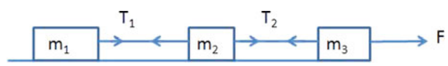
2.2 Problems

2.2.1 Motion of Blocks on a Plane

2.1 Three blocks of mass m_1, m_2 and m_3 interconnected by cords are pulled by a constant force F on a frictionless horizontal table, Fig. 2.1. Find

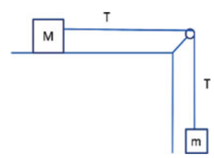
(a) Common acceleration 'a'
(b) Tensions T_1 and T_2

Fig. 2.1



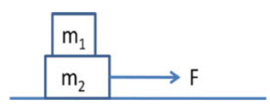
2.2 A block of mass M on a rough horizontal table is driven by another block of mass m connected by a thread passing over a frictionless pulley. Assuming that the coefficient of friction between the mass M and the table is μ, find (a) acceleration of the masses (b) tension in the thread (Fig. 2.2).

Fig. 2.2



2.3 A block of mass m_1 sits on a block of mass m_2, which rests on a smooth table, Fig. 2.3. If the coefficient of friction between the blocks is μ, find the maximum force that can be applied to m_2 so that m_1 may not slide.

Fig. 2.3



2.4 Two blocks m_1 and m_2 are in contact on a frictionless table. A horizontal force F is applied to the block m_1, Fig. 2.4. (a) Find the force of contact between the blocks. (b) Find the force of contact between the blocks if the same force is applied to m_2 rather than to m_1, Fig. 2.5.

Fig. 2.4

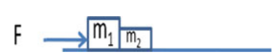


Fig. 2.5

2.5 A box of weight mg is dragged with force F at an angle θ above the horizontal. (a) Find the force exerted by the floor on the box. (b) Find the acceleration of the box if the coefficient of friction with the floor is μ. (c) How would the results be altered if the box is pushed with the same force?

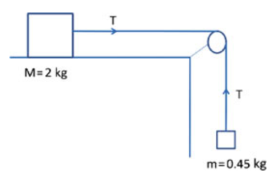
2.6 A uniform chain of length L lies on a table. If the coefficient of friction is μ, what is the maximum length of the part of the chain hanging over the table such that the chain does not slide?

2.7 A uniform chain of length L and mass M is lying on a smooth table and one-third of its length is hanging vertically down over the edge of the table. Find the work required to pull the hanging part on the table.

2.8 A block of metal of mass 2 kg on a horizontal table is attached to a mass of 0.45 kg by a light string passing over a frictionless pulley at the edge of the table. The block is subjected to a horizontal force by allowing the 0.45 kg mass to fall. The coefficient of sliding friction between the block and table is 0.2. Calculate (a) the initial acceleration, (b) the tension in the string, (c) the distance the block would continue to move if, after 2 s of motion, the string should break (Fig. 2.6).

[University of New Castle]

Fig. 2.6



2.2.2 *Motion on Incline*

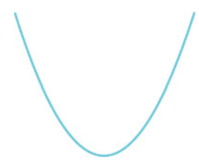
2.9 A block of mass of 2 kg slides on an inclined plane that makes an angle of 30° with the horizontal. The coefficient of friction between the block and the surface is $\sqrt{3}/2$.

(a) What force should be applied to the block so that it moves down without any acceleration?
(b) What force should be applied to the block so that it moves up without any acceleration?

[Indian Institute of Technology 1976]

2.10 A block is placed on a ramp of parabolic shape given by the equation $y = x^2/20$, Fig. 2.7. If $\mu_s = 0.5$, what is the maximum height above the ground at which the block can be placed without slipping?

Fig. 2.7



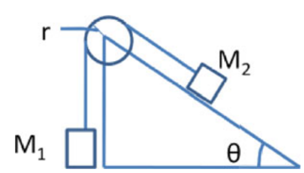
2.11 A block slides with constant velocity down an inclined plane that has slope angle $\theta = 30°$.

(a) Find the coefficient of kinetic friction between the block and the plane.
(b) If the block is projected up the same plane with initial speed $v_0 = 2.5\,\mathrm{m/s}$, how far up the plane will it move before coming to rest? What fraction of the initial kinetic energy is transformed into potential energy? What happens to the remaining energy?
(c) After the block comes to rest, will it slide down the plane again? Justify your answer.

