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Morse-Lyapunov-Based Control of Rigid Body Motion on TSE(3) via Backstepping[⋆]

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In this paper, a rigid body controller is designed on the tangent bundle TSE(3) of the Lie group SE(3) using the backstepping technique. The controller is capable of treating the rotational and translational motions of the rigid body simultaneously. The system states considered in the control design are rotational and translational displacements and velocities of the body. As a result, the states of the system are composed of the 4×4 configuration tensor of the body and its six-dimensional augmented velocity vector. In addition, the use of geometric mechanics in the control design results in almost global asymptotic stability of the resulting motion which is proved via a Morse-Lyapunov-based approach.

Nomenclature

e_i	elements of the natural basis ($i = 1, 2, 3$)
g	4×4 tensor of configuration of the rigid body
$\mathbb{I}$	inertia tensor of the body
J	matrix of inertia of the body
K_1, K_2	control gain matrices
k_3, κ	control gain scalars
m	mass of the body
r	displacement vector of the center of mass with respect to and expressed in the inertial frame
R	rotation matrix from body frame to inertial frame
$\mathfrak{so}(3)$	space of 3×3 real skew-symmetric matrices
SO(3)	Lie group of three-dimensional rotations
$\mathfrak{se}(3)$	Lie algebra of SE(3)
SE(3)	Lie group of three-dimensional rotations and translations
TSE(3)	tangent bundle of the Lie group SE(3)
v	linear velocity of the center of mass of the body
V	Lyapunov function
$\mathbb{V}$	augmented velocity of the body
u	nonlinear control input
ω	angular velocity of the body expressed in the body-fixed frame

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I. Introduction

Different parameterizations have so far been utilized for rigid body attitude and position representation and feedback control. Recent research using the framework of geometric mechanics and control has been successful in demonstrating almost globally stable control of rigid body attitude and position. Control schemes based on local attitude parameterizations (such as the Euler angle representations) for limited motion ranges cannot be applied to maneuverable vehicles with large ranges of rotational motion without encountering singularities. The most commonly used attitude parameterization set are the unit quaternions which are defined in terms of the principal rotation elements. However, the quaternions are not unique as a single attitude corresponds to two antipodal quaternions on the three-sphere S^3. When used for attitude control, quaternion-based feedback may result in large rotations by more than 180 deg, a phenomenon known as unwinding[1], although this can be avoided by using discontinuous feedback or nonlinear control laws.

Almost-global asymptotically stable Lyapunov-based feedback controllers utilize Morse-Lyapunov (M-L) functions, which have isolated critical points on rigid body attitude $\mathrm{SO}(3)$[2]. Combining rigid body attitude and translational motion with rotation matrices used for the attitude representation results in a more convenient dynamical representation of rigid body motion. The configuration space of translations and rotations in three-dimensional Euclidean space is the special Euclidean Lie group $\mathrm{SE}(3)$.

A fundamental attitude parameterization set is the principal rotation vector obtained from the multiplication of the principal rotation angle and principal rotation axis. Exponential coordinates, i.e. the principal rotation vector, associated with the linear space of the Lie algebra $\mathfrak{se}(3)$ may be used to design feedback controllers[3–5]. This representation is minimal and has a nonlinear kinematic differential equation. In Ref. [3], a tracking control scheme for decentralized spacecraft formation flying was developed, where the concept of a virtual leader was used to generate continuous-time state trajectories to be tracked by each spacecraft in the formation, and exponential coordinates were used to express state tracking errors for the relative motion. Exponential coordinates were also used in Ref. [4] to design a Lyapunov-based asymptotic tracking control of spacecraft motion relative to an asteroid using exponential coordinates associated with the Lie group $\mathrm{SE}(3)$. Later, the spacecraft motion relative to the asteroid was addressed using a continuous finite-time control scheme, where, again, the control law was expressed in terms of the exponential coordinates[5]. Note that exponential coordinates are composed of the principal rotation angle (and the translational part), and therefore they include the disadvantages associated with principal rotation angles, i.e. the ambiguity at $\theta = 180$ deg. However, control design on $\mathrm{TSE}(3)$ and the use of the Morse functions avoids such disadvantages.

There have been few papers in the literature discussing rigid body control design on $\mathrm{TSE}(3)$, the tangent bundle associated with $\mathrm{SE}(3)$. The robust feedback tracking control of autonomous underwater vehicles (AUVs) with ocean wave disturbance was studied[6], where the control design for the AUV motion was extended to yield almost global asymptotic tracking of reference trajectories in $\mathrm{TSE}(3)$ in the presence of disturbance inputs. An attitude tracking scheme was implemented in[7] to obtain a feedback control torque guaranteeing almost global asymptotic stability (AGAS) of the desired attitude and angular velocity trajectory. Feedback tracking control of an autonomous vehicle moving in a three-dimensional Euclidean space was considered, where the tracked trajectories in $\mathrm{TSE}(3)$ were created by concatenating simple motion primitives using a motion planning algorithm.

