Find
(a) the rocket thrust,
(b) net acceleration at the beginning,
(c) time to reach the burn-out velocity,
(d) the burn-out velocity.

2.63 A 5000 kg rocket is to be fired vertically. Calculate the rate of ejection of gas at exhaust speed 100 m/s in order to provide necessary thrust to

(a) support the weight of the rocket and
(b) impart an initial upward acceleration of $2\,g$.

2.64 A flexible rope of length L and mass per unit length μ slides over the edge of a frictionless table. Initially let a length y_0 of it be hanging at rest over the edge and at time t let a length y moving with a velocity $\mathrm{d}y/\mathrm{d}t$ be over the edge. Obtain the equation of motion and discuss its solution.

2.65 An open railway car of mass W is running on smooth horizontal rails under rain falling vertically down which it catches and retains in the car. If v_0 is the initial velocity of the car and k the mass of rain falling into the car per unit time, show that the distance travelled in time t is $(W v_0/k)\ln(1 + kt/W)$.
[with courtesy from R.W. Norris and W. Seymour, Mechanics via Calculus, Longmans, Green and Co., 1923]

2.66 A heavy uniform chain of length L and mass M hangs vertically above a horizontal table, its lower end just touching the table. When it falls freely, show that the pressure on the table at any instant during the fall is three times the weight of the portion on the table.
[with courtesy from R.W. Norris and W. Seymour, Mechanics via Calculus, Longmans, Green and Co., 1923]

2.67 A spherical rain drop of radius R cm falls freely from rest. As it falls it accumulates condensed vapour proportional to its surface. Find its velocity when it has fallen for t s.
[with courtesy from R.W. Norris and W. Seymour, Mechanics via Calculus, Longmans, Green and Co., 1923]

2.3 Solutions

2.3.1 Motion of Blocks on a Plane

2.1 **(a)** Acceleration $= \dfrac{\text{Force}}{\text{Total mass}}$

$$a = \frac{F}{(m_1 + m_2 + m_3)} \tag{1}$$

(b) Tension $T_1 =$ Force acting on m_1

$$T_2 = m_1 a = \frac{m_1 F}{(m_1 + m_2 + m_3)} \tag{2}$$

where we have used (1).

Applying Newton's second law to m_2

$$m_2 a = T_2 - T_1$$

or $\quad T_2 = m_2 a + T_1 = (m_1 + m_2)a$

$$T_2 = \frac{(m_1 + m_2)F}{(m_1 + m_2 + m_3)} \tag{3}$$

where we have used (1) and (2).

2.2 **(a)** The equations of motion are

$$ma = mg - T \tag{1}$$

$$Ma = T - \mu Mg \tag{2}$$

Solving (1) and (2)

$$a = \frac{(m - \mu M)g}{m + M} \tag{3}$$

$$T = \frac{Mm}{M + m}(1 + \mu)g \tag{4}$$

Thus with the introduction of friction, the acceleration is reduced and tension is increased compared to the motion on a smooth surface ($\mu = 0$).

2.3 $F_{\max} = (m_1 + m_2)a \quad (1)$

The condition that m_1 may not slide is

$$a = \mu g \tag{2}$$

Using (2) in (1)

$$F_{\max} = (m_1 + m_2)\mu g$$

2.4 **(a)** The force of contact F_c between the blocks is equal to the force exerted on m_2:

$$F_c = m_2 a \tag{1}$$

where the acceleration of the whole system is

$$a = \frac{F}{m_1 + m_2} \tag{2}$$

$$\therefore \quad F_c = \frac{m_2 F}{m_1 + m_2}$$

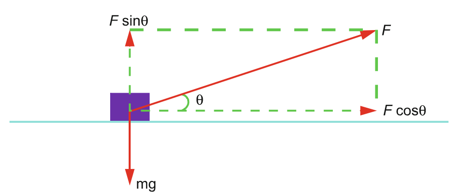
(b) Here the contact force F_c' is given by

$$F_c' = m_1 a = \frac{m_1 F}{m_1 + m_2}$$

Notice that $F_c' \neq F_c$ simply because $m_1 \neq m_2$.

2.5 **(a)** When the box is dragged, the horizontal component of F is $F\cos\theta$ and the vertical component (upward) is $F\sin\theta$ as in Fig. 2.19. The reaction force N on the box by the floor will be

Fig. 2.19



$$N = mg - F\sin\theta \tag{1}$$

(b) The equation of motion will be

$$ma = F\cos\theta - \mu N = F\cos\theta - \mu(mg - F\sin\theta)$$

$$\therefore \quad a = \frac{F}{m}(\cos\theta + \mu\sin\theta) - \mu g \tag{2}$$

(c) When the box is pushed the horizontal component of F will be $F\cos\theta$ and the vertical component $F\sin\theta$ (downwards), Fig. 2.20. The reaction force exerted by the floor on the box will be

$$N' = mg + F\sin\theta \tag{3}$$

which is seen to be greater than N (the previous case).

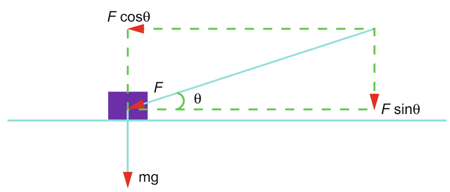
The equation of motion will be

$$ma' = F\cos\theta - \mu N' = F\cos\theta - \mu(mg + F\sin\theta)$$

$$\therefore \quad a' = \frac{F}{m}(\cos\theta - \mu\sin\theta) - \mu g \tag{4}$$

a value which is less than a (the previous case). It therefore pays to pull rather than push at an angle with the horizontal. The difference arises due to the smaller value of the reaction in pulling than in pushing. This fact is exploited in handling a manual road roller or mopping a floor, which is pulled rather than pushed.

Fig. 2.20



2.6 Let x be the length of the chain hanging over the table. The length of the chain resting on the table will be $L - x$. For equilibrium, gravitational force on the hanging part of the chain = frictional force on the part of the chain resting on the table. If M is the mass of the entire chain then

$$\frac{Mg\,x}{L} = \frac{M(L-x)}{L}\mu g$$

$$\therefore \quad x = \frac{\mu L}{\mu + 1}$$

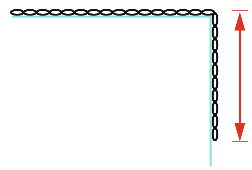
2.7 *First method:* The centre of mass of the hanging part of the chain is located at a distance $L/6$ below the edge of the table, Fig. 2.21. The mass of the hanging part of the chain is $M/3$. The work done to pull the hanging part on the table

$$W = \frac{Mg}{3}\frac{L}{6} = \frac{MgL}{18}$$

Second method: We can obtain the same result by calculus. Consider an element of length dx of the hanging part at a distance x below the edge. The mass of the length dx is $\frac{M\,\mathrm{d}x}{L}$. The work required to lift the element of length dx through a distance x is

$$\mathrm{d}W = \frac{M\,\mathrm{d}x}{L} g\,x$$

Fig. 2.21



Work required to lift the entire hanging part is

$$W = \int \mathrm{d}W = \int_{o}^{L/3} \frac{Mg}{L} x \, \mathrm{d}x = \frac{MgL}{18}$$

2.8 **(a)** The equations of motion are

$$Ma = mg - T \tag{1}$$

$$ma = T - Mg\mu \tag{2}$$

Solving (1) and (2)

$$a = \frac{(m - \mu M)g}{M + m} = \frac{(0.45 - 0.2 \times 2)9.8}{2 + 0.45} = 0.2\,\mathrm{m/s^2}$$

(b) $T = m(g - a) = 0.45(9.8 - 0.2) = 4.32\,\mathrm{N}$

(c) After 2 s, the velocity will be

$$\upsilon_1 = 0 + at = 0.2 \times 2 = 0.4\,\mathrm{m/s^1}$$

When the string breaks, the acceleration will be $a_1 = -\mu g = -0.2 \times 9.8 = -1.96\,\mathrm{m/s^2}$ and final velocity $\upsilon_2 = 0$:

$$S = \frac{\upsilon_2^2 - \upsilon_1^2}{2a_1} = \frac{0 - (0.4)^2}{(2)(-1.96)} = 0.0408\,\mathrm{m} = 4.1\,\mathrm{cm}$$

2.3.2 Motion on Incline

2.9 **(a)** Gravitational force down the incline is $Mg \sin\theta$. Frictional force up the incline is $\mu mg \cos\theta$. Net force

$$F = \mu Mg\cos\theta - Mg\sin\theta = Mg(\mu\cos\theta - \sin\theta)$$
$$= 2 \times 9.8\left(\frac{\sqrt{3}}{2}\cos 30^\circ - \sin 30^\circ\right) = 4.9\,\mathrm{N}$$

(b) $$F' = Mg\sin\theta + \mu Mg\cos\theta = Mg(\sin\theta + \mu\cos\theta)$$
$$= 2 \times 9.8\left(\sin 30^\circ + \frac{\sqrt{3}}{2}\cos 30^\circ\right) = 24.5\,\mathrm{N}$$

2.10 $y = \dfrac{x^2}{20}$ $\qquad \dfrac{\mathrm{d}y}{\mathrm{d}x} = \tan\theta = \dfrac{x}{10}$

For equilibrium, $mg\sin\theta - \mu mg\cos\theta = 0$

$$\therefore \quad \tan\theta = \mu = 0.5$$
$$x = 10\tan\theta = 10 \times 0.5 = 5$$
$$y = \frac{x^2}{20} = \frac{5^2}{20} = 1.25\,\mathrm{m}$$

2.11 **(a)** $\mu = \tan\theta = \tan 30^\circ = 0.577$

(b) $ma = -(mg\sin\theta + \mu\, mg\cos\theta)$

$$\therefore \quad a = -g(\sin\theta + \mu\cos\theta) = -g(\sin\theta + \tan\theta\cos\theta)$$
$$= -9.8(2\sin 30^\circ) = -9.8$$
$$s = \frac{v_0^2}{-2a} = \frac{(2.5)^2}{2 \times 9.8} = 0.319\,\mathrm{m}$$

Initial kinetic energy

$$K = \frac{1}{2}mv_0^2$$

Potential energy $\quad U = mgh = mgs\sin\theta$

$$\therefore \quad \frac{U}{K} = \frac{2\,mgs\sin\theta}{mv_0^2} = \frac{2 \times 9.8 \times 0.319 \times \sin 30^\circ}{(2.5)^2} = 0.5$$

The remaining energy goes into heat due to friction.