2.12 Consider a fixed inclined plane at angle θ. Two blocks of mass M_1 and M_2 are attached by a string passing over a pulley of radius r and moment of inertia I_1 as in Fig. 2.8:

(a) Find the net torque acting on the system comprising the two masses, pulley and the string.
(b) Find the total angular momentum of the system about the centre of the pulley when the blocks are moving with speed v.
(c) Calculate the acceleration of the blocks.

Fig. 2.8



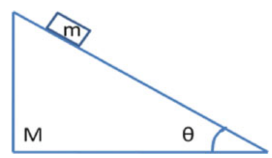
2.13 A box of mass 1 kg rests on a frictionless inclined plane which is at an angle of 30° to the horizontal plane. Find the constant force that needs to be applied parallel to the incline to move the box

(a) up the incline with an acceleration of 1 m/s^2
(b) down the incline with an acceleration of 1 m/s^2

[University of Aberystwyth, Wales 2008]

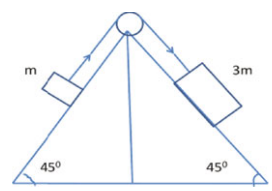
2.14 A wedge of mass M is placed on a horizontal floor. Another mass m is placed on the incline of the wedge. Assume that all surfaces are frictionless, and the incline makes an angle θ with the horizontal. The mass m is released from rest on mass M, which is also initially at rest. Find the accelerations of M and m (Fig. 2.9).

Fig. 2.9



2.15 Two smooth inclined planes of angles 45° and hinged together back to back. Two masses m and $3m$ connected by a fine string passing over a light pulley move on the planes. Show that the acceleration of their centre of mass is $\sqrt{5}/8\,g$ at an angle $\tan^{-1}$ ½ to the horizon (Fig. 2.10).

Fig. 2.10



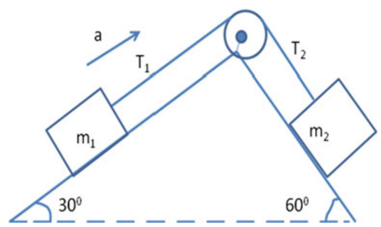
2.16 Two blocks of masses m_1 and m_2 are connected by a string of negligible mass which passes over a pulley of mass M and radius r mounted on a frictionless axle. The blocks move with an acceleration of magnitude a and direction as shown in the diagram. The string does not slip on the pulley, so the tensions T_1 and T_2 are different. You can assume that the surfaces of the inclines are frictionless. The moment of inertia of the pulley is given by $I = ½Mr^2$:

(a) Draw free body diagrams for the two blocks and the pulley.
(b) Write down the equations for the translational motion of the two blocks and the rotational motion of the pulley.
(c) Show that the magnitude of the acceleration of the blocks is given by

$$a = \frac{g(\sqrt{3}m_2 - m_1)}{M + 2(m_2 + m_1)}$$

2.17 Two masses in an Atwood machine are 1.9 and 2.1 kg, the vertical distance of the heavier body being 20 cm above the lighter one. After what time would the lighter body be above the heavier one by the same vertical distance? Neglect the mass of the pulley and the cord (Fig. 2.11).

Fig. 2.11



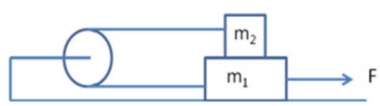
2.18 A body takes 4/3 times as much time to slide down a rough inclined plane as it takes to slide down an identical but smooth inclined plane. Find the coefficient of friction if the angle of incline is 45°.

2.19 A body slides down an incline which has coefficient of friction $\mu = 0.5$. Find the angle θ if the incline of the normal reaction is twice the resultant downward force along the incline.

2.20 Two masses m_1 and m_2 are connected by a light inextensible string which passes over a smooth massless pulley. Find the acceleration of the centre of mass of the system.

2.21 Two blocks with masses m_1 and m_2 are attached by an unstretchable string around a frictionless pulley of radius r and moment of inertia I. Assume that there is no slipping of the string over the pulley and that the coefficient of kinetic friction between the two blocks and between the lower one and the floor is identical. If a horizontal force F is applied to m_1, calculate the acceleration of m_1 (Fig. 2.12).