In this research, the state space used for feedback control is the tangent bundle, $\mathrm{TSE}(3)$. An attitude stabilization and regulation control scheme that avoids unwinding and provides AGAS of the desired attitude is developed. Since rigid body dynamics are given by its translational and attitude motions, its state is described by position, attitude, and translational and rotational velocities. To guarantee AGAS of the proposed feedback controller, a Morse-Lyapunov-based approach is utilized. In contrast to Refs. [3–5] where feedback controllers were based on exponential coordinates, here the designed feedback controller is defined as a nonlinear function of the configuration and velocity. The control design on $\mathrm{TSE}(3)$ allows for the rotational and translational motion of the spacecraft to be treated simultaneously. This is especially advantageous for motions in which perturbing forces or torques couple the translational and rotational motion. In contrast to another $\mathrm{TSE}(3)$ controller given in Ref. [7], backstepping technique is used in this control design for the proof of stability, and the performance of the proposed controller is compared with that in the literature in terms of the control effort and robustness to initial tumbling in the system.

The remainder of this paper is organized as follows. The dynamics and control design on $\mathrm{TSE}(3)$ are given in Section II. A Lyapunov stability proof of the proposed methodology is discussed in Section III. Numerical simulations of the controller design for the motion of a single rigid body are presented and discussed in Section IV. Finally, the concluding remarks are provided in Section VI.

II. Dynamics and Control Design on TSE(3)

In this section, the dynamics of a rigid body are treated within the framework of geometric mechanics, which makes it convenient to deal with the attitude dynamics and position regulation of the rigid body in a global setting. Then, a nonlinear controller is proposed on TSE(3) to drive the states of the system to the equilibrium corresponding to the identity attitude with zero angular velocity in an almost globally asymptotically stable sense.

A. System Formulation on SE(3)

The configuration (i.e. position and attitude) of a rigid body can be represented by an element

$$g = \begin{bmatrix} R & r \\ 0_{1\times 3} & 1 \end{bmatrix} \in \mathrm{SE}(3), \tag{1}$$

where $R \in \mathrm{SO}(3)$ is the rotation matrix from the rigid body frame to the inertial frame and $r \in \mathbb{R}^3$ is the position vector from the origin of the inertial frame to the center of mass of the rigid body expressed in the inertial frame.

The augmented velocity vector of the rigid body is

$$\mathbb{V} = [\omega^T, v^T]^T \in \mathbb{R}^6, \tag{2}$$

where $\omega \in \mathbb{R}^3$ and $v \in \mathbb{R}^3$ denote the angular and translational velocities, respectively, expressed in the body frame. According to Eqs. (1) and (2), the state of the vehicle can be represented by $(g, \mathbb{V}) \in \mathrm{SE}(3) \times \mathbb{R}^6 = \mathrm{TSE}(3)$, the tangent bundle of SE(3). Now, using the coupled equations, the position and attitude can be considered simultaneously, and hence control design in TSE(3) is more versatile compared to techniques that consider the translational and attitude dynamics separately, particularly when translational/rotational coupling is present.

A set of mappings required to express the dynamics in a compact form are defined in the following.

Definition 1 The adjoint action map is defined for $g = g(R, r) \in \mathrm{SE}(3)$ as

$$\mathrm{Ad}_g = \begin{bmatrix} R & 0_{3\times 3} \\ r^\times R & R \end{bmatrix} \in D \subset \mathbb{R}^6, \tag{3}$$

where, for $\Omega \in \mathbb{R}^3$,

$$\Omega^\times = \begin{bmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{bmatrix} \in \mathfrak{so}(3) \tag{4}$$

where the space of 3×3 real skew-symmetric matrices is denoted by $\mathfrak{so}(3)$, the Lie algebra of the Lie group SO(3), such that $\mathbf{e}_1^\times \mathbf{e}_2 = \mathbf{e}_1 \times \mathbf{e}_2$ for $\mathbf{e}_1, \mathbf{e}_2 \in \mathbb{R}^3$.

Note that multiplying the Adjoint operator defined in Eq. (3) by the velocity array $\mathbb{V}$ results in the angular and translational relative velocities expressed in the inertial frame with respect to the body frame.

Definition 2 The adjoint operator $\mathrm{ad}_{\mathbb{V}}$ is defined for $\mathbb{V} = [\omega^T,\ v^T]^T \in \mathbb{R}^6$ as

$$\mathrm{ad}_{\mathbb{V}} = \begin{bmatrix} \omega^\times & 0_{3\times 3} \\ v^\times & \omega^\times \end{bmatrix} \in D \subset \mathbb{R}^6 \tag{5}$$

and the co-adjoint operator is defined as

$$\mathrm{ad}^*_{\mathbb{V}} = \mathrm{ad}^T_{\mathbb{V}} = \begin{bmatrix} -\omega^\times & -v^\times \\ 0_{3\times 3} & -\omega^\times \end{bmatrix} \tag{6}$$

Definition 3 The wedge map $(.)^\vee : \mathbb{R}^6 \to \mathfrak{se}(3)$ is defined for $\mathbb{V} = [\omega^T,\ v^T]^T \in \mathbb{R}^6$ as

$$\mathbb{V}^\vee = \begin{bmatrix} \omega^\times & v \\ 0_{1\times 3} & 0 \end{bmatrix} \in \mathfrak{se}(3) \tag{7}$$

Hence, $\mathbb{R}^6$ is isomorphic to the Lie algebra $\mathfrak{se}(3)$ of SE(3).

