Central - limit - theorem

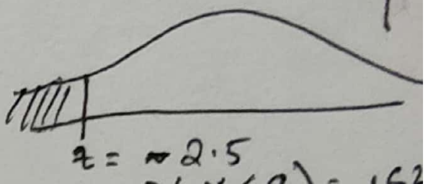
1) $\mu_{pop} = 10$

$\sigma_{pop} = 4$

$n = 100$

$\sigma_{samp} = \frac{4}{\sqrt{100}} = 0.4$

$\mu_s = \bar{x}$



$P(\mu_s < 9) = P(\bar{x} < 9) \Rightarrow z < \frac{9 - \mu}{\sigma_{sample}}$

$z = \frac{x - \mu}{\sigma}$

$\Rightarrow z < \frac{9-10}{0.4} \Rightarrow z < -2.5$

probability for $z < -2.5 \approx .0062$

Probability of sample mean less than 9

is $.621$

2) $\mu_{pop} = 50$

$\sigma_{pop} = 15$

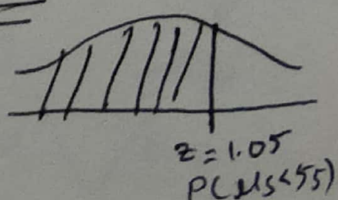
$n = 10$

$\sigma_{sam} = 15/\sqrt{10}$

Max weight is 550
So avg weight for
10 persons should be
< 55 to reach safely.

$P(\mu_s < 550) \Rightarrow z < \frac{550 - 50}{15/\sqrt{10}} = \frac{\sqrt{10}}{3} = 1.05$

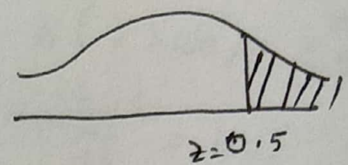
$P(\mu_s < 55) = 85.314$



3) $\mu_{pop} = 2.4$

$\sigma_{pop} = 2$

$n = 100$



if 250 tickets to be sold by 100 passengers avg tickets purchase for 100 passengers should be ≥ 2.5

$P(\bar{X} \geq 2.5) \Rightarrow Z \geq \dfrac{2.5 - 2.4}{2/\sqrt{100}} = \dfrac{0.1}{2/10} = 0.5$

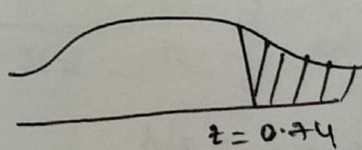
probability for $z < 0.5$ is $.69146 \Rightarrow 69.146\%$

$z > 0.5$ is $1 - .691 \Rightarrow .31 \Rightarrow 31\%$

4) $\mu_{pop} = 96$

$\sigma_{pop} = 16$

$n = 35$



$P(\bar{X} > 98) \Rightarrow Z \geq \dfrac{98 - 96}{16/\sqrt{35}} = \dfrac{\sqrt{35}}{8} = 0.74$

$P(z \leq 0.74) = 77\%$

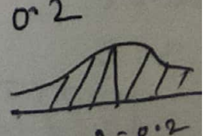
Probability of $z > 0.74 = 23\%$

5) Since the original distribution is normal even with sample sizes < 30 will be normally distributed

9) $\mu_{pop} = 6$ $\quad P(\bar{X} \leq 6.2) \Rightarrow z \leq \dfrac{6.2 - 6}{1} = 0.2$

$\sigma_{pop} = 1$

$\Rightarrow P = 57.9\%$



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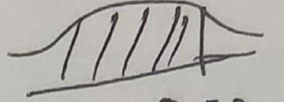
b) $n = 100$



$z=2$

97.7%

$P(\bar{x} \leq 6.2) \Rightarrow z \leq \frac{6.2-6}{1/\sqrt{100}} \Rightarrow z \leq 2$

$P(\bar{x} \leq 6.2) = \underline{97.7\%}$

(6) This kind of reasoning works only for sample sizes > 100

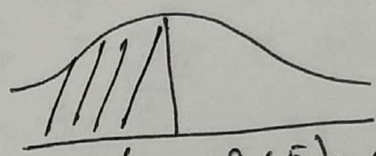
as $P(\bar{X} \leq 6.2)$ is increasing with sample size.

(7)

$\mu_{pop} = 268$

$\sigma_{pop} = 15$

$n = 25$



$P(z < -2.65) = .379$

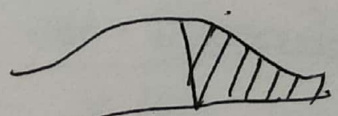
$P(\bar{x} < 260) \Rightarrow z < \frac{260-268}{15/\sqrt{25}} = -\frac{8}{3} \approx -2.67$

$P(\bar{x} < 260)$ is $\underline{0.37\%}$

(8) No, since the probability of having 260 as length of pregnancy is 0.37% which is very very less.

