

ASSIGNMENT 4

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Question: Find the probability that when a hand of 7 cards is drawn from a well shuffled deck of 52 cards, it contains

- (i) all Kings
- (ii) 3 Kings
- (iii) atleast 3 Kings

Solution: Given that a hand of 7 cards is drawn from a well shuffled deck of 52 cards.

Total number of possible hands = $\binom{52}{7}$

Let's a random variable X such that $X \in \{0, 1, 2, 3, 4\}$ denote the outcome of the given problem.

Event	Description
$X=0$	Hand of 7 cards with no Kings
$X=1$	Hand of 7 cards contain 1 King
$X=2$	Hand of 7 cards contain 2 Kings
$X=3$	Hand of 7 cards contain 3 Kings
$X=4$	Hand of 7 cards contain 4 Kings

TABLE I
RANDOM VARIABLE AND EVENT DISTRIBUTION

- (i) Number of hands with 4 Kings = $\binom{4}{4} \times \binom{48}{3}$

(Since the other 3 cards must be choosen from remaining 48 cards.)

The probability that hand of 7 cards contains all the kings is

$$\Pr(X = 4) = \frac{\binom{4}{4} \times \binom{48}{3}}{\binom{52}{7}} \quad (1)$$

$$= \frac{1}{7735} \quad (2)$$

- (ii) Number of hands with 3 Kings = $\binom{4}{3} \times \binom{48}{4}$

(Since the other 4 cards must be choosen from remaining 48 cards.)

The probability that hand of 7 cards contains

3 kings is

$$\Pr(X = 3) = \frac{\binom{4}{3} \times \binom{48}{4}}{\binom{52}{7}} \quad (3)$$

$$= \frac{9}{1547} \quad (4)$$

- (iii) The probability that hand of 7 cards contains atleast 3 kings is

$$\Pr(X \geq 3) = \Pr(X = 3) + \Pr(X = 4) \quad (5)$$

$$= \frac{1}{7735} + \frac{9}{1547} \quad (6)$$

$$= \frac{46}{7735} \quad (7)$$