where V is in cubic metres and S in square metre for the volume and surface area of the room, respectively, and K is the absorption coefficient of the material of the floor, ceiling, walls, etc. summed over these components.

Beats: When two wave trains of slightly different frequencies travel through the same region, a regular swelling and fading of the sound is heard, a phenomenon called beats.

At a given point let the displacements produced by the two waves be

$$y = A \sin \omega_1 t \tag{8.36}$$

$$y = A \sin \omega_2 t \tag{8.37}$$

By the superposition principle, the resultant displacement is given by

$$y = y_1 + y_2 = [2A \cos 2\pi (f_1 - f_2)t/2] \sin 2\pi (f_1 + f_2)t/2 \tag{8.38}$$

The resulting vibration has a frequency

$$f = (f_1 + f_2)/2 \tag{8.39}$$

and an amplitude given by the expression in the square bracket of (8.38). The beat frequency is given by $f_1 \sim f_2$.

8.2 Problems

8.2.1 Vibrating Strings

8.1 Show that the one-dimensional wave equation is satisfied by the function $y = A\sqrt{(x + vt)}$.

8.2 Show that the equation $y = 2A \sin(n\pi x/L) \cos 2\pi f t$ for a standing wave is a solution of the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F}\frac{\partial^2 y}{\partial t^2}$$

where F is the tension and μ the mass/unit length.

8.3 A cord of length L fixed at both ends is set in vibration by raising its centre a distance h and let go. Obtain an expression for the displacement y at any point x and time t as a series expansion assuming that initially the velocity is zero. Also show that even harmonics are absent.

8.4 Show that the superposition of the waves $y_1 = A \sin(kx - \omega t)$ and $y_2 = 3A \sin(kx + \omega t)$ is a pure standing wave plus a travelling wave in the negative direction along the x-axis. Find the amplitude of **(a)** the standing wave and **(b)** the travelling wave.

8.5 A sinusoidal wave on a string travelling in the $+x$ direction at 8 m/s has a wavelength 2 m. (a) Find its wave number, frequency and angular frequency. (b) If the amplitude is 0.2 m, and the point $x = 0$ on the string is at its equilibrium position ($y = 0$) at time $t = 0$, find the equation for the wave.

8.6 A sinusoidal wave on a string travelling in the $+x$ direction has wave number $3/m$ and angular frequency 20 rad/s. If the amplitude is 0.2 m and the point at $x = 0$ is at its maximum displacement and $t = 0$, find the equation of the wave.

8.7 Show that when a standing wave is formed, each point on the string is undergoing SHM transverse to the string.

8.8 The length of the longest string in a piano is 2.0 m and the wave velocity of the string is 120 m/s. Find the frequencies of the first three harmonics.

8.9 Two strings are tuned to fundamentals of $f_1 = 4800$ Hz and $f_1' = 32$ Hz. Their lengths are 0.05 and 2.0 m, respectively. If the tension in these two strings is the same, find the ratio of the masses per unit length of the two strings.

8.10 The equation of a transverse wave travelling on a rope is given by $y = 5\sin\pi(0.02x - 4.00t)$, where y and x are expressed in centimetres and t is in seconds. Find the amplitude, frequency, velocity and wavelength of the wave.

8.11 A string vibrates according to the equation $y = 4\sin\frac{1}{2}\pi x\cos 20\pi t$, where x and y are in centimetres and t is in seconds. **(a)** What are the amplitudes and velocity of the component waves whose superposition can give rise to this vibration? **(b)** What is the distance between the nodes? **(c)** What is the velocity of the particle in the transverse direction at $x = 1.0$ cm and when $t = 9/4$ s?

8.12 A wave of frequency 250 cycles/s has a phase velocity 375 m/s. **(a)** How far apart are two points 60° out of phase? **(b)** What is the phase difference between two displacements at a certain point at time 10^{-3} s apart?

8.13 Two sinusoidal waves having the same frequency and travelling in the same direction are combined. If their amplitudes are 6.0 and 8.0 cm and have a phase difference of $\pi/2$ rad, determine the amplitude of the resultant motion.