(c) It will not slide down as the coefficient of static friction is larger than the coefficient of kinetic friction.

2.12 (**a**) The torque due to the external gravitational force on M_1 will be M_1gr, and the torque due to the external gravitational force on M_2 will be the component of M_2g along the string times r, i.e. $(M_2g\sin\theta)gr$. Now, these two torques act in opposite directions. Taking the counterclockwise rotation of the pulley as positive and assuming that the mass M_1 is falling down, the net torque is

$$\tau = M_1gr - (M_2g\sin\theta)r = (M_1 - M_2\sin\theta)gr \tag{1}$$

and pointing out of the page.

(**b**) When the string is moving with speed υ, the pulley will be rotating with angular velocity $\omega = \upsilon/r$, so that its angular momentum is

$$L_{\text{pulley}} = I\omega = \frac{I\upsilon}{r}$$

and that of the two blocks will be

$$L_{M_1} = rM_1\upsilon \qquad L_{M_2} = rM_2\upsilon$$

All the angular momenta point in the same direction, positive if M_1 is assumed to fall. The total angular momentum is then given by

$$L_{\text{total}} = \upsilon\left[(M_1 + M_2)r + \frac{I}{r}\right] \tag{2}$$

(**c**) Using (1) and (2)

$$\tau = \frac{\mathrm{d}L}{\mathrm{d}t} = \frac{\mathrm{d}\upsilon}{\mathrm{d}t}\left[(M_1 + M_2)r + \frac{1}{r}\right] = [M_1 - M_2\sin\theta]\,gr$$

The acceleration $a = \dfrac{\mathrm{d}\upsilon}{\mathrm{d}t} = \dfrac{[M_1 - M_2\sin\theta]\,g}{(M_1 + M_2) + \frac{1}{r^2}}$

2.13 (**i**) $ma = F - mg\sin\theta$ (equation of motion, up the incline)

$$\begin{aligned} F = ma + mg\sin\theta &= m(a + g\sin\theta) \\ &= (1.0)(1 + 9.8 \times 0.5) = 5.9\,\text{N}\ (\theta = 30^\circ) \end{aligned}$$

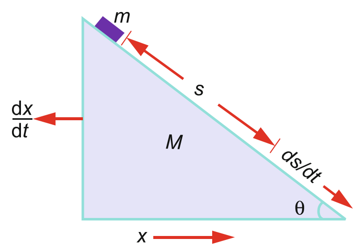
(**ii**) $ma = F + mg\sin\theta$ (equation of motion, down the incline)

$$\begin{aligned} \therefore \quad F &= ma - mg\sin\theta = m(a - g\sin\theta) \\ &= (1.0)(1 - 9.8 \times 0.5) = -3.9\,\text{N} \end{aligned}$$

The negative sign implies that the force F is to be applied up the incline.

2.14 The displacement on the edge is measured by s while that on the floor by x. As the mass m goes down the wedge the wedge itself would start moving towards left, Fig. 2.22. Since the external force in the horizontal direction is zero, the component of momentum along the x-direction must be conserved:

Fig. 2.22



$$(M+m)\frac{dx}{dt} - m\frac{ds}{dt}\cos\alpha = 0 \tag{1}$$

Since the wedge is smooth, the only force acting down the plane is $mg \sin\alpha$

$$m\left(\frac{d^2s}{dt^2} - \frac{d^2x}{dt^2}\cos\alpha\right) = mg\sin\alpha \tag{2}$$

Differentiating (1)

$$(M+m)\frac{d^2x}{dt^2} - m\cos\alpha\frac{d^2s}{dt^2} = 0 \tag{3}$$

Solving (2) and (3)

$$\frac{d^2s}{dt^2} = \frac{(M+m)g\sin\alpha}{M+m\sin^2\alpha} \quad \text{(acceleration of } m\text{)}$$

$$\frac{d^2x}{dt^2} = \frac{mg\sin\alpha\cos\alpha}{M+m\sin^2\alpha} \quad \text{(acceleration of } M\text{)}$$

2.15 The lighter body of mass $m_1 = m$ moves up the plane with acceleration a_1 and the heavier one of mass $m_2 = 3\,\text{m}$ moves down the plane with acceleration a_2. Assuming that the string is taut, the acceleration of the two masses must be numerically equal, i.e.
$a_2 = a_1 = a$. Let the tension in the string be T.
The equations of motion of the two masses are

$$F_1 = m_1a_1 = ma = T - mg\sin\theta \tag{1}$$

$$F_2 = m_2a_2 = 3ma = 3mg\sin\theta - T \tag{2}$$

Adding (1) and (2)

$$a = \frac{g}{2\sqrt{2}} \tag{3}$$

$$\boldsymbol{a}_{\mathrm{CM}} = \frac{m_1\,\boldsymbol{a}_1 + m_2\,\boldsymbol{a}_2}{m_1 + m_2} = \frac{\boldsymbol{a}_1 + 3\boldsymbol{a}_2}{4} \tag{4}$$

In Fig. 2.23 BA represents $\boldsymbol{a}_1$ and AC represents $3\boldsymbol{a}_2$. Therefore, BC the third side of the ΔABC represents $|\boldsymbol{a}_1 + 3\boldsymbol{a}_2|$. Obviously $\mathrm{B\widehat{A}C}$ is a right angle so that

$$|\boldsymbol{a}_1 + 3\boldsymbol{a}_2| = \mathrm{BC} = \sqrt{a_1^2 + (3a_2)^2} = \sqrt{10}a$$

$$\therefore \quad a_{\mathrm{CM}} = \frac{1}{4}\sqrt{10}a = \frac{\sqrt{10}}{4}\frac{g}{2\sqrt{2}} = \frac{\sqrt{5}}{8}g \tag{5}$$

In Fig. 2.23, BD is parallel to the base so that $\mathrm{A\widehat{B}D} = 45°$. Let $\mathrm{C\widehat{B}D} = \alpha$.

$$\text{Now}\ \tan(\alpha + 45°) = \frac{\tan\alpha + \tan 45°}{1 - \tan\alpha\tan 45°} = \frac{\tan\alpha + 1}{1 - \tan\alpha} \tag{6}$$

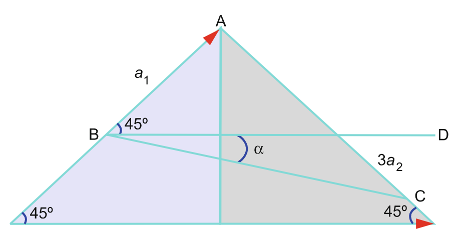
Further, in the right angle triangle ABC,

$$\tan\widehat{\mathrm{ABC}} = \tan(\alpha + 45°) = \frac{\mathrm{AC}}{\mathrm{AB}} = 3 \tag{7}$$

Combining (6) and (7) $\quad \tan\alpha = \dfrac{1}{2} \quad$ or $\quad \alpha = \tan^{-1}\left(\dfrac{1}{2}\right)$.

Thus a_{CM} is at an angle $\tan^{-1}\left(\dfrac{1}{2}\right)$ to the horizon.

