Assignment 5(CBSE 12 Example 35)

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Outline

Question

- Solution
 - PMF

Question

The probability of a shooter hitting a target is $\frac{3}{4}$. How many minimum number of times must he/she fire so that the probability of hitting the target at least once is more than 0.99?

Solution

Let the shooter fire n times. Obviously, n fires are n Bernoulli trials. In each trial,

p = probability of hitting the target $=\frac{3}{4}$ q = probability of not hitting the target $=\frac{1}{4}$.

Let X be the random variable whose probability distribution is $B(n, \frac{3}{4})$. We know that,

$$\Pr(X = k) = \binom{n}{k} q^{n-k} p^k, k = 0, 1, 2, \dots n$$
 (1)

$$= \binom{n}{k} \left(\frac{1}{4}\right)^{n-k} \left(\frac{3}{4}\right)^k \tag{2}$$

$$= \binom{n}{k} \frac{3^k}{4^n} \tag{3}$$



Solution(contd)

Now, given that,

Pr (hitting the target at least once)
$$> 0.99$$
 (4)

$$\implies \Pr(X \ge 1) > 0.99 \tag{5}$$

$$\implies 1 - \Pr(X = 0) > 0.99 \tag{6}$$

$$\implies 1 - \binom{n}{0} \frac{3^0}{4^n} > 0.99 \tag{7}$$

$$\implies \binom{n}{0} \frac{1}{4^n} < 0.01 \tag{8}$$

$$\implies \frac{1}{4^n} < 0.01 \tag{9}$$

$$\implies 4^n > 100 \tag{10}$$

The minimum value of n to satisfy the above inequality is 4. Thus, the shooter must fire 4 times.

PMF

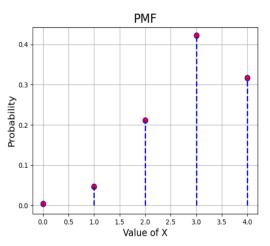


Figure: Plot of the PMF