Fig. 2.12



2.2.3 Work, Power, Energy

2.22 The constant forces $\mathbf{F}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$ N and $\mathbf{F}_2 = 4\hat{i} - 5\hat{j} - 2\hat{k}\ N$ act together on a particle during a displacement from position $\mathbf{r}_2 = 7\hat{k}$ cm to position $\mathbf{r}_1 = 20\hat{i} + 15\hat{j}$ cm. Determine the total work done on the particle.

[University of Manchester 2008]

2.23 The potential energy of an object is given by

$$U(x) = 5x^2 - 4x^3$$

where U is in joules and x is in metres.

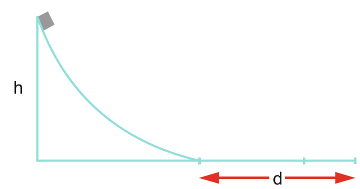
(i) What is the force, $F(x)$, acting on the object?
(ii) Determine the positions where the object is in equilibrium and state whether they are stable or unstable.

2.24 A body slides down a rough plane inclined to the horizontal at 30°. If 70% of the initial potential energy is dissipated during the descent, find the coefficient of sliding friction.

[University of Bristol]

2.25 A ramp in an amusement park is frictionless. A smooth object slides down the ramp and comes down through a height h, Fig. 2.13. What distance d is necessary to stop the object on the flat track if the coefficient of friction is μ.

Fig. 2.13



2.26 A spring is used to stop a crate of mass 50 kg which is sliding on a horizontal surface. The spring has a spring constant $k = 20\,\text{kN/m}$ and is initially in its equilibrium state. In position A shown in the top diagram the crate has a velocity of 3.0 m/s. The compression of the spring when the crate is instantaneously at rest (position B in the bottom diagram) is 120 mm.

(i) What is the work done by the spring as the crate is brought to a stop?
(ii) Write an expression for the work done by friction during the stopping of the crate (in terms of the coefficient of kinetic friction).
(iii) Determine the coefficient of friction between the crate and the surface.
(iv) What will be the velocity of the crate as it passes again through position A after rebounding off the spring (Fig. 2.14a, b)?

[University of Manchester 2007]

Fig. 2.14a

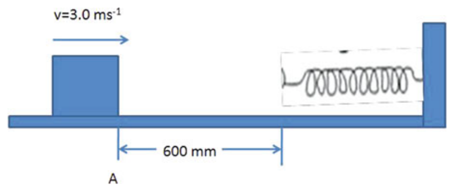
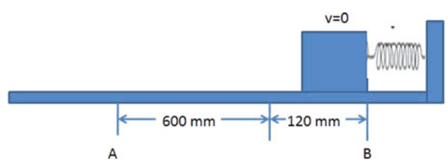


Fig. 2.14b



2.2.4 Collisions

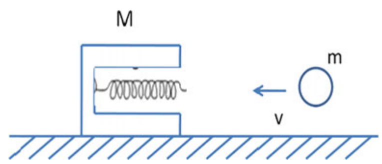
2.27 Observed in the laboratory frame, a body of mass m_1 moving at speed v collides elastically with a stationary mass m_2. After the collision, the bodies move at angles θ_1 and θ_2 relative to the original direction of motion of m_1. Find the velocity of the centre of mass (CM) frame of m_1 and m_2.
Hence show that before the collision in the CM frame m_1 and m_2 are approaching each other, m_1 with speed $m_2 v/(m_1 + m_2)$ and m_2 with speed $m_1 v/(m_1 + m_2)$.
In the CM frame after the collision m_1 moves off with speed $m_2 v/(m_1 + m_2)$ at an angle θ to its original direction. Draw a diagram showing the direction and speed of m_2 in the CM frame after the collision.
Find an expression for the speed m_1 after the collision in the laboratory frame in terms of m_1, m_2, v and the angle θ.
[University of Durham 2002]

2.28 Consider an off-centre elastic scattering of two objects of equal mass when one is initially at rest.

(a) Show that the final velocity vectors of the two objects are orthogonal.
(b) Show that neither ball can be scattered in the backward direction.