Fig. 2.23



2.16 (**a**) Free body diagram (Fig. 2.24)

(**b**) $m_1 a = T_1 - m_1 g \sin 30°$ (1)

$$m_2 a = m_2 \sin 60° - T_2 \tag{2}$$

$$(T_2 - T_1)r = I\alpha = \left(\frac{1}{2}Mr^2\right)\left(\frac{a}{r}\right) \tag{3}$$

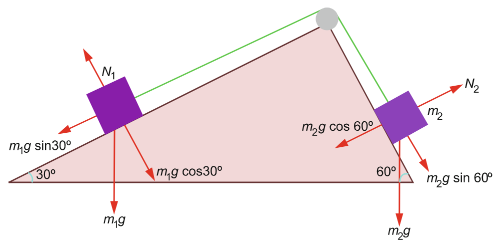


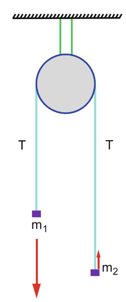
Fig. 2.24

(**c**) Combining (1), (2) and (3) and simplifying

$$a = \frac{g\left(\sqrt{3}m_2 - m_1\right)}{M + 2(m_2 + m_1)}$$

2.17 Let $m_1 = 2.1$ kg and $m_2 = 1.9$ kg, Fig. 2.25. As the pulley is weightless the tension is the same on either side of the pulley. Equations of motion are as follows:

Fig. 2.25



$$m_1 a = m_1 g - T \quad (1)$$
$$m_2 a = T - m_2 g \quad (2)$$

Adding (1) and (2)

$$(m_1 + m_2)a = (m_1 - m_2)g$$
$$\therefore \quad a = \frac{(m_1 - m_2)\,g}{m_1 + m_2} = \frac{(2.1 - 1.9)9.8}{2.1 + 1.9} = 0.49\,\text{m/s}^2$$

Distance travelled by either mass, $s = 40\,\text{cm}$. Time taken

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 0.4}{0.49}} = 1.28\,\text{s}.$$

2.18 Equations of motion are

$$\begin{aligned}
ma_1 &= mg \sin\theta - \mu mg \cos\theta \qquad &\text{(rough incline)}\\
ma_2 &= mg \sin\theta &\text{(smooth incline)}\\
\therefore \quad a_1 &= (\sin\theta - \mu\cos\theta)g
\end{aligned}$$

$$a_2 = g \sin\theta$$

$$t_1 = \sqrt{\frac{2s}{a_1}} \quad t_2 = \sqrt{\frac{2s}{a_2}} \quad \therefore \quad \frac{t_1}{t_2} = \frac{4}{3}$$

$$\sqrt{\frac{\sin\theta}{\sin\theta - \mu\cos\theta}} = \sqrt{\frac{\sin 45^\circ}{\sin 45^\circ - \mu\cos 45^\circ}} = \frac{1}{\sqrt{1-\mu}}$$

$$\therefore \quad \mu = \frac{7}{16}$$

2.19 The normal reaction $N = mg\cos\theta$
Resultant downward force $F = mg\sin\theta - \mu mg\cos\theta$
Given that $N = 2F$

$$\begin{aligned}
&mg\ \cos\theta = 2\,mg(\sin\theta - 0.5\cos\theta)\\
&\therefore \quad \tan\theta = 1 \rightarrow \theta = 45^\circ
\end{aligned}$$

2.20 By prob. (2.17), each mass will have acceleration

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2}$$

The heaver mass m_1 will have acceleration a_1 vertically down while the lighter mass m_2 will have acceleration a_2 vertically up:

$$\boldsymbol{a}_2 = -\boldsymbol{a}_1$$

The acceleration of the centre of mass of the system will be

$$\boldsymbol{a}_{\text{CM}} = \frac{m_1\,\boldsymbol{a}_1 + m_2\,\boldsymbol{a}_2}{m_1 + m_2} = \frac{(m_1 - m_2)\boldsymbol{a}_1}{m_1 + m_2}$$

$$\therefore \quad a_{\text{CM}} = \frac{(m_1 - m_2)^2 g}{(m_1 + m_2)^2}$$

2.21 Free body diagrams for the two blocks and the pulley are shown in Fig. 2.26. The forces acting on m_2 are tension T_2 due to the string, gravity, frictional force f_2 due to the movement of m_1 and the normal force which m_1 exerts on it to prevent if from moving vertically. The forces on m_1 due to m_2 are equal and opposite to those of m_1 on m_2. By Newton's third law the tensions T_1 and T_2 in the thread are not equal as the pulley has mass. The equations of motion for m_1, m_2 and the pulley are

$$m_1 a = F - f_1 - f_2 - T_1 \quad (1)$$

$$m_2 a = T_2 - f_2 \quad (2)$$

$$\alpha I = I\frac{a}{r} = r(T_1 - T_2) \quad (3)$$

Balancing the vertical forces

$$N_2 = m_2 g$$

$$N_1 = N_2 + m_1 g = (m_1 + m_2)\,\mathrm{g}$$

Frictional forces are

$$f_2 = \mu N_2 = \mu m_2 g \quad (4)$$

$$f_1 = \mu N_1 = \mu(m_1 + m_2)g \quad (5)$$

Combining (1), (2), (3), (4) and (5), eliminating f_1, f_2 and T

$$a = \frac{F - \mu(m_1 + 3m_2)g}{m_1 + m_2 + \frac{I}{r^2}}$$

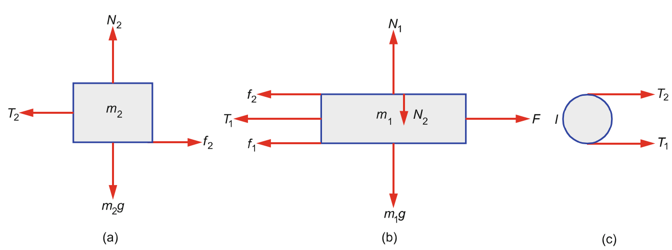


Fig. 2.26

2.3.3 Work, Power, Energy

2.22 Net force $\boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2 = (\hat{i} + 2\hat{j} + 3\hat{k}) + (4\hat{i} - 5\hat{j} - 2\hat{k})$
$= 5\hat{i} - 3\hat{j} + \hat{k}$

Displacement $\boldsymbol{r}_{12} = \boldsymbol{r}_1 - \boldsymbol{r}_2 = 7\hat{k} - (20\hat{i} + 15\hat{j}) = (-20\hat{i} - 15\hat{j} - 7\hat{k})\,\text{cm}$

Work done $W = \boldsymbol{F} \cdot \boldsymbol{r}_{12} = (5\hat{i} - 3\hat{j} + \hat{k}) \cdot (-0.20\hat{i} - 0.15\hat{j} + 0.07\hat{k})$
$= -0.48\,\text{J}.$

2.23 **(i)** $U(x) = 5x^2 - 4x^3$

$$F(x) = -\frac{\mathrm{d}U}{\mathrm{d}x} = -(10x - 12x^2) = 12x^2 - 10x$$

(ii) For equilibrium $F(x) = 0$

$x(12x - 10) = 0$ or $x = 5/6\,\text{m}$ or 0

$$\frac{\mathrm{d}F}{\mathrm{d}x} = 24x - 10$$

$$\frac{\mathrm{d}F}{\mathrm{d}x}\Big|_{x=0} = (24x - 10)|_{x=0} = -10$$

The position $x = 0$ is stable:

$$\frac{\mathrm{d}F}{\mathrm{d}x}\Big|_{x=\frac{5}{6}} = (24x - 10)\Big|_{x=\frac{5}{6}} = +10$$

The position $x = 5/6$ is unstable.

2.24 Let the body travel a distance s on the incline and come down through a height h.
Potential energy lost $= mgh = mgs \sin\theta$.
Work down against friction $W = fs = \mu mg \cos\theta \cdot s$.

By problem $\mu\, mg \cos\theta s = \dfrac{70}{100} mgs\ \sin\theta$

$\therefore\quad \mu = 0.7 \tan\theta = 0.7 \tan 30^\circ = 0.404$

2.25 At the bottom of the ramp the kinetic energy K available is equal to the loss of potential energy, *mgh*:

$K = mgh$

On the flat track the entire kinetic energy is used up in the work done against friction

$W = fd = \mu mgd$

$\therefore\quad \mu\, mgd = mgh$

$$\mu = \frac{h}{d}$$

2.26 **(i)** Work done by the spring $W_s = \frac{1}{2}kx^2 = \frac{1}{2} \times 20 \times 10^3 \times (0.12)^2 = 144\,\text{J}$

(ii) Work done by friction $W_f = \frac{1}{2}mv^2 - W_s = \frac{1}{2} \times 50 \times 3^2 - 144 = 81\,\text{J}$

(iii) $W_f = \mu mgs$

$$\therefore \quad \mu = \frac{W_f}{mgs} = \frac{81}{50 \times 9.8 \times (0.60 + 0.12)} = 0.2296$$

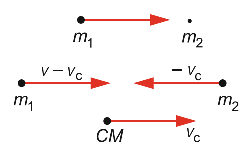
(iv) If v_1 is the velocity of the crate as it passes position A after rebonding

$$\frac{1}{2}mv_1^2 = W_s - \mu\, mgs$$
$$\frac{1}{2} \times 50v_1^2 = 144 - 0.2296 \times 50 \times 9.8 \times (0.60 + .012) = 63$$
$$\therefore \quad v_1 = 1.587\,\text{m/s}$$

2.3.4 Collisions

2.27 In the CMS the velocity of m_1 will be $v_1^* = v - v_c$ and that of m_2 will be $v_2^* = -v_c$, Fig. 2.27. By definition in the CMS total momentum is zero:

Fig. 2.27



$$m_1\vec{v}_1^* + m_2\vec{v}_2^* = 0$$
$$\therefore \quad m_1(v - v_c) - m_2 v_c = 0$$
$$\therefore \quad v_c = v_2^* = \frac{m_1 v}{m_1 + m_2} \tag{1}$$
$$\therefore \quad v_1^* = v - v_c = \frac{m_2 v}{m_1 + m_2} \tag{2}$$

Note that as the collision is elastic, the velocities of m_1 and m_2 after the collision in the CMS remain unchanged. The lab velocity v_1 of m_1 is obtained by the vectorial addition of v_1^* and v_c^*. From the triangle ABC, Fig. 2.28. After collision

$$v_1^2 = v_1^{*2} + v_c^2 - 2v_1^* v_c \cos(180^\circ - \theta) \tag{3}$$

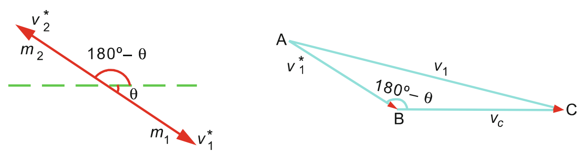


Fig. 2.28

Using (1) and (2) and simplifying

$$v_1 = \frac{v}{m_1 + m_2}\sqrt{m_1^2 + m_2^2 + 2m_1 m_2 \cos\theta} \tag{4}$$

2.28 (a) Let the initial momentum of one object be $\boldsymbol{p}$. After scattering let the momenta be $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$, with the angle θ between them, Fig. 2.29.

