**Topics: Normal distribution, Functions of Random Variables**

* The time required for servicing transmissions is normally distributed with = 45 minutes and = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
* 0.3875
* 0.2676
* 0.5
* 0.6987

**ANS: 0.2676**

**Given  *= 45minutes, = 8 minutes***

***The work begins after 10min, so the avg.time increases from 45 to 55min***

***Z = (X-)/6***

***= (60-55)/8***

***= 0.625***

***#Python Coding***

**P\_value = stats.norm.cdf(abs(0.625))**

**P\_value = 0.7340144**

**1-0.7340 = 0.266(approx..)**

* The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean = 38 and Standard deviation =6. For each statement below, please specify True/False. If false, briefly explain why.
* More employees at the processing center are older than 44 than between 38 and 44.

**False**

Given,400 clerical employees, = 38, Standard deviation =6

Z-score = (Value - Mean)/SD

Z-score for 44  = (44 - 38)/6 =1=>84.13%

People above 44 age =100-84.13%=15.87%

Which is 63 out of 400

Z-score for 38=(38-38)/6=0=50%

Hence People b/w 38 and 44 age =84.13-50=34.13%

Which is 137 out of 400

**Hence we can say that the more employees lies b/w 38 and 44 rather than older than 44.**

* A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**True.**

Z-score = (Value - Mean)/SD

Z-score for 30=(30-38)/6=-1.33=36 out of 400

**Yes,The statement above mentioned is correct and it attracts about 36 employees.**

* If *X1* ~ *N*(μ, σ2) and *X1* ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:**

Given X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2)  are two independent identically distributed random variables

According to the Central Limit Theorem, any large sum of independent, identically distributed(iid) random variables is approximately Normal

To find: 2X1

Thus, following the property of multiplication, we get

2X1 ~ N(2μ,22σ2)=> 2X1~N(2μ,4σ2)

And following property of addition

X1+X2~N(μ+ μ, σ2+ σ2)~N(2μ,2σ2)

And the difference between the two is given by

2X1-( X1+X2)~ N(2μ-2μ,2σ2+4σ2)~N(0,6σ2)

**The mean of 2X1 and X1+X2 is same but the variance of 2X1 is 2 times more than variance of X1+X2**

* Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
* 90.5, 105.9
* 80.2, 119.8
* 22, 78
* 48.5, 151.5
* 90.1, 109.9

ANS**: D.48.5, 151.5**

Ans: Given: P(a<x<b) = 0.99 ,mean =100,Standard Deviation = 20

#Using Python

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as

Z(0.5) = stats.norm.ppf(0.005)= -2.576

Z value at 99.5 percentile is given as

Z(99.5) = stats.norm.ppf(0.995) = 2.576

Z = (x - 100)/20 = > x = 20z+100

a = -(20\*2.576) + 100= 48.5

b = (20\*2.576)+100= 151.5

**Two values symmetric about mean for the given standard normal distribution are[48.5,151.5]**

* Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
* Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
* Specify the 5th percentile of profit (in Rupees) for the company
* Which of the two divisions has a larger probability of making a loss in a given year?

*Ans:* Given $1=Rs. 45

*a*). 95% of the probability lies between 1.96 standard deviations of the mean.

print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')

Range is Rs (99.00810347848784, 980.9918965215122) in Millions

b). 5th percentile of profit (in Rupees) for the company

X= 540+(-1.645)\*(225)

print('5th percentile of profit (in Million Rupees) is',np.round(X,))

5th percentile of profit (in Million Rupees) is 170.0

c). Loss is when profit < 0

# Probability of Division 1 making a loss P(X<0)

stats.norm.cdf(0,5,3)

0.0477903522728147

# Probability of Division 2 making a loss P(X<0)

stats.norm.cdf(0,7,4)

0.040059156863817086