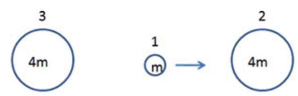
2.29 A small ball of mass m is projected horizontally with velocity v. It hits a spring of spring constant k attached inside an opening of a block resting on a frictionless horizontal surface. Find the compression of the spring noting that the block will slide due to the impact (Fig. 2.15).

Fig. 2.15



2.30 Two equal spheres of mass 4 m are at rest and another sphere of mass m is moving along their lines of centres between them. How many collisions will there be if the spheres are perfectly elastic (Fig. 2.16)?

Fig. 2.16



2.31 Two particles of mass m_1 and m_2 and velocities u_1 and $\alpha u_2 (\alpha > 0)$ make an elastic collision. If the initial kinetic energies of the two particles are equal, what should be the ratios u_1/u_2 and m_1/m_2 so that m_1 will be at rest after the collision?

2.32 Two bodies A and B, having masses m_A and m_B, respectively, collide in a totally inelastic collision.

(i) If body A has initial velocity v_A and B has initial velocity v_B, write down an expression for the common velocity of the merged bodies after the collision, assuming there are no external forces.
(ii) If $\boldsymbol{v}_A = 5\hat{i} + 3\hat{j}$ m/s and $\boldsymbol{v}_B = -\hat{i} + 4\hat{j}$ m/s and $m_A = 3m_B/2$, show that the common velocity after the collision is

$$v = 2.6\hat{i} + 3.4\hat{j} \text{ m/s}$$

(iii) Given that the mass of body A is 1200 kg and that the collision lasts for 0.2 s, determine the average force vectors acting on each body during the collision.
(iv) Determine the total kinetic energy after the collision.

2.33 A particle has an initial speed v_0. It makes a glancing collision with a second particle of equal mass that is stationary. After the collision the speed of the first particle is v and it has been deflected through an angle θ. The velocity of the second particle makes an angle β with the initial direction of the first particle.
Using the conservation of linear momentum principle in the x- and y-directions, respectively, show that $\tan\beta = v\sin\theta/(v_0 - v\cos\theta)$ and show that if the collision is elastic, $v = v_0\cos\theta$ (Fig. 2.17a,b).

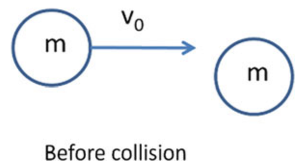


Fig. 2.17a

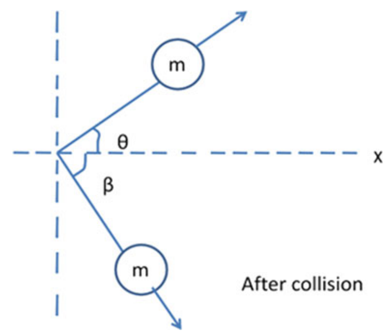


Fig. 2.17b

2.34 A carbon-14 nucleus which is radioactive decays into a beta particle, a neutrino and N-14 nucleus. In a particular decay, the beta particle has momentum p and the nitrogen nucleus has momentum of magnitude $4p/3$ at an angle of $90°$ to $\boldsymbol{p}$. In what direction do you expect the neutrino to be emitted and what would be its momentum?

2.35 If a particle of mass m collides elastically with one of mass m at rest, and if the former is scattered at an angle θ and the latter recoils at an angle φ with respect to the line of motion of the incident particle, then show that $\dfrac{m}{M} = \dfrac{\sin(2\varphi + \theta)}{\sin\theta}$.

2.36 A body of mass M rests on a smooth table and another of mass m moving with a velocity u collides with it. Both are perfectly elastic and smooth and no rotations are set up by this collision. The body M is driven in a direction at angle φ to the initial line of motion of the body m. Show that the velocity of M is $\dfrac{2m}{M+m}u\cos\varphi$.

2.37 A nucleus A of mass 2 m moving with velocity u collides inelastically with a stationary nucleus B of mass 10 m. After collision the nucleus A travels at 90°

with the incident direction while B proceeds at an angle 37° with the incident direction.

(a) Find the speeds of A and B after the collision.
(b) What fraction of the initial kinetic energy is gained or lost due to the collision.