$\boldsymbol{p} = \boldsymbol{p}_1 + \boldsymbol{p}_2$ (momentum conservation)

$\therefore \quad (\boldsymbol{p} \cdot \boldsymbol{p}) = \boldsymbol{p}^2 = (\boldsymbol{p}_1 + \boldsymbol{p}_2) \cdot (\boldsymbol{p}_1 + \boldsymbol{p}_2)$

$$= \boldsymbol{p}_1 \cdot \boldsymbol{p}_1 + \boldsymbol{p}_2 \cdot \boldsymbol{p}_2 + \boldsymbol{p}_1 \cdot \boldsymbol{p}_2 + \boldsymbol{p}_2 \cdot \boldsymbol{p}_1 = \boldsymbol{p}_1^2 + \boldsymbol{p}_2^2 + 2\boldsymbol{p}_1 + \boldsymbol{p}_2 \tag{1}$$

$$\frac{\boldsymbol{p}^2}{2m} = \frac{\boldsymbol{p}_1^2}{2m} + \frac{\boldsymbol{p}_2^2}{2m} \text{ (energy conservation) or } \boldsymbol{p}^2 = \boldsymbol{p}_1^2 + \boldsymbol{p}_2^2 \tag{2}$$

Combining (1) and (2), $2\boldsymbol{p}_1 \cdot \boldsymbol{p}_2 = 0$

$\therefore \quad \boldsymbol{p}_1$ and $\boldsymbol{p}_2$ are orthogonal

Fig. 2.29a

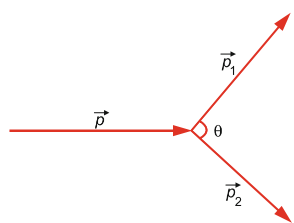


Fig. 2.29b

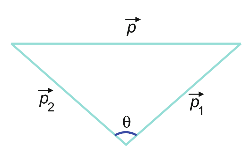
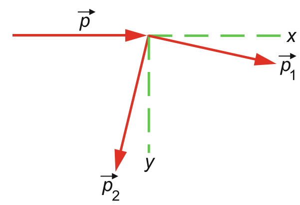


Fig. 2.30



(b) Suppose one of the objects (say 2) is scattered in the backward direction then the momenta would appear as in Fig. 2.30, and because by (a) the angle between $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ has to be a right angle, both the objects will be scattered on the same side of the incident direction (x-axis). In that case, the y-component of momentum cannot be conserved as initially $\sum p_y = 0$. Both the objects cannot be scattered in the backward direction. In that case the x-component of momentum cannot be conserved.

2.29 We work out in the CM system. The total kinetic energy available in the CMS is

$$= K^* = \frac{1}{2}\mu v^2 \tag{1}$$

where
$$\mu = \frac{m\,M}{m + M} \tag{2}$$

is the reduced mass.

If the compression of the spring is x then the spring energy would be $\frac{1}{2}kx^2$. Equating the total kinetic energy available in the CM–system to the spring energy

$$\frac{1}{2}\mu v^2 = \frac{1}{2}kx^2$$

$$x = v\sqrt{\frac{\mu}{k}} = v\sqrt{\frac{mM}{k(m+M)}}$$

2.30 After the first collision (head-on) with the sphere 2 on the right-hand side, sphere 1 moves with a velocity

$$v_1 = \frac{2m_2u_2 + u_1(m_1 - m_2)}{m_1 + m_2} = \frac{0 + u_1(m - 4m)}{m + 4m} = -0.6u_1 \tag{1}$$

and the sphere 2 moves with a velocity

$$v_2 = \frac{2m_1u_1 + u_2(m_2 - m_1)}{m_1 + m_2} = \frac{2mu_1 + 0}{m + 4m} = 0.4u_1 \tag{2}$$

where u and v with appropriate subscripts refer to the initial and final velocities.
The negative sign shows that the ball 1 moves toward left after the collision and hits ball 3. After the second collision with ball 3, ball 2 acquires a velocity v_2 and moves toward right

$$v_1' = \frac{2m_3u_3 + v_1(m_1 - m_3)}{m_1 + m_3} = \frac{0 - u_1(m_1 - 4m)}{m + 4m} = 0.36u_1 \tag{3}$$

But $v_1' < v_2$. Therefore ball 1 will not undergo the third collision with ball 2. Thus in all there will be only two collisions.

2.31 Momentum conservation gives $m_1\,\boldsymbol{u}_1 + m_2\,\boldsymbol{u}_2 = m_2\,\boldsymbol{v}_2$ (1)

Conservation of kinetic energy in elastic collision gives

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_2v_2^2 \tag{2}$$

By the problem $\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_2u_2^2$ (3)

$$u_2 = \alpha u_1 \tag{4}$$

$$\therefore \quad \alpha^2 = \frac{m_1}{m_2} \tag{5}$$

Using (3) in (2)

$$m_2u_2^2 = \frac{1}{2}m_2v_2^2$$

$$\therefore \quad v_2 = \sqrt{2}u_2 \tag{6}$$

Using (4) and (6) in (1)

$$m_1u_1 + m_2\alpha u_1 = \sqrt{2}m_2u_2 = \sqrt{2}m_2\alpha u_1$$

or $\quad m_1 + m_2\alpha = \sqrt{2}\alpha m_2$

Dividing by m_2 and using (5) and rearranging

$$\alpha\left[\alpha - \left(\sqrt{2} - 1\right)\right] = 0$$

since $\alpha \neq 0, \alpha = \sqrt{2} - 1$

$$\therefore \quad \alpha = \frac{u_2}{u_1} = \sqrt{2} - 1$$

$$\therefore \quad \frac{u_1}{u_2} = \frac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$$

From (5)

$$\frac{m_1}{m_2} = \alpha^2 = \left(\sqrt{2} - 1\right)^2 = 3 - 2\sqrt{2}$$

2.32 (**i**) If the common velocity of the merged bodies is v then momentum conservation gives

$$(m_A + m_B)\boldsymbol{v} = m_A \boldsymbol{v}_A + m_B \boldsymbol{v}_B$$
$$\therefore \quad \boldsymbol{v} = \frac{m_A \boldsymbol{v}_A + m_B \boldsymbol{v}_B}{m_A + m_B}$$

(**ii**) $$\boldsymbol{v} = \frac{\frac{3}{2} m_B (5\hat{i} + 3\hat{j}) + m_B(-\hat{i} + 4\hat{j})}{\frac{3}{2} m_B + m_B} = 2.6\hat{i} + 3.4\hat{j}$$

(**iii**) $$\begin{aligned}\Delta \boldsymbol{p}_A = m_A(\boldsymbol{v} - \boldsymbol{v}_A) &= m_A \left[2.6\hat{i} + 3.4\hat{j} - (5\hat{i} + 3\hat{j})\right] \\ &= m_A \left[-2.4\hat{i} + 0.4\hat{j}\right]\end{aligned}$$

$$\Delta p_A = 1200\sqrt{(-2.4)^2 + (0.4)^2} = 2920\,\text{N m}$$
$$F_A = \frac{\Delta p_A}{\Delta t} = \frac{2920}{0.2} = 14{,}600\,\text{N}$$
$$\begin{aligned}\Delta \vec{\boldsymbol{p}}_B = m_B(\vec{\boldsymbol{v}} - \vec{\boldsymbol{v}}_B) &= m_B \left[2.6\hat{i} + 3.4\hat{j} - (-\hat{i} + 4\hat{j})\right] \\ &= m_B \left[3.6\hat{i} - 0.6\hat{j}\right]\end{aligned}$$
$$m_B = \frac{2}{3} m_A = \frac{2}{3} \times 1200 = 800\,\text{kg}$$
$$\Delta p_B = 800\sqrt{(3.6)^2 + (-0.6)^2} = 2920\,\text{N m}$$
$$F_B = \frac{\Delta p_B}{\Delta t} = \frac{2920}{0.2} = 14{,}600\,\text{N}$$

(**iv**) $$K' = \frac{1}{2}(m_A + m_B)v^2 = \frac{1}{2}(1200 + 800)\left[(2.6)^2 + (3.4)^2\right] = 18,320\,\text{J}$$

2.33 Let the velocity of the particle moving below the x-axis be v'. Momentum conservation along x- and y-axis gives

$$mv_0 = mv\cos\theta + mv'\cos\beta \tag{1}$$
$$0 = mv\sin\theta - mv'\sin\beta \tag{2}$$