B. Rigid body dynamics using SE(3) formulation

The kinematic and kinetic equations of motion of rigid body with respect to the inertial frame expressed in the body frame of the rigid body can be written as

$$\dot{g} = g\mathbb{V}^{\vee} \tag{8a}$$

$$\dot{\mathbb{V}} = \mathbb{I}^{-1}\mathrm{ad}^{*}_{\mathbb{V}}\mathbb{I}\mathbb{V} + \mathbb{I}^{-1}u, \tag{8b}$$

where all the states are time dependent and $\mathbb{I}$ is the inertia tensor given by

$$\mathbb{I} = \begin{bmatrix} J & 0_{3\times3} \\ 0_{3\times3} & mI_3 \end{bmatrix} \in \mathbb{R}^{6\times6} \tag{9}$$

in which J is the inertia tensor of rotational motion and m is the scalar mass.

C. Proposed Controller

To design the controller, a nonlinear function of the configuration and velocity is defined as

$$\psi(g, \mathbb{V}) = \mathbb{V} + K_1 l(g) \tag{10}$$

where $K_1 = \mathrm{blockdiag}(\mathrm{k}_{11}, \mathrm{k}_{12}) \in \mathbb{R}^{6\times6}$ is a positive definite control gain matrix such that $k_{12} \neq I_3$ and

$$l(g) = [s^T(R),\ r^T]^T \tag{11}$$

and the functions $s(\cdot) : \mathrm{SO}(3) \to \mathfrak{S} \subset \mathbb{R}^3$ and $\dot{s}(\cdot) : \mathrm{SO}(3) \to \mathfrak{S}$ are defined as[9]

$$s(R) = \sum_{i=1}^{3} a_i (R^T e_i)^{\times} e_i = \sum_{i=1}^{3} \left(R^T A^T e_i\right)^{\times} e_i, \tag{12a}$$

$$\dot{s}(R, \omega) = (\mathrm{tr}(AR)I_3 - R^T A)\omega \tag{12b}$$

where e_i, $i = 1, 2, 3$, are the elements of the natural basis in $\mathbb{R}^3$, and $A = [\mathrm{diag}(a_1, a_2, a_3)]$ with the scalars a_1, a_2, and a_3 selected such that $a_1 > a_2 > a_3 \geq 1$. The nonlinear controller is proposed on TSE(3) as

$$u(t) = -\mathbb{I}K_1\dot{l} - \mathrm{ad}^{*}_{\psi - K_1 l}\mathbb{I}(\psi - K_1 l) - \mathbb{I}K_2\psi - \mathbb{I}\kappa[0_{1\times3}, r^T R^T]^T \tag{13}$$

where K_2 is a positive definite control gain matrix and κ is a scalar control gain. The stability of the closed-loop dynamics obtained from Eqs. (8) and (13) is studied below.

III. Lyapunov Stability Analysis of the Controller on TSE(3)

Considering the definition of $\psi(g, \mathbb{V})$ in Eq. (10), driving the state ψ to zero is insufficient to guarantee that $\mathbb{V}$ and $l(g)$ both go to zero. That is, a quadratic Lyapunov function defined in the form of $\psi^T P \psi$ for a positive definite matrix P is only positive semidefinite in $l(g)$ and $\mathbb{V}$. Therefore, a positive definite Lyapunov function should be sought by adding a positive semidefinite Morse function to a positive definite quadratic function in order to drive $\psi(g, \mathbb{V})$ and its derivatives to zero. Hence, the total Lyapunov function is

$$V = V_1 + V_2(R) > 0,\ \forall (g, \mathbb{V}) \neq (I, 0) \tag{14}$$

where

$$V_1 = \psi^T P \psi + \kappa(1 - k_{12}) r^T P_{22} r, \quad V_2 = \phi(\mathrm{tr}(A - AR)), \tag{15}$$

where $P = \mathrm{blockdiag}(\mathrm{P}_{11}, \mathrm{P}_{22}) > 0$ is a diagonal matrix, and $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ is a $\mathcal{C}^2$ function such that $\phi(0) = 0$ and $0 < \phi'(x) \leq \alpha(x)$ for all $x \in \mathbb{R}^+$ where $\alpha(\cdot)$ is a class-$\mathcal{K}$ function[7,10]. The attitude-dependent Morse function $V_2(R)$ has one minimum (at $R = I_3$) on SO(3), as well as one maximum and two saddle points, as shown in[9]. For positive definite diagonal matrix A with distinct diagonal entries $a_1 > a_2 > a_3 \geq 1$, the Morse function $V_2(R)$ is positive definite on SO(3) and vanishes only when $R = I_3$.