2.38 A neutron moving with velocity v_0 collides head-on with carbon nucleus of mass number 12. Assuming that the collision is elastic

(a) calculate the fraction of neutron's kinetic energy transferred to the carbon nucleus and
(b) calculate the velocities of the neutron and the carbon nucleus after the collision.

2.39 Show that in an elastic collision between a very light body and a heavy body proceeds with twice the initial velocity of the heavy body.

2.40 A moving body makes a completely inelastic collision with a stationary body of equal mass at rest. Show that half of the original kinetic energy is lost.

2.41 A bullet weighing 5 g is fired horizontally into a 2 kg wooden block resting on a horizontal table. The bullet is arrested within the block which moves 2 m. If the coefficient of kinetic friction between the block and surface of the table is 0.2, find the speed of the bullet.

2.42 A particle of mass m with initial velocity u makes an elastic collision with a particle of mass M initially at rest. After the collision the particles have equal and opposite velocities. Find (a) the ratio M/m; (b) the velocity of centre of mass; (c) the total kinetic energy of the two particles in the centre of mass; and (d) the final kinetic energy of m in the laboratory system.

2.43 Consider an elastic collision between an incident particle of mass m with M initially at rest ($m > M$). Show that the largest possible scattering angle $\theta_{\max} = \sin^{-1}(M/m)$.

2.44 The ballistic pendulum is a device for measuring the velocity v of a bullet of mass m. It consists of a large wooden block of mass M which is supported by two vertical cords. When the bullet is fired at the block, it is dislodged and the block is set in motion reaching maximum height h. Show that $v = (1 + M/m)\sqrt{2gh}$

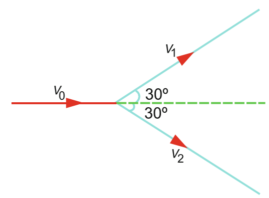
2.45 A fire engine directs a water jet onto a wall at an angle θ with the wall. Calculate the pressure exerted by the jet on the wall assuming that the collision with the wall is elastic, in terms of ρ, the density of water, A the area of the nozzle, and v the jet velocity.

2.46 Repeat the calculation of (2.45) assuming normal incidence and completely inelastic collision.

2.47 A ball moving with a speed of 9 m/s strikes an identical stationary ball such that after collision, the direction of each ball makes an angle 30° with the original line of motion (see Fig. 2.18). Find the speeds of the two balls after the collision. Is the kinetic energy conserved in the collision process?

[Indian Institute of Technology 1975]

Fig. 2.18



2.48 A ball is dropped from a height h onto a fixed horizontal plane. If the coefficient of restitution is e, calculate the total time before the ball comes to rest.

2.49 In prob. (2.48), calculate the total distance travelled.

2.50 In prob. (2.48), calculate the height to which the ball goes up after it rebounds for the nth time.

2.51 In the case of completely inelastic collision of two bodies of mass m_1 and m_2 travelling with velocities u_1 and u_2 show that the energy that is imparted is proportional to the square of the relative velocity of approach.

2.52 A projectile is fired with momentum p at an angle θ with the horizontal on a plain ground at the point A. It reaches the point B. Calculate the magnitude of change in momentum at A and B.

2.53 A shell is fired from a cannon with a velocity v at angle θ with the horizontal. At the highest point in its path, it explodes into two pieces of equal masses. One of the pieces retraces its path towards the cannon. Find the speed of the other fragment immediately after the explosion.

2.54 A helicopter of mass 500 kg hovers when its rotating blades move through an area of 45 m^2. Find the average speed imparted to air (density of air = 1.3 kg/m^3 and $g = 9.8\,\mathrm{m/s^2}$)

2.55 A machine gun fires 100 g bullets at a speed of 1000 m/s. The gunman holding the machine gun in his hands can exert an average force of 150 N against the gun. Find the maximum number of bullets that can be fired per minute.

2.56 The scale of balance pan is adjusted to read zero. Particles fall from a height of 1.6 m before colliding with the balance. If each particle has a mass of 0.1 kg and collisions occur at 441 particles/min, what would be the scale reading in kilogram weight if the collisions of the particles are perfectly elastic?

2.57 In prob. (2.56), assume that the collisions are completely inelastic. In this case, what would be the scale reading after time t?