8.14 Show that the one-dimensional wave equation is satisfied by the following functions:

(a) $y = A\ln(x + vt)$ and **(b)** $y = A\cos(x + vt)$.

8.15 **(a)** A cord of length L is rigidly attached at both ends and is plucked to a height h at a point 1/3 from one end and let it go. Show that the displacement y at any distance x along the string at time t in the subsequent motion is given by

$$y = \frac{3^{5/2}}{2\pi^2}\left[\sin\frac{\pi x}{L}\cos\frac{\pi vt}{L} + \frac{1}{4}\sin\frac{2\pi x}{L}\cos\frac{2\pi vt}{L} - \frac{1}{16}\sin\frac{4\pi x}{L}\cos\frac{4\pi vt}{L}\ldots\right]$$

(b) and that the third, sixth and ninth harmonics are absent.

8.16 Given the amplitude $A = 0.01$ m, frequency $f = 170$ vibrations/s, the wave velocity $v = 340\,\text{m/s}$, write down the equation of the wave in the negative x-direction.

8.17 **(a)** Show that the superposition of the waves $y_1 = A\sin(kx - \omega t)$ and $y_2 = +A\sin(kx + \omega t)$ is a standing wave. **(b)** Where are its nodes and antinodes?

8.18 The wave function for a harmonic wave travelling in the positive x-direction with amplitude A, angular frequency ω and wave number k is $y_1 = A\sin(kx - \omega t)$.
The wave interferes with another harmonic wave travelling in the same direction with the same amplitude, frequency and wave number, but with a phase difference δ. By using the principle of superposition, obtain an expression for the wave function of the resultant wave and show its amplitude is $|2A\cos\frac{1}{2}\delta|$.
If each wave has an amplitude of 6 cm and they differ in phase by $\pi/2$, what is the amplitude of the resultant wave?
For what phase differences would the resultant amplitude be equal to 6 cm?
Describe the effects that would be heard if the two waves were sound waves, but with slightly different frequencies. How could you determine the difference between the frequencies of the two harmonic sound sources? [you may use the result $\sin\theta_1 + \sin\theta_2 = 2\cos\frac{1}{2}(\theta_1 - \theta_2)\sin\frac{1}{2}(\theta_1 + \theta_2)$].

[University of Durham]

8.19 Show that the average rate of energy transmission $\bar{P}$, of a travelling sine of velocity v, angular frequency ω, amplitude A, along a stretched string of mass per unit length, μ, is $\bar{P} = \frac{1}{2}\mu v\omega^2 A^2$.

8.20 A fork and a monochord string of length 100 cm give 4 beats/s. The string is made shorter, without any change of tension, until it is in unison with the fork. If its new length is 99 cm, what is the frequency of the fork?

8.21 $y(x,t) = \frac{0.10}{4+(2x-t)^2}$ represents a moving pulse, where x and y are in metres and t in seconds. Find out the velocity of the pulse (magnitude and direction) and point out whether it is symmetric or not.

[adapted from Hyderabad Central University 1995]

8.22 **(a)** A piano string of length 0.6 m is under a tension of 300 N and vibrates with a fundamental frequency of 660 Hz. What is the mass density of the string?
(b) What are the frequencies of the first two harmonics?
(c) A flute organ pipe (opened at both ends) also plays a note of 660 Hz. What is the length of the pipe? (you may take the speed of sound as $V = 340\,\text{m/s}$).

[University of Manchester 2006]

8.23 **(a)** Sketch the first and second harmonic standing waves on a stretched string of length L. Deduce an expression for the frequencies of the family of standing waves that can be excited on the string.

(b) The wave function of a standing wave on a string that is fixed at both ends is given in SI units by $y(x, t) = (0.024)\sin(62.8x)\cos(471t)$.
Find the speed of the waves on the string, and the distance between nodes for the standing wave.
[hint: You may need to use $\sin\theta_1 + \sin\theta_2 = 2\cos\frac{1}{2}(\theta_2 - \theta_1)\sin\frac{1}{2}(\theta_1 + \theta_2)$].

