

Assignment 6

AKHILA, CS21BTECH11031

Question:

Let $X \sim U(0, 1)$. Show that $Y = -2 \log X$ is $\chi^2(2)$.

Solution:

Given, $X \sim U(0, 1)$

$$Y = -2 \log X$$

$$\implies x_1 = e^{-y/2}$$

Cumulative distribution function of Y

$$F_Y(y) = \Pr(Y \leq y) = \Pr(-2 \log X \leq y)$$

$$= \Pr(X \leq e^{-y/2})$$

We have,

$$F_Y(y) = F_X(e^{-y/2})$$

To find the Probability density function(pdf) of Y we simply differentiate both sides with respect to y:

$$f_Y(y) = \frac{-1}{2} e^{-y/2} f_X(e^{-y/2})$$

$$f_Y(y) = \frac{-1}{2} e^{-y/2} U(y)$$

$$\sim \text{Exponential}(2) \equiv \chi^2(2).$$

(or)

$$Y = -2 \log X$$

$$\implies x_1 = e^{-y/2}$$

$$\frac{dy}{dx} = \frac{-2}{x} = -2e^{y/2}$$

$$f_Y(y) = \frac{1}{\left| \frac{dy}{dx} \right|} f_X(x_1) = \frac{1}{2} e^{-y/2} U(y)$$

$$\sim \text{Exponential}(2) \equiv \chi^2(2).$$