By substituting Eq. (13) into Eq. (8), the closed-loop dynamics can be obtained as

$$\dot{\psi} = -K_2\psi - \kappa[0_{1\times 3},\ (Rr)^T]^T \tag{16}$$

Differentiating the Lyapunov function V_1 and using Eq. (16) yields

$$\begin{aligned}\dot{V}_1 &= 2\left(\psi^T P\dot{\psi} + \kappa(1-k_{12})r^T P_{22}\dot{r}\right)\\ &= \psi^T(-2PK_2)\psi - 2\kappa v^T P_{22}Rr - 2k_{12}\kappa r^T P_{22}Rr + 2\kappa(1-k_{12})r^T P_{22}\dot{r}\end{aligned} \tag{17}$$

Using the backstepping technique[10], it is supposed that the kinematic differential equation in Eq. (8a) can be stabilized by a state feedback control law of the form

$$\mathbb{V} = -l(g) \tag{18}$$

Therefore, the time derivative of V_1 in Eq. (17) becomes

$$\begin{aligned}\dot{V}_1 &= \psi^T(-2PK_2)\psi - 2(\kappa - \kappa k_{12} - \kappa(1-k_{12}))r^T P_{22}Rr\\ &= \psi^T(-2PK_2)\psi = \psi^T(-2P\ \mathrm{diag}(k_{21}I_3, k_{22}I_3))\psi\end{aligned} \tag{19}$$

where $\dot{r} = Rv$ is used.

Also, since $\frac{d}{dt}\mathrm{tr}(.) = \mathrm{tr}(\frac{d(.)}{dt})$, and using the chain rule, the Morse function V_2 in Eq. (15) can be differentiated as

$$dV_2 = -\phi'(\mathrm{tr}(A - AR))\mathrm{tr}(AR\omega^\times)dt \tag{20}$$

where the property $d(\mathrm{tr}(A - AR)) = d(\mathrm{tr}(-AR)) = -\mathrm{tr}(A\dot{R}dt)$ and the rotational kinematics $\dot{R} = R\omega^\times$ from Eq. (8) are used. Now, without loss of generality, let $\phi(\cdot)$ in Eq. (15) be defined as $\phi(x) = k_3 x$ ($k_3 > 0$). Using this assumption, Eq. (20) becomes

$$dV_2 = -k_3\mathrm{tr}(AR\omega^\times)dt \tag{21}$$

However, according to the virtual kinematic control input defined in Eq. (17),

$$dV_2 = k_3\mathrm{tr}(ARs^\times(R))dt \tag{22}$$

It can be shown that the following properties hold for all $A \in R^{3\times 3}$ and $b \in \mathbb{R}^3$:

$$\mathrm{tr}(Ab^\times) = b^T(A^T - A)^{|}, \quad (.)^{|\times} = (.) \tag{23a}$$

$$\mathrm{tr}(A) = \sum_{i=1}^{3} e_i^T A e_i \tag{23b}$$

$$\left(A^T - A\right)^{|} = \sum_{i=1}^{3} e_i^\times A^T e_i \tag{23c}$$

where $(.)^{|} : \mathbb{R}^3$ is the unskew operator, i.e. $(b^\times)^{|} = b$ ($b \in \mathbb{R}^3$). Using the properties given in Eq. (23), and the definition of $s(R)$ in (12a),

$$\begin{aligned}\mathrm{tr}(ARs^\times(R)) &= s^T(R)\left((AR)^T - AR\right)^{|}\\ &= s^T(R)\sum_{i=1}^{3} e_i^\times (AR)^T e_i = -s^T(R)\sum_{i=1}^{3}\left(R^T A^T e_i\right)^\times e_i\\ &= -s^T(R)s(R)\end{aligned} \tag{24}$$

Therefore, the Morse function derivative on the trajectory can be evaluated from Eqs. (22) and (24) as

$$\dot{V}_2 = -k_3 s^T(R)s(R) \le 0 \tag{25}$$

which is negative semidefinite on SO(3). Note that V_2 vanishes when $R = I_3$ (the global minimum), as well as at $R = \mathrm{diag}(1, -1, -1)$, $\mathrm{diag}(-1, 1, -1)$, and $\mathrm{diag}(-1, -1, 1)$ (one of which is the global maximum while the other two are saddles).

Finally, the time derivative of the M-L function can be obtained from Eqs. (19) and (25) as

$$\dot{V} = \dot{V}_1 + \dot{V}_2 = -k_3 s^T(R)s(R) - 2\psi^T(PK_2)\psi \tag{26}$$

which is negative for all $(g, \mathbb{V})$ except at $(g, \mathbb{V}) = (I, 0)$ (which is an asymptotically stable equilibrium) and $(\text{diag}(1, -1, -1, 1), 0)$, $(\text{diag}(-1, 1, -1, 1), 0)$, and $(\text{diag}(-1, -1, 1, 1), 0)$ (which are three saddle equilibria with stable manifolds of zero Lebesgue measure). Note that the three non-degenerate unstable attitudes on SO(3) $\text{diag}(1, -1, -1)$, $\text{diag}(-1, 1, -1)$, and $\text{diag}(-1, -1, 1)$ are associated with the 180 deg rotations of the body frame with respect to the inertial frame about either of the body axes. According to the Morse lemma[11], these critical points are isolated. Therefore, since the stable manifolds of these equilibria have zero measure in TSE(3), the equilibrium state $(g^*, \mathbb{V}^*) = (I, 0)$ is almost globally asymptotically stable. Since, according to Theorem 8.4 in Ref. [10], the angular velocity converges globally to zero, the union of $(\mathcal{E}, 0)$, where the set $\mathcal{E}$ is defined as