8.24 A progressive wave travelling along a string has maximum amplitude $A = 0.0821\,\text{m}$, angular frequency $\omega = 100\,\text{rad/s}$ and wave number $k = 22.0\,\text{rad/m}$. If the wave has zero amplitude at $t = 0$ and $x = 0$ for its starting conditions

(i) State the wave function that represents the progressive wave motion for this wave travelling in the negative x-direction.
(ii) State the wave function for this wave travelling in the positive x-direction.
(iii) Find the wavelength (λ), period (T) and the speed (v) of this wave.
(iv) Find its amplitude at a time $t = 2.5\,\text{s}$ at a distance $x = 3.2\,\text{m}$ from its origin, for this wave travelling in the negative x-direction.

[University of Wales 2008]

8.25 The speed of a wave on a string is given by $v = \sqrt{\frac{F}{\mu}}$. Show that the right-hand side of this equation has the units of speed.

8.26 For a sinusoidal wave travelling along a string show that at any time t the slope $\frac{\partial y}{\partial x}$ at any point x is equal to the negative of the instantaneous transverse velocity $\frac{\partial y}{\partial t}$ of the string at x divided by the wave velocity v.

8.27 **(a)** Consider a small segment of a string upon which a wave pulse is travelling.
Using this diagram, or otherwise, show that the wave equation for transverse waves on a stretched string is

$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F}\frac{\partial^2 y}{\partial t^2}$$

where μ is the mass per unit length and F is the tension.
(b) Show that the wave function representing a wave travelling in the positive x-direction, $y(x - vt)$, is a solution of the wave equation. Obtain an expression for the velocity, v, of the wave (Fig. 8.1).

8.28 Two wires of different densities are joined as in Fig. 8.2. An incident wave $y_1 = A_1\sin(\omega t - k_1 x)$ travelling in the positive x-direction along the wire at the boundary is partly transmitted. (a) Find the reflected and transmitted amplitudes in terms of the incident amplitude. (b) When will the amplitude of the reflected wave be negative?

Fig. 8.1

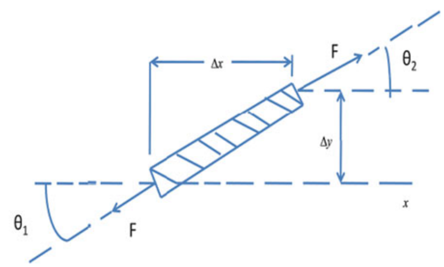
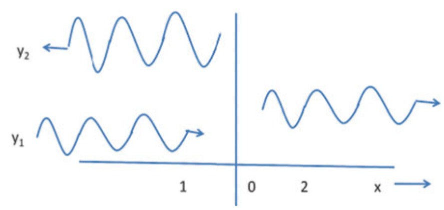


Fig. 8.2



8.29 For the wave shown in Fig. 8.3 find its frequency and wavelength if its speed is 24 m/s. Write the equation for this wave as it travels along the $+x$-axis if its position at $t = 0$ is as shown in Fig. 8.3.

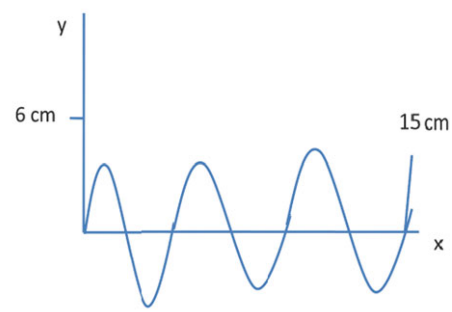


Fig. 8.3

8.30 In prob. (8.29) if the linear density of the string is 0.25 g/m, how much energy is sent down the string per second?

8.31 **(a)** Show that when a string of length L plucked at the centre through height h, the energy in the nth mode is given by $E_n = \frac{16Mh^2v^2}{n^2\pi^2L^2}$, where v is the wave velocity and M is the total mass of the string.
(b) Compare the energies in the first and the third harmonics of a string plucked at the centre.

