

Assignment 13

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Question

Classify the states of the Markov chains with the following transition probabilities

$$P = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

$$P = \begin{pmatrix} 0 & 0 & 1/3 & 2/3 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$P = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 2/3 & 0 \\ 0 & 0 & 2/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \end{pmatrix}$$

Solution

The chain represented by

$$P = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

is irreducible(every state in a Markov chain is accessible from every other state (possibly in different number of transitions)) and aperiodic(A chain is aperiodic if it is irreducible and if all states are aperiodic, which is ensured by one state being aperiodic).

The chain represented by

$$P = \begin{pmatrix} 0 & 0 & 1/3 & 2/3 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

is also irreducible and aperiodic.

The chain represented by

$$P = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 2/3 & 0 \\ 0 & 0 & 2/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 & 0 & 1/3 \end{pmatrix}$$

has two aperiodic closed sets $\{e_1, e_2\}$ and $\{e_3, e_4\}$ and a transient state e_5 .