Cancelling m and reorganizing (1) and (2)

$$v' \cos \beta = v_0 - v \cos \theta \quad (3)$$

$$v' \sin \beta = v \sin \theta \quad (4)$$

Dividing (4) by (3)

$$\tan \beta = \frac{v \sin \theta}{v_o - v \cos \theta} \quad (5)$$

If the collision is elastic, kinetic energy must be conserved:

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 + \frac{1}{2} m v'^2 \quad (6)$$

or $$v_0^2 = v^2 + v'^2 \quad (7)$$

Squaring (3) and (4) and adding

$$v'^2 = v^2 + v_0^2 - 2 v_0 v \cos \theta \quad (8)$$

Eliminating v'^2 between (7) and (8) and simplifying

$$v = v_0 \cos \theta$$

2.34 The momenta of β and ^{14}N are indicated in both magnitude and direction in Fig. 2.31. The resultant R of these momenta is given from the diagonal AC of the rectangle (parallelogram law). The momentum of u is obtained by protruding CA to E such that AE = AC:

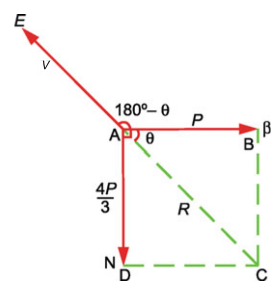


Fig. 2.31 Decay of ^{14}C at rest

$$R = \sqrt{\mathrm{AB}^2 + \mathrm{BC}^2} = \sqrt{p^2 + (4p/3)^2} = \frac{5p}{3}$$

$$\tan\theta = \frac{\mathrm{BC}}{\mathrm{AB}} = \frac{4p/3}{p} = \frac{4}{3}$$

$$\therefore \quad \theta = 53^\circ$$

Thus the neutrino is emitted with momentum $5p/3$ at an angle $(180 - 53^\circ)$ or 127° with respect to the β particle.

2.35 Denoting the angles with (*) for the CM system transformation of angles for CMS to LS is given by

$$\tan\theta = \frac{\sin\theta^*}{\cos\theta^* + \frac{m}{M}} \tag{1}$$

But $\quad \theta^* = \pi - \phi^* = \pi - 2\phi$

$$\therefore \quad \sin\theta^* = \sin(\pi - 2\phi) = \sin 2\phi$$

$$\cos\theta^* = \cos(\pi - 2\phi) = -\cos 2\phi$$

(**i**) becomes

$$\tan\theta = \frac{\sin 2\phi}{\frac{m}{M} - \cos 2\phi}$$

Furthermore $\quad \dfrac{\sin\theta}{\cos\theta} = \dfrac{\sin 2\phi}{\frac{m}{M} - \cos 2\phi}$

Cross-multiplying and rearranging

$$\frac{m}{M}\sin\theta = \sin\theta\cos 2\phi + \cos\theta\sin 2\phi = \sin(\theta + 2\phi)$$

$$\therefore \quad \frac{m}{M} = \frac{\sin(2\phi + \theta)}{\sin\theta}$$

2.36 In the lab system let M be projected at an angle ϕ with velocity v. In the CMS the velocity v^* for the struck nucleus will be numerically equal to v_c, the centre of mass velocity. Therefore, M is projected at an angle 2ϕ with velocity $v^* = \dfrac{mv}{M+m}$. The CM system velocity $v_c = \dfrac{mv}{M+m}$. The velocities v^* and v_c must be combined vectorially to yield v, Fig. 2.32. Since $v_c = v^*$ the velocity triangle ABC is an isosceles triangle. If BD is perpendicular on AC, then

$$\text{AC} = 2\text{AD} = 2\text{AB}\cos\phi$$

$$\therefore \quad v = 2v^*\cos\phi = \frac{2mu\cos\phi}{M+m}$$

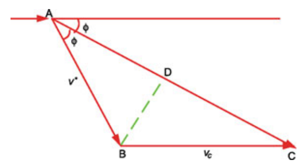


Fig. 2.32

2.37 **(a)** Kinetic energy of A before collision $K_A = \frac{1}{2}(2m)u^2 = mu^2$. Since B is initially stationary, its kinetic energy $K_B = 0$. Hence before collision, total kinetic energy $K_0 = mu^2 + 0 = mu^2$.
Let A and B move with velocity υ_A and υ_B, respectively, after the collision, Fig. 2.33. Total kinetic energy after the collision,

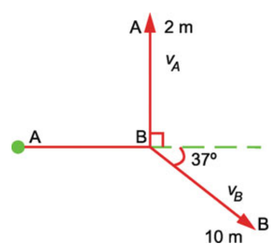


Fig. 2.33

$$K' = K'_A + K'_B = \frac{1}{2}(2m)\upsilon_A^2 + \frac{1}{2}(10m)\upsilon_B^2 = m\upsilon_A^2 + 5m\upsilon_B^2$$

If an energy Q is lost in the collision process, conservation of total energy gives

$$mu^2 = m\upsilon_A^2 + 5m\upsilon_B^2 + Q \tag{1}$$

Applying momentum conservation along the incident direction and perpendicular to it

$$2mu = 10m\upsilon_B\cos 37° = 8m\upsilon_B \tag{2}$$
$$2m\upsilon_B = 10m\upsilon_B\sin 37° = 6m\upsilon_B \tag{3}$$

From (2) and (3) we find

$$v_{\mathrm{A}} = \frac{3u}{4}; \quad v_{\mathrm{B}} = \frac{u}{4}$$

(b) From (1), $Q = m(u^2 - v_{\mathrm{A}}^2 - 5v_{\mathrm{B}}^2)$

$$= m\left(u^2 - \frac{9u^2}{16} - \frac{5u^2}{16}\right) = \frac{mu^2}{8}$$

$$\therefore \quad \frac{Q}{k_{\mathrm{A}}} = \frac{mu^{2/8}}{mu^2} = \frac{1}{8}$$

Since Q is positive, energy is lost in the collision process.

2.38 **(a)** In the elastic collision (head-on) of a particle of mass m_1 and kinetic energy K_1 with a particle of mass m_2 initially at rest, the fraction of kinetic energy imparted to m_2 is

$$\frac{K_2}{K_0} = \frac{4m_1 m_2}{(m_1 + m_2)^2} = \frac{4 \times 1 \times 12}{(1 + 12)^2} = \frac{48}{169}$$

(b) $$\frac{\frac{1}{2}m_2 v_2^2}{\frac{1}{2}m_1 v_1^2} = \frac{12}{1}\frac{v_2^2}{v_0^2} = \frac{48}{169}$$

$$\therefore \quad v_2 = \frac{2}{13}v_0$$

$$K_1 = K_0 - K_2 = K_0 - \frac{48}{169}K_0 = \frac{121}{169}K_0$$

$$\therefore \quad \frac{1}{2}m_1 v_1^2 = \frac{121}{169} \times \frac{1}{2}m_1 v_0^2$$

$$\therefore \quad v_1 = -\frac{11}{13}v_0$$

Negative sign is introduced because neutron being lighter then the carbon nucleus will bounce back.

2.39 Let the heavy body of mass M with momentum P_0 collide elastically with a very light body of mass m be initially at rest. After the collision both the bodies will be moving in the direction of incidence, the heavier one with velocity v_{H} and the lighter one with velocity v_L.
Momentum conservation gives

$$p_0 = p_L + p_H \tag{1}$$

Energy conservation gives

$$\frac{p_0^2}{2M} = \frac{p_{\mathrm{L}}^2}{2m} + \frac{p_{\mathrm{H}}^2}{2M} \tag{2}$$

Eliminating p_H between (1) and (2) and simplifying

$$P_L = \frac{2p_0 m}{M+m}$$

$$m v_L = \frac{2Mum}{M+m}$$

$$\text{or} \quad v_L = \frac{2uM}{M+m} = 2u \quad (\because m << M)$$

2.40 Let a body of mass m_1 moving with velocity u make a completely inelastic collision with the body of mass m_2 initially at rest. Let the combined mass moves with a velocity v_c given by

$$v_c = \frac{m_1 u}{m_1 + m_2} = \frac{u}{2} \quad (\because m_1 = m_2)$$

$$\text{Energy lost} \quad = \frac{1}{2}mu^2 - \frac{1}{2}(2m)\left(\frac{u}{2}\right)^2 = \frac{1}{4}mu^2 = \frac{1}{2}K_0$$

where $K_0 = \frac{1}{2}mu^2$ is the initial kinetic energy.

