

Assignment 9

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Outline

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Question

Suppose the Conditional distribution of X given $Y = n$ is binomial with parameters n and p_1 . Further, Y is a binomial random variable with parameters M and p_2 . Show that the distribution of X is also binomial. Find its parameters.

Solution

If X has the binomial distribution $B(m, p)$ given by

$$p_n = \Pr(X = n) = \binom{m}{n} p^n q^{m-n}, 0 \leq n \leq m \quad (1)$$

then

Moment generating function

$$\Gamma(z) = E\{z^X\} = \sum_{n=0}^m \binom{m}{n} p^n q^{m-n} z^n = (pz + q)^m \quad (2)$$

Given that the Conditional distribution of X given $Y = n$ is binomial with parameters n and p_1 .

$$\Pr(X = k|Y = n) = \binom{n}{k} p_1^k q_1^{n-k}, k = 0, 1, 2, \dots, n$$

$$\begin{aligned} E[z^X|Y = n] &= \sum_{k=0}^n z^k \Pr(X = k|Y = n) \\ &= (p_1 z + q_1)^n \quad (3) \end{aligned}$$

Also, We have

$$\Gamma(z) = E \{ z^X \} = E \{ E[z^X | Y = n] \} \quad (4)$$

$$= \sum_{n=0}^M E[z^X | Y = n] \Pr(Y = n) \quad (5)$$

$$= \sum_{n=0}^M (p_1 z + q_1)^n \binom{M}{n} p_2^n q_2^{M-n} \quad (6)$$

$$= \sum_{n=0}^M \binom{M}{n} [p_2 (p_1 z + q_1)]^n q_2^{M-n} \quad (7)$$

$$= (p_1 p_2 z + p_2 q_1 + q_2)^M \quad (8)$$

But

$$1 - p_1 p_2 = 1 - (1 - q_1)(1 - q_2) = q_1 p_2 + q_2 \quad (9)$$

Hence

$$\Gamma(z) = (pz + q)^M \quad (10)$$

Where $p = p_1 p_2$.

Therefore, $X \sim \text{Binomial}(M, p_1 p_2)$

i.e, The distribution of X is also binomial with parameters M and $p_1 p_2$.