$$\mathcal{E} = \{(\text{diag}(1, -1, -1, 1)), \quad (\text{diag}(-1, 1, -1, 1)), \quad (\text{diag}(-1, -1, 1, 1))\}, \tag{27}$$

is a zero measure embedded sub-manifold of TSE(3). This latter fact is due to the non-increasing nature of the Lyapunov function away from those equilibria. This can be shown in more details by the fact that $\dot{V}$ vanishes when both $\dot{V}_1$ and $\dot{V}_2$ go to zero. When $\dot{V}_2$ goes to zero, the attitude approaches either of the following:

$$R \to I, \quad R \to \text{diag}(1, -1, -1), \quad R \to \text{diag}(-1, 1, -1), \quad R \to \text{diag}(-1, -1, 1) \tag{28}$$

the last three of which correspond to the attitude part of $\mathcal{E}$ given in Eq. (27). Now, assuming that $\dot{V}_2 = 0$ (i.e. $s(R) = 0$), then, according to the quadratic form of $\dot{V}_1$ given in Eq. (19) and the definitions of $\psi(\cdot)$ and $l(\cdot)$ given in Eqs. (10) and (11), $\dot{V}_1 \to 0$ implies that

$$\omega \to 0 \tag{29}$$

and that

$$v + k_{12} r = 0 \tag{30}$$

However, according to the backstepping transformation given in Eq. (18), Eq. (30) yields that

$$(r, v) \to (0, 0) \tag{31}$$

when $k_{12} \neq 1$. Therefore, according to Eqs. (28), (29), and (31), the system states $(g, \mathbb{V})$ defined in TSE(3) go to either the desired equilibrium $(I, 0)$ or any of the members of $\mathcal{E}$ (with zero probability), and hence the AGAS of $(g^*, \mathbb{V}^*) = (I, 0)$ is guaranteed.

In the next section, the performance of the proposed controller is compared with that of the TSE(3) control scheme given in Ref. [7] in which the control problem of tracking a desired continuous trajectory for an AUV was addressed in the presence of gravity, buoyancy, and fluid dynamic forces and moments. For the sake of comparison, the dynamics presented in Ref. [7], are simplified for the case of regulation without considering gravity, buoyancy, and fluid dynamic forces and moments as

$$M\dot{v} = mv^\times \omega + \phi_c \tag{32a}$$

$$J\dot{\omega} = -\omega^\times J\omega - v^\times mv + \tau_c \tag{32b}$$

where, ϕ_c and τ_c are the control force and moment, respectively. Furthermore, the control laws become simplified as

$$\phi_c = -L_v v - R^T N r \tag{33a}$$

$$\tau_c = -L_\omega \omega - k \sum_{i=1}^{3} a_i e_i^\times A^T e_i \tag{33b}$$

where L_v, L_ω, and N are control gain matrices, and k is a scalar control gain[7]. It can be seen that the system dynamics are given in the present work in a more compact form than those in Ref. [7]. As a result, fewer Lyapunov functions were required in this work than in the other work (see [7] for their stability proof).

IV. Numerical simulations and discussions

This section describes numerical simulations for regulation control of rigid body pose, using the proposed AGAS TSE(3) control scheme given in Eq. (13). The proposed controller has been evaluated in two cases: a) without initial tumbling and b) with initial tumbling. The numerical values of control gains are given in Table 1. The scalars a_1, a_2, and a_3 are selected as 1.2, 1.1 and 1, respectively. The rigid body is required to obtain the desired configuration in the inertial frame, which equals to the 4×4 identity matrix. The mass and moment of inertia of the rigid body are selected as $m = 60$ kg and $J = \mathrm{diag}(4.97, 6.16, 8.37)$ kg.m^2, respectively. The initial conditions for the displacement vector of the mass center, and translational velocity are selected as $r_0 = [10, -1, 1]^T$ and $v_0 = [1, -0.2, -0.3]^T$. The initial orientations for cases (a) and (b) mentioned above are

$$R_0 = \begin{bmatrix} 0.1840 & -0.8602 & 0.4755 \\ 0.8602 & 0.3750 & 0.3455 \\ -0.4755 & 0.3455 & 0.809 \end{bmatrix}, \quad \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

respectively, and the initial angular velocities for those cases are $\omega_0 = [0.2, -0.3, 0.4]^T$ rad/s and $[20, -30, 40]^T$ rad/s, respectively. In the simulation results, the rigid body attitude is represented by the principal rotation angle θ. The logarithm map is used to convert the rotation matrix R to its associated exponential coordinate Θ, i.e. $\log(\cdot) : \mathrm{SO}(3) \to \mathfrak{so}(3)$. The principal rotation angle $\theta = \|\Theta\|$ is defined as

$$\begin{aligned} \Theta^\times &= \log R = \begin{cases} \frac{\theta}{2\sin\theta}(R - R^T), & \theta \in (-\pi, \pi)\backslash\{0\} \\ 0, & \theta = 0 \end{cases} \\ \theta &= \cos^{-1}\left(\frac{\mathrm{tr}(R) - 1}{2}\right) \end{aligned} \tag{34}$$

In the remainder of this section, the performance of the proposed controller is evaluated for the two aforementioned cases, where a 2% settling time envelope is considered. Initial tumbling of the body depends on the choice of the initial angular velocity. The results are then compared to the M-L-based controller on TSE(3) introduced in Ref. [7] for the same settling time.