2.41 Let the speed of the bullet be u. Let the block + bullet system be travelling with initial speed v. If m and M are the masses of the bullet and the block, respectively, then momentum conservation gives

$$mu = (M+m)v \tag{1}$$

$$\therefore \quad v = \frac{mu}{M+m} \tag{2}$$

The initial kinetic energy of the block + bullet system

$$K = \frac{1}{2}(M+m)v^2 = \frac{1}{2}\frac{m^2u^2}{(M+m)}$$

Work done to bring the block + bullet system to rest in distance s is

$$W = \mu(M+m)gs = \frac{1}{2}\frac{m^2u^2}{(M+m)}$$

$$\therefore \quad u = \frac{(M+m)}{m}\sqrt{2\mu\, gs} = \frac{(2.000+0.005)}{0.005}\sqrt{2 \times 0.2 \times 9.8 \times 2}$$

$$= 1123 \text{m/s}$$

2.42 **(a)** Let $m_1 = m$ with velocity u collide with $m_2 = M$, initially at rest. For elastic collision the final velocities will be

$$v_1 = \frac{(m_1 - m_2)}{m_1 + m_2}u = \frac{(m-M)}{m+M}u \qquad (m < M) \tag{1}$$

$$v_2 = \frac{2m_1}{m_1 + m_2}u = \frac{2mu}{m + M} \tag{2}$$

By problem $-v_1 = v_2$ (3)

Combining (1), (2) and (3)

$$\frac{M}{m} = 3 \tag{4}$$

(b) $$v_c = \frac{mu}{M + m} = \frac{mu}{3m + m} = \frac{u}{4} \tag{5}$$

(c) $K^* = K_1{}^* + K_2{}^* = \frac{1}{2}mv_1{}^{*2} + \frac{1}{2}Mv_2{}^{*2}$

But $v_1{}^* = \frac{Mu}{M + m} = \frac{3mu}{3m + m} = \frac{3u}{4}$

$v_2{}^* = -v_c = -\frac{u}{4}$

$\therefore \quad K^* = \frac{1}{2}m\left(\frac{3u}{4}\right)^2 + \frac{1}{2}3m\left(\frac{u}{4}\right)^2 = \frac{3}{8}mu^2$

(d) $K_1(\text{final}) = \frac{1}{2}mv_1^2 = \frac{1}{8}mu^2$

where we have used (1) and (4).

2.43 We can work out this problem in the lab system. But we prefer to use the centre of mass system. The CMS and LS scattering angles are related by

$$\tan\theta = \frac{\sin\theta^*}{\cos\theta^* + \frac{M}{m}} \tag{1}$$

$\theta_{\max}$ is obtained from the condition

$$\frac{\mathrm{d}\tan\theta}{\mathrm{d}\theta^*} = 0 \tag{2}$$

This gives $\quad \cos\theta^* = \frac{m}{M}$ (3)

$$\therefore \quad \sin\theta^* = \frac{\sqrt{M^2 - m^2}}{M} \tag{4}$$

Use (3) and (4) in (1) to get

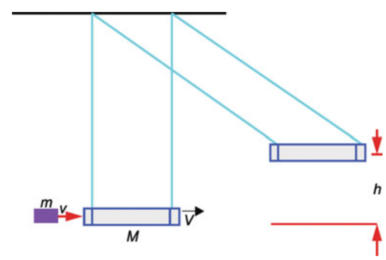
$$1 + \cot^2\theta = \operatorname{cosec}^2\theta = \frac{M^2}{m^2}$$

$$\therefore \quad \sin\theta_{\max} = \frac{m}{M}$$

$$\text{or} \quad \theta_{\max} = \sin^{-1}\left(\frac{m}{M}\right)$$

2.44 Momentum of the bullet before collision = momentum of the block + bullet system immediately after collision, Fig. 2.34:

Fig. 2.34



$$mv = (m + M)V \tag{1}$$

where V is the initial speed of the block + bullet system. The kinetic energy of the system immediately after the impact is

$$K = \frac{1}{2}(m + M)V^2 \tag{2}$$

Due to the impact, the pendulum would swing to the right and would be raised through the maximum height h vertically above the rest position of the pendulum, Fig. 2.34. At this point, the kinetic energy of the pendulum is entirely converted into gravitational potential energy:

$$\frac{1}{2}(m + M)V^2 = (m + M)gh \tag{3}$$

$$\therefore \quad V = \sqrt{2gh} \tag{4}$$

Using (4) in (1)

$$v = \left(1 + \frac{M}{m}\right)\sqrt{2gh} \tag{5}$$

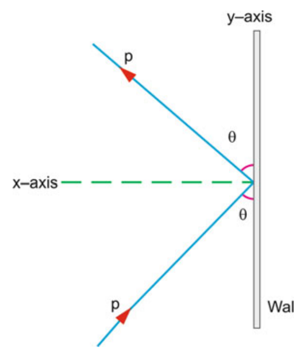
By measuring h and knowing m and M, the original velocity of the bullet can be calculated.

2.45 Let the area of the nozzle through where the jet comes be A m^2. The mass of water in the jet per second is $\rho A\ v$, where ρ is the density of water and v the get velocity.
The momentum associated with this volume of water is

$$p = (\rho A v)v = \rho A v^2 \tag{1}$$

The momentum after hitting the wall will also be equal to $\rho A v^2$ since the collision is assumed to be elastic. Resolve the momentum along the x-axis and y-axis, Fig. 2.35.

Fig. 2.35



The change of the x-component of momentum is

$$\Delta p_x = p \sin\theta - (-p \sin\theta) = 2p \sin\theta \tag{2}$$

The change in the y-component of momentum is

$$\Delta p_y = p \cos\theta - p \cos\theta = 0 \tag{3}$$

Then $\Delta p = \Delta p_x = 2p \sin\theta = 2\rho A v^2 \sin\theta$ (4)

Pressure exerted on the wall will be

$$P = \frac{\Delta p}{A} = 2\rho v^2 \sin\theta \tag{5}$$

For normal incidence, $\theta = 90°$ and

$$P = 2\rho v^2 \tag{6}$$

2.46 For completely inelastic collision there is no rebounding of the jet. The pressure on the wall is given by

$$P = \rho v^2 \sin\theta \tag{1}$$

For normal incidence, $\theta = 90°$ and

$$P = \rho v^2 \tag{2}$$

2.47 Resolve the momentum mv_1 and mv_2 along the original line of motion and in a direction perpendicular to it. Along the original line of motion, the initial momentum must be equal to the sum of the components of momentum after the collision:

$$mv_0 = mv_1 \cos 30° + mv_2 \cos 30° \tag{1}$$

In the direction perpendicular to the original direction of motion, the sum of components of momentum after the collision must be equal to zero because before collision the balls do not have any component of momentum in the perpendicular direction:

$$mv_1 \sin 30° - mv_2 \sin 30° = 0$$
$$\text{or} \quad v_1 = v_2 \tag{2}$$

This result could have been anticipated from symmetry.
Using (2) in (1)

$$v_0 = 2v_1 \cos 30° = \sqrt{3}v_1$$
$$\text{or} \quad v_1 = v_2 = \frac{v_0}{\sqrt{3}} = \frac{9}{\sqrt{3}} = 5.19\,\text{m/s}$$

Total kinetic energy of the two balls before collision

$$K_0 = \frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv_0^2 \tag{3}$$

Total kinetic energy after the collision

$$K' = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = mv_1^2 = \frac{1}{3}mv_0^2 \tag{4}$$

On comparing (3) and (4) we conclude that kinetic energy is not conserved. The collision is said to be inelastic.

2.48 Time taken for the ball to reach the plane in the initial fall

$$t_0 = \sqrt{\frac{2h}{g}} \tag{1}$$

Velocity with which it reaches the plane

$$u_1 = \sqrt{2gh} \tag{2}$$

The velocity with which it rebounds from the plane

$$v_1 = eu_1 = e\sqrt{2gh} \tag{3}$$

Time to reach the plane again

$$t_1 = \frac{2v_1}{g} = 2e\sqrt{\frac{2h}{g}} = 2et_0$$

If this process is repeated indefinitely the total time

$$T = t_0 + t_1 + t_2 + \cdots + t_\infty = t_0 + 2et_0 + 2e^2 t_0 + \cdots$$
$$= t_0[1 + 2e(1 + e + e^2 + \cdots)]$$

$$= t_0\left[1 + \frac{2e}{1-e}\right] = \sqrt{\frac{2h}{g}}\frac{1+e}{1-e}$$

where we have used the formula for the sum of infinite number of terms of a geometric series.