8.2.2 *Waves in Solids*

8.32 **(a)** A steel bar of density 7860 kg/m^3 and Young's modulus 2×10^{11} N/m^2 and of length 0.25 m is rigidly clamped at one end and free to move at the other end. Determine the fundamental frequency of the bar for longitudinal harmonic vibrations.
(b) How do the frequencies compare with **(i)** rod free at both ends; **(ii)** bar clamped at the midpoint; and **(iii)** bar clamped at both the ends.

8.33 A 2 kg mass is hung on a steel wire of 1×10^{-5} m^2 cross-sectional area and 1.0 m length. **(a)** Calculate the fundamental frequency of vertical oscillations of the mass by considering it to be a simple oscillator and **(b)** calculate the fundamental frequency of vertical oscillations of the mass by regarding it as a system of longitudinally vibrating bar fixed at one end and mass-loaded at the other. Assume $Y = 210^{11}$ N/m^2 and $\rho = 7800$ kg/m^2 for steel.

8.34 Show that for $kl < 0.2$, the frequency equation derived for the mass loaded system for the bar of length l clamped at one end and loaded at the other reduces to that of a simple harmonic oscillator (you may assume that the frequency condition for this system is, $kl \tan kl = M/m$).

8.2.3 *Waves in Liquids*

8.35 **(a)** Find the velocity of long waves for a liquid whose depth is $\lambda/4$ and compare it with **(b)** the velocity for a similar wavelength λ in a deep liquid and **(c)** that for canal waves.

8.36 Find the maximum depth of liquid for which the formula $v^2 = gh$ represents the velocity of waves of length λ within 1%. You may assume that the velocity of surface waves is given by $v = \sqrt{\frac{g \tanh(kh)}{k}}$ which is valid for relatively deep waters.

8.37 In an experiment to measure the surface tension of water by the ripple method, the waves were created by a tuning fork of frequency 100 Hz and the wavelength was 3.66 mm. Calculate the surface tension of water.

8.38 Compare the minimum velocities of surface waves at 10°C for mercury and water if the surface tensions are 544 and 74 dyne/cm, respectively, and the specific gravity of mercury is 13.56.

8.39 It is only when a string is perfectly flexible that the phase velocity of a wave on a string is given by $\sqrt{T/\mu}$. The dispersion relations for the real piano wire can be written as

$$\frac{\omega^2}{k^2} = \frac{T}{\mu} + ak^2$$

where α is a small positive quantity which depends on the stiffness of the string. For perfectly flexible string, $\alpha = 0$. Obtain expressions for phase velocity (v_p) and group velocity v_g and show that v_p increases as wavelength decreases.

8.40 The dispersion relation for water waves of very short wavelength in deep water is $\omega^2 = \frac{S}{\rho}k^3$, where S is the surface tension and ρ is the density.

(a) What is the phase velocity of these waves?
(b) What is the group velocity?
(c) Is the group velocity greater or less than the phase velocity?

8.41 The general dispersion relation for water waves can be written as

$$\omega^2 = \left(gk + \frac{s}{\rho}k^3\right) \tanh\, kh$$

where g is acceleration due to gravity, ρ is the density of water, S is the surface tension and h is the water depth. Use the properties of tanh x function viz. for $x >> 1$, $\tanh x = 1$ and for $x << 1$, $\tanh x = x$.
Show that **(a)** in shallow water the group velocity and the phase velocity are both equal to $\sqrt{gh}$ if the wavelength is long enough to ensure that $Sk^2/v = 4\pi^2 S/\lambda^2\rho << g$. **(b)** Show that for deep water the phase velocity is given by $v_\mathrm{p} = \sqrt{\frac{g}{k} + Sk/\rho}$ and find the group velocity.

8.42 For water $\rho = 10^3\,\mathrm{kg/m^3}$ and $S = 0.075\,\mathrm{N/m}$. Calculate v_p and v_g in deep water for small ripples with $\lambda = 1$ cm and for large waves with $\lambda = 1$ m.

