Assignment 6(Papoulis chap 5 Ex 5.18)

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LATEX

Outline

Question

Let $X \sim U(0, 1)$. Show that $Y = -2 \log X$ is $\chi^2(2)$.

Solution

Given, $X \sim U(0, 1)$

$$Y = -2 \log X$$
$$\implies x_1 = e^{-y/2}$$

Cumulative distribution function of Y

$$F_Y(y) = \Pr(Y \le y) = \Pr(-2 \log X \le y)$$
$$= \Pr\left(X \le e^{-y/2}\right)$$

We have,

$$F_Y(y) = F_X(e^{-y/2})$$

. To find the Probability density function(pdf) of Y we simply differentiate both sides with respect to y:

$$f_Y(y) = \frac{-1}{2} e^{-y/2} f_X(e^{-y/2})$$

$$f_Y(y) = \frac{-1}{2} e^{-y/2} U(y)$$

$$\sim \text{Exponential}(2) \equiv \chi^2(2).$$

(or)

$$Y = -2 \log X$$

$$\implies x_1 = e^{-y/2}$$

$$\frac{dy}{dx} = \frac{-2}{x} = -2e^{y/2}$$

$$f_Y(y) = \frac{1}{|\frac{dy}{dx}|} f_X(x_1) = \frac{1}{2} e^{-y/2} U(y)$$

$$\sim \text{Exponential}(2) \equiv \chi^2(2).$$