In the first case, the controller is implemented to the rigid body without initial tumbling and its performance is compared to the controller in Ref. [7]. In the second case, the controller is implemented to the tumbling rigid body. Figure 1 compares the performance of the proposed controller and the TSE(3) controller introduced in Ref. [7] for the control gains given in Table 1. In this figure, the principal rotation angle, the norm of the position vector r of the rigid body mass center, and angular- and translational velocity norms are compared. The control gains of both controllers are chosen such that the settling times of the principal rotation angle and position vector norm are almost identical, i.e. the rotational and translational motions are suppressed almost simultaneously. As can be seen in Fig. 1, the principal rotation angle gradually decreases until it goes to zero. As demonstrated, both the angular and translational velocities vanish as a result of which the augmented velocity vector of the rigid body $\mathbb{V}$ defined in Eq. (2) goes to zero. Based on Eq. (10), ψ is a function of $\mathbb{V}$ and $l(g)$. The vector of position in inertial frame also becomes zero and, according to the definitions of $l(g)$ and $s(R)$ in Eqs. (11) and (12a), when the rotation matrix becomes identity, $l(g)$ vanishes. In other words, as the system states $(g, \mathbb{V})$ go to $(I, 0)$, ψ goes to zero, which agrees with the results of the time plot of ψ depicted in Fig. 2.

Table 1. Control gains of the controller in Eq. (13) and those of the controller in Ref. [7]

Present Work	Ref. [7]
$k_{11} = 0.134$	$L_v = 20[\mathrm{diag}(1,1,1)]$
$k_{12} = 1.1$	$N = 0.07[\mathrm{diag}(1,1,1)]$
$k_{21} = 1.1$	$L_\omega = [\mathrm{diag}(1,1,1)]$
$k_{22} = 0.024$	k = 0.1
$\kappa = 0.02$	

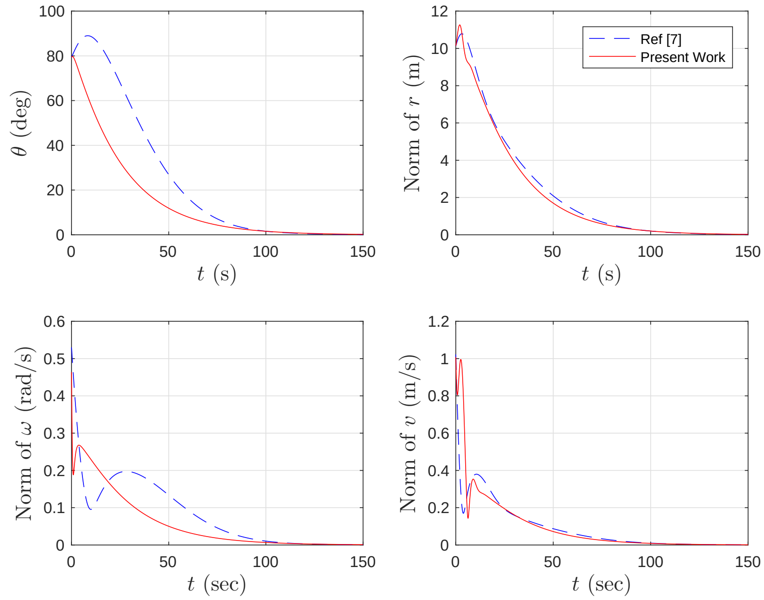

Figure 1. Principal rotation angle (top-left), norm of position vector of the mass center (top-right), angular velocity norm (bottom-left), and translational velocity norm (bottom-right) of the rigid body for the proposed controller (solid) and the $\mathrm{TSE}(3)$ **controller introduced in Ref. [7] (dashed) in the case of no initial tumbling for the gains shown in Table 1.**

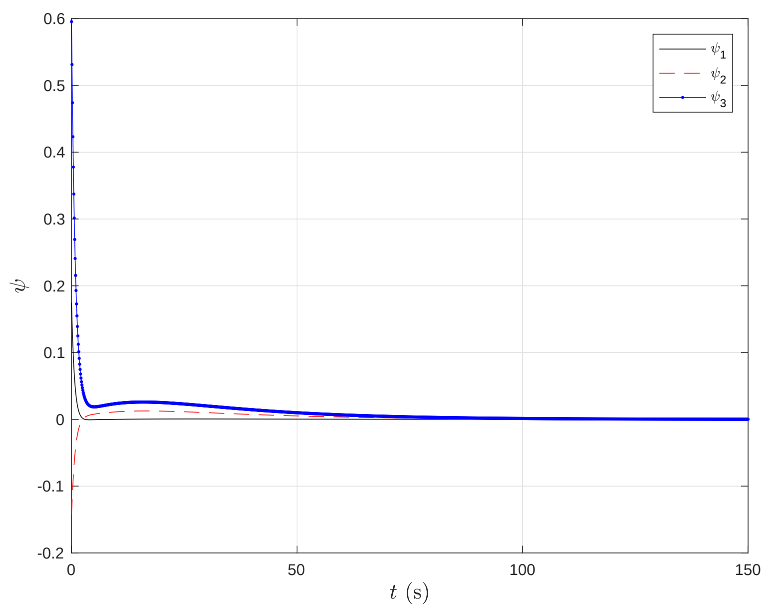

Figure 2. Components of the ψ **function given in Eq. (10) for the proposed controller in the case of no initial tumbling.**