8.43 The relation for total energy E and momentum p for a relativistic particle is $E^2 = c^2p^2 + m^2c^4$, where m is the rest mass and c is the velocity of light. Using the relations, $E = \hbar\omega$ and $p = \hbar k$, where ω is the angular frequency and k is the wave number and $\hbar = h/2\pi$, h being Planck's constant. Show that the product of group velocity v_g and the phase velocity v_p, $v_\mathrm{p}v_\mathrm{g} = c^2$.

8.44 Taking the surface tension of water as 0.075 N/m its density as $1000\,\mathrm{kg/m^3}$, find the wavelength of surface waves on water with a velocity of 0.3 m/s.

Which one of these would be preferable to use in determining the surface tension by means of ripples?

8.45 Waves in deep water travel with phase velocity given by $v_p{}^2 = g/k$, where g is the acceleration due to gravity and k is the wave number, $2\pi/\lambda$. Obtain an expression for the group velocity and show that it is equal to $v_p/2$.

[University of Manchester 2006]

8.46 The dispersion relation for sound waves in air is $\omega = \sqrt{\frac{\gamma RT}{M}}k$. Find the phase velocity and the group velocity.

8.47 The phase velocity v_p for deep water waves is given by $v_p{}^2 = (g/k + Sk/\rho)$. Show that the phase velocity is minimum at $\lambda = 2\pi\sqrt{\frac{s}{\rho g}}$.

8.2.4 Sound Waves

8.48 Let both displacement and pressure of a plane wave vary harmonically. Obtain a relation between pressure amplitude and displacement amplitude. Also show that the displacement is 90° out of phase with the pressure wave.

8.49 Assuming $\rho = 1.29\,\text{kg/m}^3$ for the density of air and $v = 331\,\text{m/s}$ for the speed of sound, find the pressure amplitude corresponding to the threshold of hearing intensity of $10^{-12}\,\text{W/m}^2$.

8.50 For ordinary conversation, the intensity level is given as 60 dB. What is the intensity of the wave?

8.51 A small source of sound radiates energy uniformly at a rate of 4 W. Calculate the intensity and the intensity level at a point 25 cm from the source if there is no absorption.

8.52 The maximum pressure variation that the ear can tolerate is about $29\,\text{N/m}^2$. Find the corresponding maximum displacement for a sound wave in air having a frequency of 2000 Hz. Assume the density of air as $1.22\,\text{kg/m}^3$ and the speed of sound as 331 m/s.

8.53 If two sound waves, one in air and the other in water, have equal pressure amplitudes, what is the ratio of the intensities of the waves? Assume that the density of air is $1.293\,\text{kg/m}^3$, and the speed of sound in air and water is 330 and 1450 m/s, respectively.

8.54 The pressure in a progressive sound wave is given by the equation $P = 2.4\sin\pi(x - 330t)$, where x is in metres, t in seconds and P in N/m^2. Find **(a)** the pressure amplitude, **(b)** frequency, **(c)** wavelength and **(d)** speed of the wave.

8.55 A note of frequency 1200 vibrations/s has an intensity of $2.0\,\mu\,\text{W/m}^2$. What is the amplitude of the air vibrations caused by this sound?

8.56 Show that a plane wave having an effective acoustic pressure of a microbar in air has an intensity level of approximately 74 dB. Assume that the density and speed of air is $1.293\,\mathrm{kg/m^3}$ and sound velocity is 330 m/s.

8.57 Calculate the energy density and effective pressure of a plane wave in air of 70 dB intensity level. Assume the velocity of sound in air to be 331 m/s and the air density $1.293\,\mathrm{kg/m^3}$.

8.58 Find the pressure amplitude for an intensity of $1\,\mathrm{W/m^2}$ at the pain threshold. Assume that sound velocity is 331 m/s and gas density is $1.293\,\mathrm{kg/m^3}$.

