For half-wave plate

$$(n_{\text{slow}} - n_{\text{fast}})t = (2m + 1)\lambda/2 \tag{15.55}$$

This implies that if two components enter the slab in phase, they emerge from the slab 180° out of phase. The half-wave plate rotates the polarization direction but otherwise leaves the polarization unaffected.

For quarter wave plate

$$(n_{\text{slow}} - n_{\text{fast}})t = (2m + 1)\lambda/4 \tag{15.56}$$

Here, the two components have the phase difference of 90° as they emerge from the slab of thickness t.

If two quarter wave plates are put together the combination acts as a half-wave plate.

When an unpolarized beam is incident on a birefringent material, it is split up into an ordinary ray which travels in a normal way, obeying Snell's law, and an extraordinary ray which is displaced, the two emerging rays being linearly polarized at right angles to each other.

Polarimeter: When a polarized beam of light is passed through an optically active liquid such as sugar solution then the polarizing plane rotates through an angle θ:

$$\theta = \alpha L D \tag{15.57}$$

where α is the specific rotation, L is the length of the tube in decimetres and D is the amount of solvent in grams per 100 c.c.

15.2 Problems

15.2.1 Geometrical Optics

General

15.1 Show that the fraction F of light that escapes from a point source within a medium across a flat surface is given by

$$F = \frac{1}{2}\left[1 - \frac{1}{\mu}\sqrt{\mu^2 - 1}\right]$$

where μ is the refractive index.

15.2 Assuming that a 1000 W light bulb radiates equally in all directions, calculate the radiation pressure on a perfectly absorbing surface at a distance of 2 m.

15.3 Define optical path and state Fermat's principle. Using *Fermat's principle*, derive Snell's law of refraction at the plate interface between two materials of refractive index n and n'.

15.4 Use the concept of optical path to briefly explain why a mirage occurs.
Early in the morning, on a sunny day, the heat of the sun produces a thin layer of warm air above the surface of a long straight road. Consider a possible light ray path such as that illustrated in Fig. 15.5. This connects an eye-level point on the tree with an observer of height $h = 2\,\text{m}$. If the layer of hot air has refractive index $n_2 = 1.00020$, while the cold air has refractive index $n_1 = 1.00030$.

(a) Show that the *optical path length* along ABCD is approximately

$$n_2 x + n_1\sqrt{(d-x)^2 + 4h^2}$$

(b) By using Fermat's principle, determine the actual distance that the ray travels in the layer of hot air when $d = 500\,\text{m}$.

(c) As the observer walks towards the tree, she finds that the mirage disappears. At what distance from the tree does this occur?

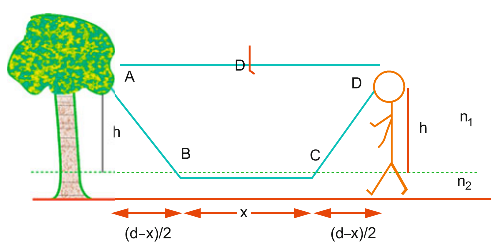


Fig. 15.5

15.5 An optical fibre consists of an inner material (the fibre) with refractive index n_f and an outer material of lower refractive index n_c, known as cladding, as in Fig. 15.6.

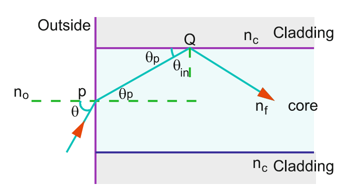


Fig. 15.6

(a) What is the purpose of cladding?
(b) Show that the maximum acceptance angle $\theta_{\max}$ is given by

$$n_0 \sin\theta_{\max} = \sqrt{n_f^2 - n_c^2}$$

(c) Discuss two main fibre loss mechanisms.

15.2.2 Prisms and Lenses

15.6 A triangular glass prism ($n = 1.6$) is immersed in a liquid ($n = 1.1$) as shown in Fig. 15.7. A thin ray of light is incident as shown on face AB making an angle of 20° with the normal. Calculate the angle that the ray emerging from AC makes with the ground when it leaves AC and strikes the ground.

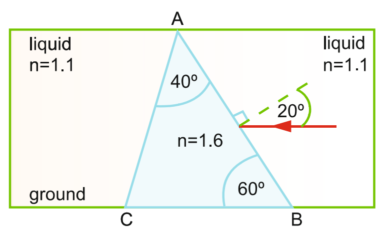


Fig. 15.7

15.7 **(a)** What is the critical angle for a block of glass, with refractive index $n_g = 1.45$, in air?
(b) Two narrow beams of microwave radiation are incident normal to one surface of a large wedge of chocolate as shown in Fig. 15.8. If the index of refraction for chocolate relative to air for these microwaves is 1.2, calculate the angle between the two emerging beams, shown as α on the

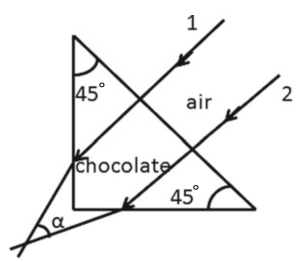


Fig. 15.8

diagram. (You can assume that the chocolate does *not* melt because of the microwaves.)