The control input vector norms of the proposed controller and that proposed in Ref. [7] are compared together in Fig. 3. Also, the numerical values of the integrated translational and attitude control efforts are provided in Table 2 for the both controllers. It can be seen that the integrated translational control effort of the controller in Ref. [7] is less than that proposed here for the gains given in Table 1, while the attitude control effort are close. Therefore, the controller in Ref. [7] has a better overall performance in terms of the control effort for the same settling time for those sets of gains.

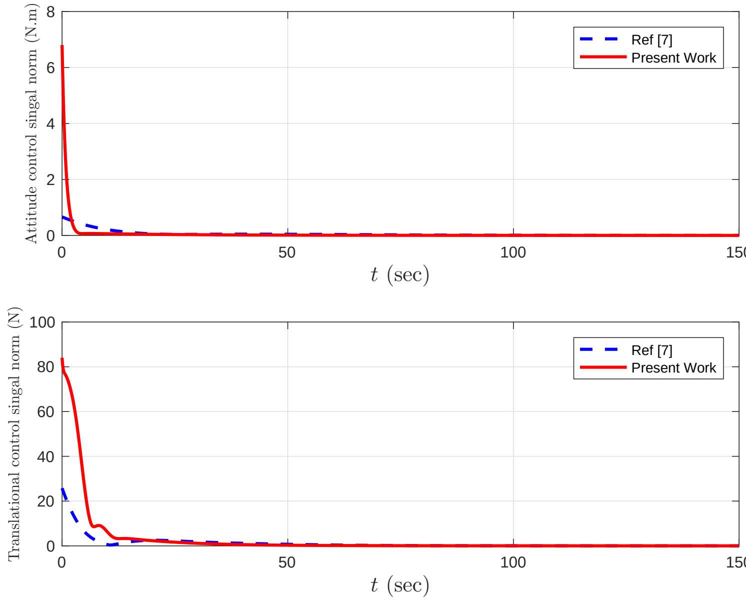

Figure 3. Control signal norm of the proposed controller (solid) and the TSE(3) controller introduced in Ref. [7] (dashed) in the case of no initial tumbling.

Table 2. Integrated control efforts

	Translational	Attitude
Present Work	419.84 (N s)	7.43 (N m s)
Ref. [7]	159.60 (N s)	7.56 (N m s)

For further investigation of the performance of the two controllers, they are implemented to the system with initial tumbling. In the second case, the controller (13) is applied to the rigid body with initial tumbling, i.e. when the initial angular velocity norm is large enough to cause tumbling. In this case, the simulations are done with the same control gains as in the first case. By implementing the TSE(3) controller in Ref. [7] for the system with initial tumbling, divergence was obtained as shown in Fig. 4. However, the controller introduced in Eq. (13) is capable of treating the system with initial tumbling without the necessity to re-adjust the control gains. In order for the controller in Ref. [7] to be able to overcome the initial tumbling and have a satisfying performance, its control gains need to be tuned again. Hence, the proposed controller shows more robustness than that in Ref. [7] when initial tumbling exists. Numerical simulations analogous to those in Fig. 1 are given in Fig. 5 for the proposed controller when applied to the system with initial tumbling. As demonstrated, because of large initial tumbling, the principal rotation angle reaches 180 deg in the beginning. Furthermore, since this angle varies between −180 and 180 deg, it does not surpass 180 deg. Based on the same explanation pointed out in the first case, it can be seen that as the augmented velocity vector and $l(g)$ reach zero, ψ also goes to zero, which is shown in the time plot of ψ in Fig. 6.

V. Acknowledgment

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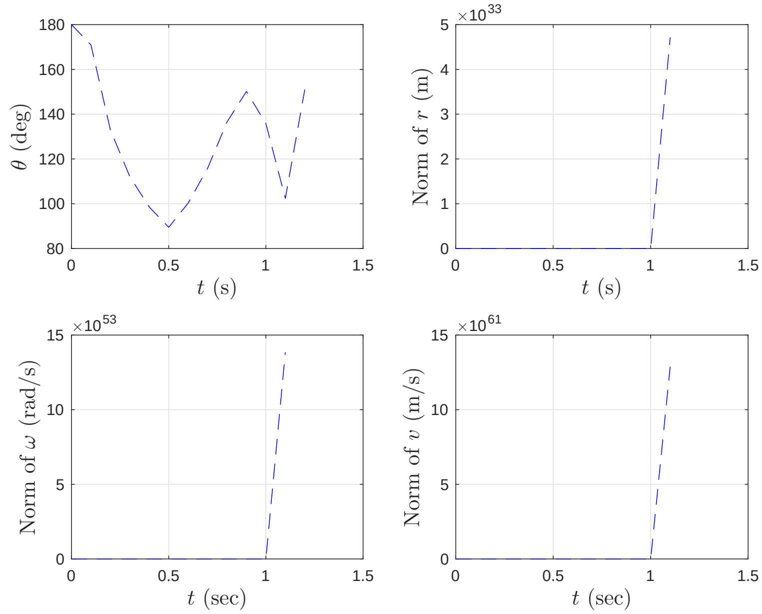

Figure 4. Principal rotation angle (top-left), norm of position vector of the mass center (top-right), angular velocity norm (bottom-left), and translational velocity norm (bottom-right) of the rigid body for the controller given in Ref. [7] in the case of initial tumbling for the gains shown in Table 1.

