)} \tag{5}$$

and since $P_{n-1}(z)$ is free of zeros in $|z| \leq 1$,we have $R(\theta) > 0, 0 < \theta < 2\pi$, and once again substituting these into

$$\sqrt{1-s_n^2}P_n(z) = P_{n-1}(z) - zs_n\tilde{P}_{n-1}(z) \tag{6}$$

We obtain

$$\sqrt{1-s_n^2}P_n(e^{j\theta}) = R(\theta)e^{j\psi(\theta)} - e^{j(\theta+\alpha)}e^{j(n-1)\theta}R(\theta)e^{-j\psi(\theta)} \tag{7}$$

$$= R(\theta)[e^{j\psi(\theta)} - e^{j(n\theta+\alpha)}e^{-j\psi(\theta)}] \tag{8}$$

$$= 2jR(\theta)e^{j(n\theta+\alpha)/2}sin\left(\psi(\theta) - \frac{n\theta}{2} - \frac{\alpha}{2}\right) \tag{9}$$

Due to the strict Hurwitz nature of $P_{n-1}(z)$, as θ varies from 0 to 2π,there is no net increment in the phase term $\psi(\theta)$, and the entire argument of the sine term above increases by $n\pi$.
Consequently $P_n(e^{j\theta})$ equals zero atleast at n distinct points $\theta_1, \theta_2, ...\theta_n,\ 0 < \theta_i < 2\pi$.
However $P_n(z)$ is a polynomial odd degree n is z and can have atmost n zeros. Thus all the above zeros are simple and they all lie on the unit circle.

(b) Suppose $P_n(z)$ and $P_{n-1}(z)$ has a common zero at $z = z_0$. Then $|z_0| > 1$ and we obtain

$$z_0 s_n \tilde{P}_{n-1}(z_0) = 0 \tag{10}$$

which gives $s_n = 0$, since $\tilde{P}_{n-1}(z_0) = 0$,($\tilde{P}_{n-1}(z)$ has all its zeros in $|z| < 1$).
Hence $s_n \neq 0$ implies $P_n(z)$ and $P_{n-1}(z)$ do not have a common zero.