8.59 Find the theoretical speed of sound in hydrogen at 0°C. For a diatomic gas $\gamma = 1.4$ and for hydrogen $M = 2.016\,\mathrm{g/mol}$; the universal gas constant $R = 8.317\,\mathrm{J/mol/K}$.

8.60 The density of oxygen is 16 times that of hydrogen. For both $\gamma = 1.4$. If the speed of sound is 317 m/s in oxygen at 0°C what is the speed in hydrogen at the same pressure?

8.61 Two sound waves have intensities 0.4 and $10\,\mathrm{W/m^2}$, respectively. How many decibels is one louder than the other?

8.62 If one sound is 6.0 dB higher than another, what is the ratio of their intensities?

8.63 A small source radiates uniformly in all directions at a rate of 0.009 W. If there is no absorption, how far from the source is the sound audible?

8.64 For the faintest sound that can be heard at 1000 Hz the pressure amplitude is about $2 \times 10^5\,\mathrm{N/m^2}$. Find the corresponding displacement amplitude. Assume that the velocity of sound is 331 m/s and the air density is $1.22\,\mathrm{kg/m^3}$.

8.65 Two sound waves of equal pressure amplitudes and frequencies traverse two liquids for which the velocities of propagation are in the ratio 3:2 and the densities of the liquids are in the ratio 3:4. Compare the **(a)** displacement amplitudes, **(b)** intensities and **(c)** energy densities.

8.66 One sound wave travels in air and the other in water, their intensities and frequencies being equal. Calculate the ratio of their **(a)** wavelength, **(b)** pressure amplitudes and **(c)** amplitudes of vibration of particles in air and water. Assume that the density of air is $1.293\,\mathrm{kg/m^3}$, and sound velocity in air and water is 331 and 1450 m/s, respectively.

8.67 Show that the characteristic impedance ρv of a gas is inversely proportional to the square root of its absolute temperature T. What is the characteristic impedance at **(a)** 0°C and **(b)** 80°C?

8.68 A beam of plane waves in water contains 50 W of acoustic power distributed uniformly over a circular cross-section of 50 cm diameter. The frequency of the waves is 25 kc/s. Determine **(a)** the intensity of the beam, **(b)** the sound pressure amplitude, **(c)** the acoustic particle velocity amplitude, **(d)** the acoustic particle displacement amplitude and **(e)** the condensation amplitude. Assume that the velocity of sound in water is 1450 m/s.

8.69 Derive Laplace formula for the sound velocity in a gas.

8.70 An empirical formula giving the velocity of sound in distilled water as a function of temperature at a pressure of one atmosphere in the range $0-60°$C is $v = 1403 + 5t - 0.06t^2 + 0.0003t^3$ where t is the temperature of water in °C and v is in m/s. **(a)** Determine the velocity of sound in distilled water at 20°C and **(b)** find the change of velocity of sound per degree Celsius at this temperature.

8.2.5 Doppler Effect

8.71 A railway engine whistles as it approaches a tunnel, and the sound is reflected back by the wall of the rock at the opening. If the train is proceeding at a speed of 72 km/h and if the effect of the wind be neglected, find the ratio of the relative frequencies of the reflected and direct sounds as heard by the driver of the engine.

[University of Aberystwyth, Wales]

8.72 Two trains move towards each other at a speed of 90 km/h relative to the earth's surface. One gives a 520 Hz signal. Find the frequency heard by the observer on the other train.

8.73 Two trains move away from each other at a speed of 25 m/s relative to the earth's surface. One gives a 520 Hz signal. Find the frequency heard by the observer on the other train (sound velocity = 330 m/s).

8.74 A whistle of frequency 540 Hz rotates in a circle of radius 2 m at an angular speed of 15.0 rad/s. What are the maximum and minimum frequencies heard by a listener, standing at a long distance away at rest from the centre of the circle (sound velocity = 330 m/s).

