# ASSIGNMENT 2

AKHILA, CS21BTECH11031

**Question:** If $x = \tan(\log y)$, prove that

$$\left(1+x^2\right)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$$

**Solution:** Given, $x = \tan(\log y)$

$$\implies \tan^{-1} x = \log y \tag{1}$$

$$\implies y = e^{\tan^{-1} x} \tag{2}$$

Differentiating on both sides with respect to x..

$$\implies \frac{dy}{dx} = \frac{d}{dx}\left(e^{\tan^{-1} x}\right) \tag{3}$$

$$\implies \frac{dy}{dx} = \left(e^{\tan^{-1} x}\right)\frac{d}{dx}\left(\tan^{-1} x\right) \tag{4}$$

$$\implies \frac{dy}{dx} = \left(e^{\tan^{-1} x}\right)\left(\frac{1}{1+x^2}\right) \tag{5}$$

$$\implies \frac{dy}{dx} = \frac{e^{\tan^{-1} x}}{1+x^2} \tag{6}$$

From (2)...

$$\therefore \frac{dy}{dx} = \frac{y}{1+x^2} \tag{7}$$

Again differentiating on both sides with respect to x..

$$\implies \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{y}{1+x^2}\right) \tag{8}$$

$$= \frac{d}{dx}\left(\frac{1}{1+x^2}\right)y + \frac{1}{1+x^2}\frac{dy}{dx} \tag{9}$$

$$= \frac{-1(2x)}{\left(1+x^2\right)^2}(y) + \frac{1}{1+x^2}\frac{dy}{dx} \tag{10}$$

$$= \frac{-2x}{\left(1+x^2\right)}\frac{y}{\left(1+x^2\right)} + \frac{1}{1+x^2}\frac{dy}{dx} \tag{11}$$

From (7)...

$$\implies \frac{d^2y}{dx^2} = \frac{-2x}{\left(1+x^2\right)}\frac{dy}{dx} + \frac{1}{\left(1+x^2\right)}\frac{dy}{dx} \tag{12}$$

$$\implies \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{dy}{dx}(1-2x) \tag{13}$$

$$\implies \left(1+x^2\right)\frac{d^2y}{dx^2} = -\frac{dy}{dx}(2x-1) \tag{14}$$

$$\therefore \left(1+x^2\right)\frac{d^2y}{dx^2} + \frac{dy}{dx}(2x-1) = 0 \tag{15}$$

Hence proved.

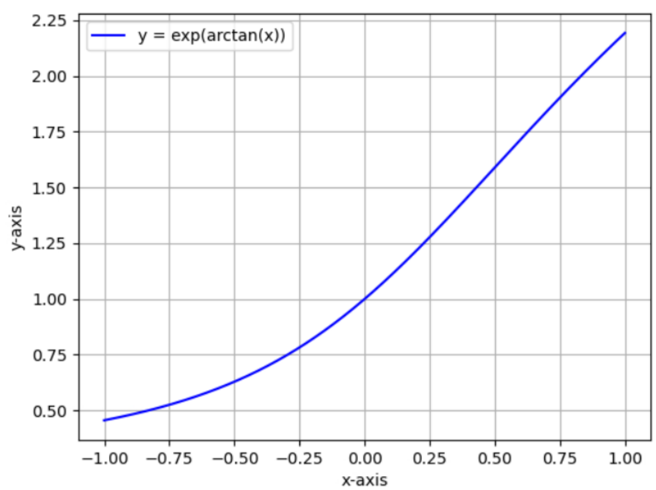


Fig. 1: Graph of $y = e^{\tan^{-1} x}$