Assignment 5

AKHILA, CS21BTECH11031

Question: The probability of a shooter hitting a target is $\frac{3}{4}$. How many minimum number of times must he/she fire so that the probability of hitting the target at least once is more than 0.99?

Solution: Let the shooter fire n times. Obviously, n fires are n Bernoulli trials.
In each trial,
p = probability of hitting the target $=\frac{3}{4}$
q = probability of not hitting the target $=\frac{1}{4}$.

Let X be the random variable whose probability distribution is $\text{B}\left(n, \frac{3}{4}\right)$.
We know that,

$$\Pr\left(X = k\right) = \binom{n}{k} q^{n-k} p^k, k = 0, 1, 2, \ldots n \tag{1}$$

$$= \binom{n}{k} \left(\frac{1}{4}\right)^{n-k} \left(\frac{3}{4}\right)^{k} \tag{2}$$

$$= \binom{n}{k} \frac{3^k}{4^n} \tag{3}$$

Now, given that,

$$\Pr\left(\text{hitting the target at least once}\right) > 0.99 \tag{4}$$

$$\implies \Pr\left(X \geq 1\right) > 0.99 \tag{5}$$

$$\implies \Pr\left(X = 1\right) + \Pr\left(X = 2\right) + \ldots \\ .. + \Pr\left(X = n\right) > 0.99 \tag{6}$$

$$\implies 1 - \Pr\left(X = 0\right) > 0.99 \tag{7}$$

$$\implies 1 - \binom{n}{0} \frac{3^0}{4^n} > 0.99 \tag{8}$$

$$\implies 1 - \binom{n}{0} \frac{1}{4^n} > 0.99 \tag{9}$$

$$\implies \binom{n}{0} \frac{1}{4^n} < 0.01 \tag{10}$$

$$\implies \frac{1}{4^n} < 0.01 \tag{11}$$

$$\implies 4^n > 100 \tag{12}$$

The minimum value of n to satisfy the above inequality is 4.
Thus, the shooter must fire 4 times.

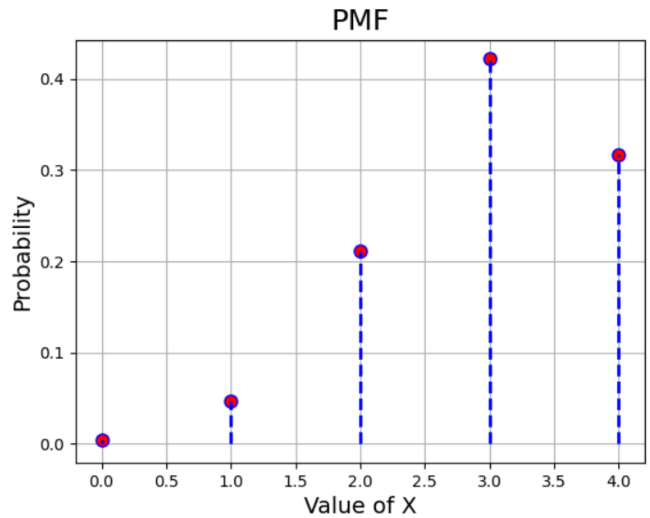

Fig. 1: Plot of the PMF