# Assignment 8

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# Outline

# Question

We denote by $X_m$ a random variable equal to the number of tosses of a coin until heads shows for the $m$th time. Show that if P$\{h\} =$ p. then $E\left\{x_m\right\} = \frac{m}{p}$.

# Solution

$$1 + X + X^2 + ... + X^n ... = \frac{1}{1-X}, |X| < 1 \tag{1}$$

Differentiating on both sides we get..

$$\implies 1 + 2X + 3X^2 + ....nX^{n-1} + ... = \sum_{k=1}^{\infty} kX^{k-1} \tag{2}$$

$$= \frac{1}{(1-X)^2} \tag{3}$$

The RV $X_1$ equals to the number of tosses until head shows for the first time.
Hence, $X_1$ takes the values 1,2,.... with $\Pr\{X_1 = k\} = pq^{k-1}$.
Hence,

$$E\{X_1\} = \sum_{k=1}^{\infty} k \Pr\{X_1 = k\} = \sum_{k=1}^{\infty} kpq^{k-1} \tag{4}$$

$$= \frac{p}{(1-q)^2} = \frac{1}{p} \tag{5}$$

Starting the count after the first head shows,we conclude that the RV $X_2 - X_1$ has the same statistics as the RV $X_1$. Hence,

$$E\left\{X_2 - X_1\right\} = E\left\{X_1\right\} \tag{6}$$

$$E\left\{X_2\right\} = 2E\left\{X_1\right\} = \frac{2}{p} \tag{7}$$

Reasoning similarly,we conclude that

$$E\left\{X_n - X_{n-1}\right\} = E\left\{X_1\right\} \tag{8}$$

Hence,

$$E\left\{X_n\right\} = E\left\{X_{n-1}\right\} + E\left\{X_1\right\} = \frac{n-1}{p} + \frac{1}{p} = \frac{n}{p} \tag{9}$$