8.75 In the Kundt's tube experiment, the length of the steel rod which is stroked is 120 cm long and the distance between heaps of cork dust is 8 cm when the rod is caused to vibrate longitudinally in air. If the ends of the tube are sealed and the air replaced by a gas and the experiment repeated, the distance between heaps is observed to be 10 cm. **(a)** What is the velocity of sound in the gas if the velocity in air is 340 m/s. **(b)** What is the velocity of sound in the rod?

8.76 A sound source from a motionless train emits a sinusoidal wave with a source frequency of $f_s = 514$ Hz. Given that the speed of sound in air is 340 m/s and that you are a stationary observer. Find the wavelength of the wave you observe

(i) When the train is at rest
(ii) When the train is moving towards you at 15 m/s

(iii) When the train is moving away from you at 15 m/s>

[University of Aberystwyth, Wales 2007]

8.2.6 Shock Wave

8.77 **(a)** What is a shock wave?
(b) What is the Mach number when a plane travels with a speed twice the speed of the sound?
(c) Calculate the angle of Mach cone in (b).

8.2.7 Reverberation

8.78 Calculate the reverberation time of a room, 10 m wide by 18 m long by 4 m high. The ceiling is acoustic, the walls are plastered, the floor is made of concrete and there are 50 persons in the room. Sound absorption coefficients are acoustic ceiling 0.60, plaster 0.03, concrete 0.02, the absorbing power per person is 0.5.

8.2.8 Echo

8.79 A man standing in front of mountain at a certain distance beats a drum at regular intervals. The drumming rate is gradually increased and he finds the echo is not heard distinctly when the rate becomes 40/min. He then moves nearer to the mountain by 90 m and finds that the echo is again not heard when the drumming rate becomes 60/min. Calculate **(a)** the distance between the mountain and the initial position of the man and the mountain and **(b)** the velocity of sound.

[Indian Institute of Technology 1974]

8.80 A rifle shot is fired in a valley formed between two parallel mountains. The echo from one mountain is heard 2 s after the first one.

(a) What is the width of the valley?
(b) Is it possible to hear the subsequent echoes from the two mountains simultaneously, at the same point? If so, after what time, given sound velocity $= 360\,\mathrm{m/s}$.

[Indian Institute of Technology 1973]

8.2.9 Beat Frequency

8.81 Two whistles are sounded with frequencies of 548 and 552 cycles/s, respectively. A man directly in the line between them walks towards the lower pitched whistle at 1.5 m/s. Find the beat frequency that he hears. Assume the sound velocity of 330 m/s.

8.82 A tuning fork of frequency 300 c/s gives 2 beats/s with another fork of unknown frequency. On loading the unknown fork the beats increase to 5/s, while transferring the load to the fork of known frequency increases the number of beats per second to 9. Calculate the frequency of the unknown fork (unloaded) assuming the load produces the same frequency change in each fork.

[University of Newcastle]

8.2.10 Waves in Pipes

8.83 An open organ pipe sounding its fundamental note is in tune with a fork of frequency 439 cycles/s. How much must the pipe be shortened or lengthened in order that 2 beats/s shall be heard when it sounded with the fork? Assume the speed of sound is 342 m/s.

[University of Durham]

8.84 A light pointer fixed to one prong of a tuning fork touches a vertical plate. The fork is set vibrating and the plate is allowed to fall freely. Eight complete oscillations are counted when the plate falls through 10 cm. What is the frequency of the tuning fork?

[Indian Institute of Technology 1997]

8.85 Air in a tube closed at one end vibrates in resonance with tuning fork whose frequencies are 210 and 350 vibrations/s, when the temperature is 20°C. Explain how this is possible and find the effective length of the tube. Assume that the velocity in air at 0°C is 33, 150 cm/s.

[University of London]

8.86 An open organ pipe is suddenly closed with the result that the second overtone of the closed pipe is found to be higher in frequency by 100 vibrations/s than the first overtone of the original pipe. What is the fundamental frequency of the open pipe?

[University of Bristol]

8.3 Solutions

8.3.1 Vibrating Strings

8.1 The one-dimensional wave equation is

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \tag{1}$$