(9) $\mu_{pop} = 172$

$\sigma_{pop} = 29$



$z = 0.62$

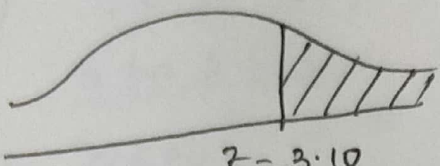
a) $n=1$

$P(\bar{x} > 190) \Rightarrow z > \frac{190-172}{29} = \frac{18}{29} = 0.62$.

$P(\bar{x} \leq 190)$ is 73.23% $\Rightarrow$ $P(\bar{x} > 190) = \underline{26.77\%}$

b) $n = 25$

$P(\bar{X} > 190) \Rightarrow Z > \frac{190-172}{29/\sqrt{25}} = \frac{18 \times 5}{29} = 3.10$



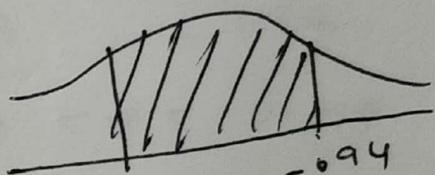
$z = 3.10$

$\rightarrow P(\bar{X} \leq 190)$ is 99.90%

$\Rightarrow P(\bar{X} > 190)$ is $\underline{\underline{.10\%}}$

c) $\frac{4750}{25} = 190$

$P(\bar{X} > 190)$ is $\underline{\underline{.10\%}}$



-2.35 $-.94$

$.94\%$ 17.3%

(10) $\mu_{pop} = 4$

$\sigma_{pop} = 1.5$

$n = 50$

$P(3.5 < \bar{X} < 3.8) \Rightarrow \frac{3.5-4}{1.5/\sqrt{50}} < Z < \frac{3.8-4}{1.5/\sqrt{50}}$

$-\frac{\sqrt{50}}{3} < Z \quad \frac{2 \times \sqrt{50}}{15} \Rightarrow -2.35 < Z < -.94$

$0.00939 \qquad .17361$

$\Rightarrow P(3.5 < \bar{X} < 3.8) = 17.3\% - .94\%$

$\approx \underline{\underline{16.35\%}}$

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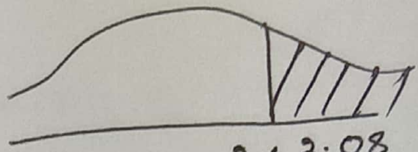
(11)

$\mu_{pop} = 23.1$

$\sigma_{pop} = 3.1$

$n = 6$



$z \leq 3.08$

Prob = 99.89

$P(\bar{x} > 27) \Rightarrow z > \dfrac{27 - 23.1}{3.1/\sqrt{6}} = \dfrac{3.9 \times \sqrt{6}}{3.1}$

$= 3.08$

$\Rightarrow P(\bar{x} > 27) = .11\%$

(12)

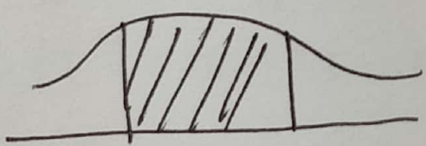
$\mu_{pop} = 21.50$

$\sigma_{pop} = 2.22$

$n = 8$

$P(20 < \bar{x} < 23) \Rightarrow \dfrac{20 - 21.50}{2.22/\sqrt{8}} < z < \dfrac{23 - 21.50}{2.22/\sqrt{8}}$

$= -0.616 < z < 0.616$



$z < (-0.616)$ Prob $= 26.9\%$

$z < (0.616)$ Prob $= 73.07\%$

Here for $z = .616$ Took avg of $z = .61 \ \& \ .62$

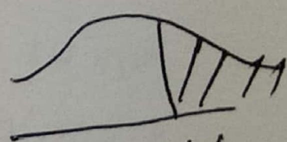
$\Rightarrow P(20 < \bar{x} < 23) \approx 46.17\%$

(13)

$\mu_{pop} = 75$

$\sigma_{pop} = 5$

a) $P(\bar{x} > 83) \Rightarrow z > \dfrac{83 - 75}{5} = 1.6$



$z > 1.6$

$1 - .94520$

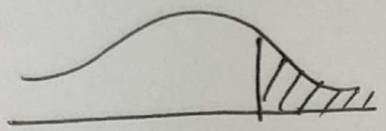
$\approx .055$

$\Rightarrow 5.5\%$

b) $P(\bar{X} > 83) \Rightarrow z > \dfrac{83-75}{5/\sqrt{5}} = 1.6 \times \sqrt{5}$

$= 3.577$

≈ 3.58



$z > 3.58$

$1 - .99983 = 0.00017 \Rightarrow P(\bar{X} > 83)$

$= \underline{\underline{0.017\%}}$

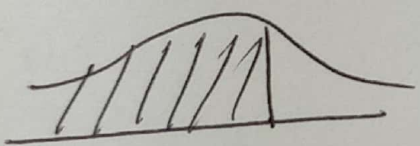
(14)

$\mu_{pop} = 28.3$

$\sigma_{pop} = 2.3$

$n = 10$

$P(\bar{X} < 27) \Rightarrow z < \dfrac{27 - 28.3}{2.3/\sqrt{10}} = -1.78$



$z < -1.78$

Prob = $\underline{\underline{3.754}}$