2.49 Total distance traversed

$$S = h + 2h_1 + 2h_2 + \cdots \quad = h + 2e^2 h + 2e^4 h + 2e^6 h + \cdots$$
$$= h\left[1 + \frac{2e^2}{1-e^2}\right] = h\frac{(1+e^2)}{1-e^2}$$

2.50 On the first bounce, $v_1 = e\sqrt{2gh}$

On the second bounce, $v_2 = e^2\sqrt{2gh}$

On the nth bounce, $v_n = e^n\sqrt{2gh}$

$$h_n = \frac{v_n^2}{2g} = e^{2n}h$$

2.51 Let the two bodies of mass m_1 and m_2 be travelling with the velocities u_1 and u_2 before the impact, and the combined body of mass $m_1 + m_2$ with velocity v after the impact is

$$v = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2} \tag{1}$$

Energy wasted = total kinetic energy before the collision minus total kinetic energy after the collision

$$W = \left(\frac{1}{2} m_1 {u_1}^2 + \frac{1}{2} m_2 {u_2}^2\right) - \frac{1}{2}(m_1 + m_2) v^2$$

$$= \frac{1}{2} m_1 {u_1}^2 + \frac{1}{2} m_2 {u_2}^2 - \frac{1}{2} \frac{(m_1 u_1 + m_2 u_2)^2}{m_1 + m_2} = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2$$

2.52 Resolve the momentum along x- and y-axes at points A and B, Fig. 2.36. Take the downward direction as positive:

$p_x(\mathrm{A}) = p \cos\theta \quad p_x(\mathrm{B}) = p \cos\theta$

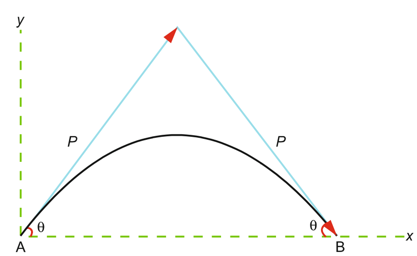
$p_y(\mathrm{A}) = -p \sin\theta \quad p_y(\mathrm{B}) = p \sin\theta$

Then $\Delta p_x = p_x(\mathrm{B}) - p_x(\mathrm{A}) = p \cos\theta - p \cos\theta = 0$

$\Delta p_y = p_y(\mathrm{B}) - p_y(\mathrm{A}) = p \sin\theta - (-p \sin\theta) = 2p \sin\theta$

$\therefore \quad \Delta p = \Delta p_y = 2p \sin\theta$

Fig. 2.36



2.53 Let the shell of mass $2m$ explode into two pieces each of mass m. At the highest point the entire velocity consists of the horizontal component $(v \cos\theta)$ alone. Since one of the components retraces its path, it follows that it has velocity $- v \cos\theta$, and therefore a momentum $-mv \cos\theta$. Let the momentum of the other pieces be p. Now, the momentum of the shell just before the explosion was $2mv \cos\theta$ momentum conservation gives

$$p - mv\cos\theta = 2mv\cos\theta$$

$$\therefore \quad p = 3mv\cos\theta$$

$$\therefore \quad \text{velocity} = \frac{p}{m} = 3v\cos\theta$$

2.54 Volume of air moving down per second = Av, where v is the air velocity moving down through an area A.
Mass of air moving down per second = ρAv

$$F = \frac{\Delta p}{\Delta t} = \left(\frac{\text{mass}}{\text{sec}}\right)(\Delta v) = \rho A v^2$$

Reaction force upward = Helicopter's weight

$$\rho A v^2 = Mg$$

$$v = \sqrt{\frac{Mg}{\rho A}} = \sqrt{\frac{500 \times 9.8}{1.3 \times 45}} = 9.15\,\text{m/s}$$

2.55 If v is the velocity of each bullet of mass m and n the number of bullets that can be fired per second then rate of change of momentum will be

$$\frac{\Delta p}{\Delta t} = mnv \tag{1}$$

$$\therefore \quad \frac{\Delta p}{\Delta t} = F = mnv \tag{2}$$

$$n = \frac{F}{mv} = \frac{150}{(0.1)(1000)} = 1.5/\text{s}$$

Thus the number of bullets that can be fired per minute will be $60 \times 1.5 = 90$.

2.56 If v is the velocity with which a particle of mass m falls on the balance pan, momentum before impact is mv and after impact $-mv$ so that

$$\Delta p = -mv - mv = -2mv \tag{1}$$

If height of fall is h then

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 1.6} = 5.6\ \text{m/s} \tag{2}$$

If n particles fall per second, the force exerted on the pan is

$$F = +2mnv = (2)(0.1)\left(\frac{441}{60}\right)(5.6) = 8.232\,\mathrm{N}$$
$$= \frac{8.232}{9.8}\,\text{kg wt} = 0.84\,\text{kg wt}$$

2.57 In this case, the particles will stick to the pan. Therefore the scale reading will increase due to the weight of the particles that get accumulated in the pan. For complete inelastic collision $\Delta p = mv$ as the final momentum is zero. Net force on the scale = weight of the particle + force of impact. At time t, scale reading (in newtons)

$$= mngt + mn\sqrt{2gh}$$
$$= mng\left[t + \sqrt{\frac{2h}{g}}\right]$$

Scale reading in kg wt $= mn\left[t + \sqrt{\frac{2h}{g}}\right]$

2.58 Let a sphere of mass m_1 travelling with velocity u_1 collide with the second sphere of mass m_2 at rest, with their centres in straight line. After the collision let the final velocities be v_1 and v_2, respectively, for m_1 and m_2. By definition the coefficient of restitution e is given by the ratio

$$e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}} = \frac{v_2 - v_1}{v_1} \tag{1}$$

Momentum conservation requires that total momentum before collision = total momentum after collision:

$$m_1u_1 = m_1v_1 + m_2v_2 \tag{2}$$

Eliminating v_2 between (1) and (2),

$$v_1 = \frac{(m_1 - em_2)u_1}{m_1 + m_2} \tag{3}$$

$$v_2 = \frac{m_1(1 + e)u_1}{m_1 + m_2} \tag{4}$$

(i) Putting $u_1 = u, \quad m_1 = m$ and $m_2 = \frac{m}{2}$

$$v_1 = \frac{u}{3}(2 - e) \tag{5}$$

$$v_2 = \frac{2u}{3}(1 + e) \tag{6}$$

Total energy after the collision

$$K' = K_1 + K_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{m}{2}\right)v_2^2 \tag{7}$$

Using (5) and (6) in (7) and simplifying

$$K' = \frac{mu^2}{6}(2 + e^2) \tag{8}$$

(ii) Kinetic energy lost during the collision

$$\Delta K = K_0 - K' = \frac{1}{2}mu^2 - \frac{mu^2}{6}(2 + e^2) = \frac{mu^2}{6}(1 - e^2)$$

2.59 **(a)** Distance traversed by the car before it falls off, $s = 18 - 2 = 16\,\text{m}$:

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 16}{4}} = 2\sqrt{2}\,\text{s}$$

(b) By Newton's third law, the force exerted by the car is equal to that by boat + car

$$(M + m)a_{\text{B}} = ma$$

where $M = 8000\,\text{kg}$, $m = 1200$, $a = 4\,\text{m/s}^2$
The acceleration of the boat $a_{\text{B}} = \frac{ma}{M+m} = 0.26\,\text{m/s}^2$
The distance travelled by the boat in the opposite direction

$$s_{\text{B}} = \frac{1}{2}a_{\text{B}}t^2 = \frac{1}{2} \times 0.26 \times \left(2\sqrt{2}\right)^2 = 104\,\text{m}$$

(c) Momentum conservation gives

$$mv_{\text{c}} = (M + m)v_{\text{B}}$$
$$\frac{v_{\text{B}}}{v_{\text{C}}} = \frac{m}{M + m} = \frac{1200}{8000 + 1200} = 0.13$$

which is independent of the car's acceleration.

2.3.5 Variable Mass

2.60 **(a)** Resultant force on rocket = (upward thrust on rocket) − (weight of rocket)

$$\therefore \quad m\frac{dv}{dt} = -v_{\text{r}}\frac{dm}{dt} - g \tag{1}$$

Setting $\alpha = -\frac{\mathrm{d}m}{\mathrm{d}t}$ and $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$, minimum exhaust velocity

$$v_\mathrm{r} = \frac{g}{\alpha}$$

(b) Dividing (1) by m

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{v_\mathrm{r}}{m}\frac{\mathrm{d}m}{\mathrm{d}t} - g \tag{2}$$

$$\therefore \quad \mathrm{d}v = -v_\mathrm{r}\frac{\mathrm{d}m}{m} - g\,\mathrm{d}t$$

Assuming that v_r and g remain constant, and at $t = 0$, $v = 0$ and $m = m_0$,

$$\int_0^{v_\mathrm{B}} \mathrm{d}v = -v_\mathrm{r}\int_{m_0}^{m_\mathrm{B}} \frac{\mathrm{d}m}{m} - g\int_0^t \mathrm{d}t$$

$$\therefore \quad v_\mathrm{B} = -v_\mathrm{r}\ln\left(\frac{m_0}{m_\mathrm{B}}\right) - gt \tag{3}$$

where m_0 is the initial mass of the system and m_B the mass at burn-out velocity v_B

(c) Setting $g = 0$ in (2)

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{v_\mathrm{r}}{m}\frac{\mathrm{d}m}{\mathrm{d}t} \tag{4}$$

$$-\frac{\mathrm{d}m}{m} = \alpha = \text{Positive constant}$$

$$\mathrm{d}m = -\alpha\mathrm{d}t$$

$$\therefore \quad \int \mathrm{d}m = -\alpha\int \mathrm{d}t + C$$

where C is the constant of integration

$$m = -\alpha t + C$$

When $t = 0$, $m = m_0$. Therefore, $C = m_0$

$$m(t) = m_0 - \alpha t \tag{5}$$

Using (5) in (4)

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{\alpha\ v_\mathrm{r}}{m_o - \alpha t}$$

$$\mathrm{d}v = \frac{(v_\mathrm{r}\alpha/m_0)\mathrm{d}t}{1 - \dfrac{\alpha}{m_0}t}$$

Integrating between $v = 0$ and v

$$v = -v_\mathrm{r} \ln\left(1 - \frac{\alpha t}{m_0}\right) \tag{6}$$

Writing $\left(1 - \dfrac{\alpha t}{m_0}\right) = \dfrac{m}{m_0}$ in (6) with the aid of (5), the rocket equation simplifies to

$$v = -v_\mathrm{r} \ln \frac{m}{m_0}$$

$$\text{or } m = m_0 e^{-v/v_\mathrm{r}} \tag{7}$$

(d) Time taken for the rocket to reach the burn-out velocity is given by (5):

$$t = t_0 = \frac{m_0 - m}{\alpha} \tag{8}$$

2.61 $a = 0.5\,g = \dfrac{v_\mathrm{r}}{m}\dfrac{\mathrm{d}m}{\mathrm{d}t} - g$

$$\frac{\mathrm{d}m}{\mathrm{d}t} = 1.5\frac{mg}{v_\mathrm{r}} = \frac{1.5 \times 10^6 \times 9.8}{2000} = 7350\,\mathrm{kg/s}$$

2.62 **(a)** Rocket thrust $= v_\mathrm{r}\dfrac{\mathrm{d}m}{\mathrm{d}t} = 55 \times 10^3 \times 1290 = 71 \times 10^6\,\mathrm{N}$.