2.58 A smooth sphere of mass m moving with speed v on a smooth horizontal surface collides directly with a second sphere of the same size but of half the mass that is initially at rest. The coefficient of restitution is e.

(i) Show that the total kinetic energy after collision is $\frac{mv^2}{6}\left(2+e^2\right)$.
(ii) Find the kinetic energy lost during the collision.

[University of Aberystwyth, Wales 2008]

2.59 A car of mass $m = 1200$ kg and length $l = 4$ m is positioned such that its rear end is at the end of a flat-top boat of mass $M = 8000$ kg and length $L = 18$ m. Both the car and the boat are initially at rest and can be approximated as uniform in their mass distributions and the boat can slide through the water without significant resistance.

(a) Assuming the car accelerates with a constant acceleration $a = 4\,\text{m/s}^2$ relative to the boat, how long does it take before the centre of mass of the car reaches the other end of the boat (and therefore falls off)?
(b) What distance has the boat travelled relative to the water during this time?
(c) Use momentum conservation to find a relation between the velocity of the car relative to the boat and the velocity of the boat relative to the water. Hence show that the distance travelled by the boat, until the car falls off, is independent of the acceleration of the car.

[University of Durham 2005]

2.2.5 Variable Mass

2.60 A rocket has an initial mass of m and a burn rate of

$$a = -\mathrm{d}m/\mathrm{d}t$$

(a) What is the minimum exhaust velocity that will allow the rocket to lift off immediately after firing? Obtain an expression for (b) the burn-out velocity; (c) the time the rocket takes to attain the burn-out velocity ignoring g; and (d) the mass of the rocket as a function of rocket velocity.

2.61 A rocket of mass 1000 t has an upward acceleration equal to $0.5\,g$. How many kilograms of fuel must be ejected per second at a relative speed of 2000 m/s to produce the desired acceleration.

2.62 For the Centaur rocket use the data given below:
Initial mass $m_0 = 2.72 \times 10^6$ kg
Mass at burn-out velocity, $m_B = 2.52 \times 10^6$ kg
Relative velocity of exhaust gases $v_r = 55$ km/s
Rate of change of mass, $\mathrm{d}m/\mathrm{d}t = 1290$ kg/s.

Find
(a) the rocket thrust,
(b) net acceleration at the beginning,
(c) time to reach the burn-out velocity,
(d) the burn-out velocity.

2.63 A 5000 kg rocket is to be fired vertically. Calculate the rate of ejection of gas at exhaust speed 100 m/s in order to provide necessary thrust to

(a) support the weight of the rocket and
(b) impart an initial upward acceleration of $2\,g$.

2.64 A flexible rope of length L and mass per unit length μ slides over the edge of a frictionless table. Initially let a length y_0 of it be hanging at rest over the edge and at time t let a length y moving with a velocity $\mathrm{d}y/\mathrm{d}t$ be over the edge. Obtain the equation of motion and discuss its solution.

2.65 An open railway car of mass W is running on smooth horizontal rails under rain falling vertically down which it catches and retains in the car. If v_0 is the initial velocity of the car and k the mass of rain falling into the car per unit time, show that the distance travelled in time t is $(Wv_0/k)\ln(1 + kt/W)$.
[with courtesy from R.W. Norris and W. Seymour, Mechanics via Calculus, Longmans, Green and Co., 1923]

2.66 A heavy uniform chain of length L and mass M hangs vertically above a horizontal table, its lower end just touching the table. When it falls freely, show that the pressure on the table at any instant during the fall is three times the weight of the portion on the table.
[with courtesy from R.W. Norris and W. Seymour, Mechanics via Calculus, Longmans, Green and Co., 1923]

2.67 A spherical rain drop of radius R cm falls freely from rest. As it falls it accumulates condensed vapour proportional to its surface. Find its velocity when it has fallen for t s.
[with courtesy from R.W. Norris and W. Seymour, Mechanics via Calculus, Longmans, Green and Co., 1923]

2.3 Solutions

2.3.1 Motion of Blocks on a Plane

2.1 **(a)** Acceleration $= \dfrac{\text{Force}}{\text{Total mass}}$

$$a = \frac{F}{(m_1 + m_2 + m_3)} \tag{1}$$