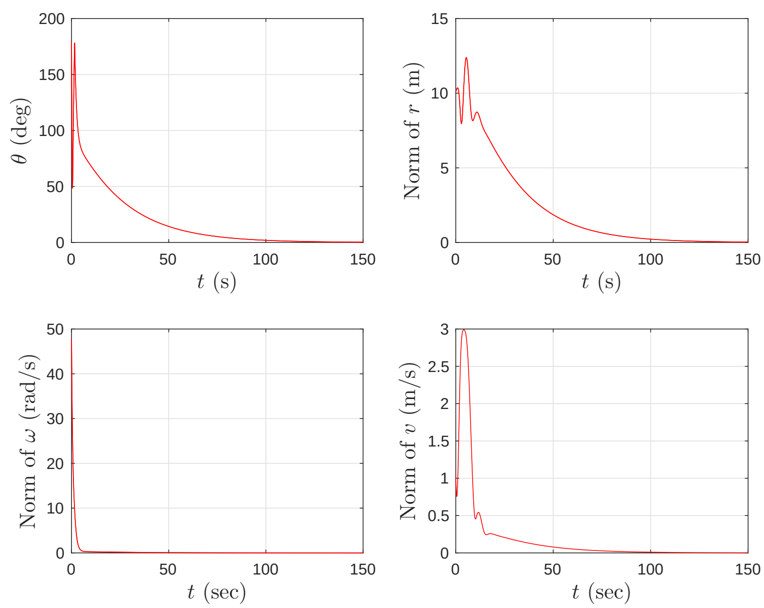

Figure 5. Principal rotation angle (top-left), norm of position vector of the mass center (top-right), angular velocity norm (bottom-left), and translational velocity norm (bottom-right) of the rigid body for the proposed controller in the case of initial tumbling for the gains shown in Table 1.

VI. Conclusions

In this paper, an almost globally asymptotically stable Morse-Lyapunov (M-L)-based controller was designed on TSE(3), the tangent bundle of the Lie group SE(3), using the backstepping technique. The

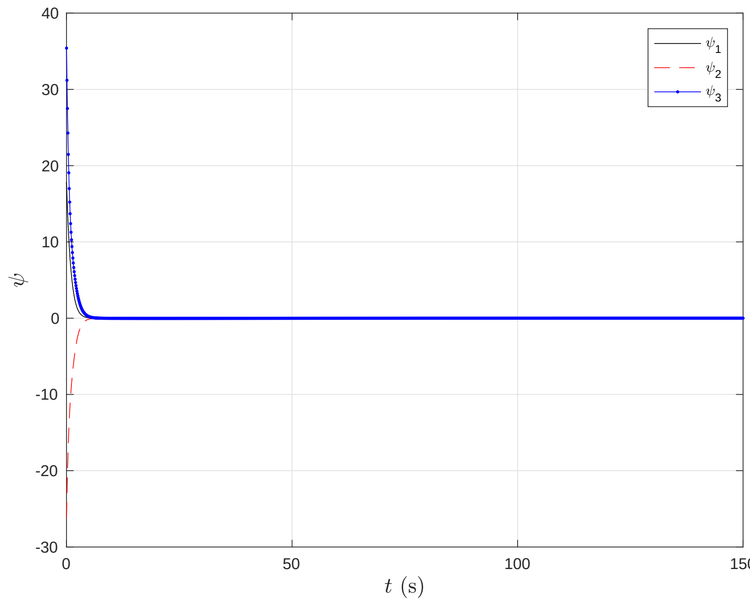

Figure 6. Components of the ψ function given in Eq. (10) for the proposed controller in the case of initial tumbling.

proposed controller is capable of treating the rotational and translational motions of the rigid body simultaneously, despite the presence of rotational-translational coupling terms. This control design is particularly advantageous for motions in the presence of forces and torques that induce coupling between translational and rotational motion. Furthermore, almost global asymptotic stability of the proposed controller was proved by the use of a M-L function. Results obtained from numerical simulations demonstrate the excellent regulation performance of this control scheme, regardless of the initial tumbling of the body. The performance of the proposed controller was compared to another M-L-based TSE(3) controller presented in Ref. [7]. According to the results, the proposed controller showed more robustness than that in Ref. [7] when initial tumbling exists. However, it has higher translational control effort (but with slightly less attitude control effort) for the same settling time of the response than the TSE(3) controller in the literature. Nonlinear control design on TSE(3) enables us to treat coupled translational and rotational regulation simultaneously, and both translational and rotational motions are almost globally asymptotically stabilized using a single controller. Therefore, this controller is more versatile than the other controllers that deal with the translational and rotational motions separately.

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