(b) Net acceleration $a = \dfrac{v_\mathrm{r}}{m}\dfrac{\mathrm{d}m}{\mathrm{d}t} - g = \dfrac{71 \times 10^6}{2.72 \times 10^6} - 9.8 = 16.3\,\mathrm{m/s^2}$.

(c) Time to reach the burn-out velocity $t = \dfrac{m_0 - m_\mathrm{B}}{\alpha}$

$$= \frac{2.72 \times 10^6 - 2.52 \times 10^6}{1290} = 155\,\mathrm{s}.$$

(d) Burn-out velocity $v_\mathrm{B} = v_i + v_\mathrm{r}\ \ln\ \dfrac{m_0}{m_\beta} - gt$

$$= 0 + 55,000\ \ln\ \frac{2.72 \times 10^6}{2.52 \times 10^6} - (9.8 \times 155) = 2714\,\mathrm{m/s} = 2.7\,\mathrm{km/s}$$

2.63 **(a)** Weight of the rocket

$$M_0 g = 5000 \times 9.8 = 49,000\,\mathrm{N}$$

let x kg of gas be ejected per second. Then

$$x v_e = M_0 g$$
$$\therefore \quad x = \frac{M_0 g}{v_e} = \frac{49,000}{1000} = 49\,\text{kg/s}$$

(b) Upward acceleration required, $a = 2g$. Upward thrust required

$$F = M_0 a = (M_0)(2g) = 2\,M_0 g$$

Weight of the rocket $W = M_0 g$

Total force required $= F + W = 2\,M_0 g + M_0 g = 3\,M_0 g$

Let x' kg gas be ejected per second with $v_e = 1000\,\text{m/s}$

$$1000x' = 147,000 \quad x' = 147\,\text{kg/s}$$

2.64 At any time, the total kinetic energy of the system is

$$K = \frac{1}{2}(\mu L)\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 \tag{1}$$

Let the potential energy at the surface of the table be zero. The potential energy of the portion of the rope hanging down is

$$U = -(\mu y)g\left(\frac{y}{2}\right) = \frac{1}{2}\mu\ g\ y^2 \tag{2}$$

Total mechanical energy

$$E = K + U = \text{constant}$$
$$\frac{1}{2}\mu L\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 - \frac{1}{2}\mu\ g\ y^2 = \text{constant}$$

Differentiating with respect to time

$$\frac{1}{2}\mu\ L2\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\frac{\mathrm{d}y}{\mathrm{d}t} - -\frac{1}{2}\mu g2y\frac{\mathrm{d}y}{\mathrm{d}t} = 0$$

Cancelling the common factors,

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - \frac{g}{L}y = 0 \tag{3}$$

Calling $\beta^2 = g/L$, (3) becomes

$$\frac{d^2y}{dt^2} - \beta^2 y = 0 \tag{4}$$

which has the solution

$$y = Ce^{\beta t} + De^{-\beta t} \tag{5}$$

where C and D are constants. When $t = 0$, $y = y_0$

$$\therefore \quad y_0 = C + D \tag{6}$$

Further $\frac{dy}{dt} = \beta\left(Ce^{\beta t} - De^{-\beta t}\right)$

When $t = 0$, $\frac{dy}{dt} = 0$

$\therefore \quad 0 = C - D$

$$\therefore \quad C = D = \frac{y_0}{2} \tag{7}$$

Using (7) in (5)

$$y = \frac{y_0}{2}\left(e^{\beta t} + e^{-\beta t}\right) \tag{8}$$

Thus the complete solution is

$$y = y_0 \cosh(\beta t) \tag{9}$$

Note that initially both the terms in the parenthesis of (8) are important. As t increases, the second term becomes vanishingly small and the first term alone dominates. Thus y (length of the rope hanging down) increases exponentially with time.

From (3) the acceleration

$$\frac{d^2y}{dt^2} = \frac{g}{L}y$$

Thus acceleration continuously increases with increasing value of y. This then is the case of non-uniform acceleration.

2.65 Consider the equation for the variable mass

$$m\frac{\mathrm{d}v}{\mathrm{d}t} + v\frac{\mathrm{d}m}{\mathrm{d}t} = F = 0 \tag{1}$$

$$\frac{\mathrm{d}m}{\mathrm{d}t} = k \tag{2}$$

$$\therefore \quad m\frac{\mathrm{d}v}{\mathrm{d}t} + kv = 0 \tag{3}$$

Integrating (2)

$$m = \int \mathrm{d}m = \int k\mathrm{d}t = kt + C_1$$

where $C_1 =$ constant.

$$\text{At} \quad t = 0, m = W$$

$$\therefore \quad m = kt + W \tag{4}$$

Using (4) in (3)

$$\frac{\mathrm{d}v}{v} = -\frac{k}{m}\mathrm{d}t = -\frac{k\mathrm{d}t}{kt + W} \tag{5}$$

$$\therefore \quad \int \frac{\mathrm{d}v}{v} = -\int \frac{k\mathrm{d}t}{kt + W}$$

$$\therefore \quad \ln\ v = -\ln(kt + W) + C_2$$

where $C_2 =$ constant.

$$\text{At} \quad t = 0, v = v_0, C_2 = \ln v_0 + \ln W$$

$$\therefore \quad \ln\left(\frac{v_0}{v}\right) = \ln\left(1 + \frac{kt}{W}\right)$$

$$\therefore \quad \frac{v_0}{v} = 1 + \frac{kt}{W}$$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} = \frac{v_0}{1 + \frac{kt}{W}}$$

The distance travelled in time t

$$S = \int_0^s \mathrm{d}s = v_0 \int_0^t \frac{\mathrm{d}t}{1 + \frac{kt}{W}} = \frac{Wv_0}{k} \ln\left(1 + \frac{kt}{W}\right)$$

2.66 The pressure on the table consists of two parts:

(a) The weight of the coil on the table producing the pressure and
(b) the destruction of momentum producing the pressure.

First consider part (b).
Let a length x be coiled up on the table. Since the chain is falling freely under gravity, the velocity of the chain will be $\sqrt{2gx}$. In a small time interval δt, the length which reaches the table is $\delta t\sqrt{2gx}$.
$\therefore$ The momentum destroyed in time δt is

$$\delta p = \delta t \frac{M}{L}\sqrt{2gx}\sqrt{2gx} = \delta t \frac{M}{L} 2gx$$

$\therefore$ The rate of destruction of momentum is

$$\frac{\delta p}{\delta t} = \frac{M}{L} 2gx$$

Pressure due to part (a) will be $\frac{Mg}{L}x$
$\therefore$ Total pressure on the table $= \frac{M}{L}2gx + \frac{Mgx}{L} = \frac{3Mgx}{L}$ = three times the weight of the coil on the table.

2.67 Measuring x vertically down, the equation of motion is

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(m\frac{\mathrm{d}x}{\mathrm{d}t}\right) = mg \tag{1}$$

where m is the mass of the rain drop after time t and x the distance through which the drop has fallen. If ρ is the density and r the radius after time t:

$$m = \frac{4}{3}\pi r^3 \rho \tag{2}$$

$$\therefore \quad \frac{\mathrm{d}m}{\mathrm{d}t} = \frac{\mathrm{d}m}{\mathrm{d}r}\frac{\mathrm{d}r}{\mathrm{d}t} = 4\pi\rho r^2 \frac{\mathrm{d}r}{\mathrm{d}t} \tag{3}$$

By problem $\frac{\mathrm{d}m}{\mathrm{d}t} = k\rho 4\pi r^2$ (4)

Comparing (3) and (4) $\frac{\mathrm{d}r}{\mathrm{d}t} = k$ (5)

Integrating $\quad r = kt + C_1$ (6)

where C_1 = constant.

At $t = 0, r = R$

$\therefore \quad C_1 = R$

$$\therefore \quad r = kt + R \tag{7}$$

Using (2) and (7) in (1)

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{4}{3}\pi\rho(R+kt)^3\frac{\mathrm{d}x}{\mathrm{d}t}\right\} = \frac{4}{3}\pi\rho(R+kt)^3 g$$

Integrating $(R+kt)^3\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{(R+kt)^4 g}{4k} + C_2$

But $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$ when $t = 0$. $C_2 = -\frac{R^4}{4k}g$

The velocity after time t is therefore

$$v(t) = \frac{g}{4k}\left\{a + kt - \frac{R^4}{(R+kt)^3}\right\}$$