

1. Determine all functions $f:R \rightarrow R$ such that the equality
$$f(\lfloor x \rfloor y) = f(x) \lfloor f(y) \rfloor$$
holds for all $x, y \in R$. (Here $\lfloor z \rfloor$ denotes the greatest integer less than or equal to z .)
2. Let I be the incentre of triangle ABC and let Γ be its circumcircle. Let the line AI intersect Γ again at D . Let E be a point on the arc \overline{BDC} and F a point on the side BC such that
$$\angle BAF = \angle CAE \angle \frac{1}{2} \angle BAC.$$
Finally, let G be the midpoint of the segment IF . Prove that the lines DG and EI intersect on Γ .
3. Let N be the set of positive integers. Determine all functions $g:N \rightarrow N$ such that
$$(g(m) + n)(m + g(n))$$
is a perfect square for all $m, n \in N$.
4. Let P be a point inside the triangle ABC . The lines AP , BP and CP intersect the circumcircle Γ of triangle ABC again at the points K, L and M respectively. The tangent to Γ at C intersects the line AB at S . Suppose that $SC=SP$. Prove that $MK=ML$.
5. In each of six boxes $B_1, B_2, B_3, B_4, B_5, B_6$ there is initially one coin. There are two types of operation allowed:
Type 1: Choose a nonempty box B_j with $1 \leq j \leq 5$. Remove one coin from B_j and add two coins to B_{j+1} .
Type 2: Choose a nonempty box B_k with $1 \leq k \leq 4$. Remove one coin from B_k and exchange the contents of (possibly empty) boxes B_{k+1} and B_{k+2} .
Determine whether there is a finite sequence of such operations that results in boxes B_1, B_2, B_3, B_4, B_5 being empty and box B_6 containing exactly $2010^{2010^{2010}}$ coins. (Note that $a^{(b^c)}$.)
6. Let a_1, a_2, a_3, \dots be a sequence of positive real numbers. Suppose that for some positive integer s , we have
$$a_n = \max\{a_k + a_{n-k} | 1 \leq k \leq n-1\}$$
for all $n > s$. Prove that there exist positive integers l and N , with $l \leq s$ and such that $a_n = a_l + a_{n-l}$ for all $n \leq N$.