[University of Aberystwyth, Wales]

15.8 Each of the angles of a prism is 60° and the refraction index for sodium light is 1.5. Sodium light is incident at the correct angle for minimum deviation. Calculate the deviation of that portion of the light, which finally emerges from the prism, after having suffered one internal reflection.

[University of Durham]

15.9 Write down the lens maker's formula relating the focal length of a lens to the object and image distances. Explain the sign convention used for the distances involved.
Show that as two lenses are brought into contact, the focal length of the combined system, f, can be expressed as

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

where f_1 and f_2 are the focal lengths of the two separate lenses.

[University of Durham 2001]

15.10 An object is placed at a fixed distance D from the screen. Real images of the object are formed on the screen for two positions of a lens, separated by a distance d. Show that

(a) the ratio between the sizes of the two images will be $\dfrac{(D-d)^2}{(D+d)^2}$
(b) The object size $= \sqrt{I_1 I_2}$, where I_1 and I_2 are the sizes of the images
(c) $f = \dfrac{D^2 - d^2}{4D}$
(d) $D > 4f$

15.11 **(a)** Derive the lens maker's formula

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

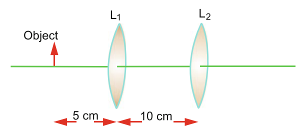
for a thin lens.

(b) A biconvex lens of plastic of refractive index $n = 1.2$ is immersed in water ($n = 1.33$). Would the lens act as a converging lens or diverging lens?

15.12 A combination of two thin convex lenses are placed as in Fig. 15.9. An object is placed 5 cm in front of the first lens which has a focal length of 10 cm. The second lens is 10 cm behind the first lens and has a focal length of 12 cm.

(a) Locate the image of the first lens with the aid of a ray diagram.
(b) Is the image real or virtual? Erect or inverted?
(c) Locate the final image for the lens combination.
(d) Is the final image real or virtual? erect or inverted?

Fig. 15.9



15.13 Let a glass sphere of radius r lie with its centre on the x-axis. A ray of light parallel to the x-axis will form an image on the other side of the sphere. Show that the distance of the image from the centre of the sphere will be equal to $\frac{\mu r}{2(\mu - 1)}$, where μ is the refractive index of the glass.

15.14 The light from a 100 W bulb uniformly spreads out in all directions. Find the intensity I of the electromagnetic waves and the amplitude E_0 at a distance of 5 m from the bulb.

15.15 An astronomical telescope has the focal lengths of objective and eyepiece in the ratio 8:1. Both the lenses are convex. A tower 100 m tall is at a distance of 10,000 m: (a) locate the image and (b) find the height of the image.

15.16 A 1000 W laser beam is concentrated by a lens of a cross-sectional area of 10^{-5} cm^2. Find the corresponding (a) intensity and (b) the amplitude of the electric field.

15.2.3 Matrix Methods

15.17 Derive expressions for the refraction matrix and translation matrix for a single lens.

15.18 Obtain the matrix equation for a pair of surfaces of radii r_1 and r_2 and refractive index n, separated by distance d and placed in air.

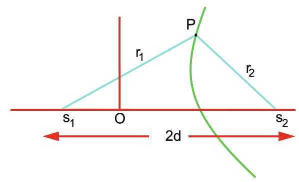
15.19 Using the results of prob. (15.18) show that for a thin lens,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

15.2.4 Interference

15.20 Consider two point sources s_1 and s_2, Fig. 15.10, which emit coherent waves. Show that curves such as that traced by p, for which the phase difference for rays r_1 and r_2 is a constant, are hyperboloids in three dimensions.

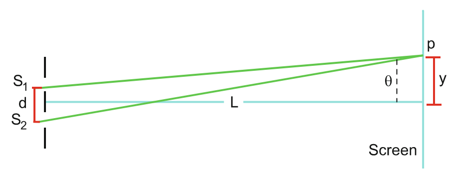
Fig. 15.10



15.21 A beam of monochromatic light with wavelength λ is incident upon two slits S_1 and S_2 a distance d apart, as shown in Fig. 15.11. Derive an expression, in terms of λ, L, d, and m for the distance y_m from the central point to the mth bright fringe of the interference pattern on a screen a distance L away when $L >> y_m$.

Two narrow slits separated by 2 mm are illuminated with a helium–neon laser of wavelength 612 nm. Calculate the spacing of the fringes observed on a screen 4 m.

Fig. 15.11



15.22 When a thin transparent plate of thickness t and refractive index μ is introduced in the path of one of the two interfering monochromatic beams of wavelength λ in Young's double-slit experiment, then fringes are shifted. Show that $(\mu - 1)t = n\lambda$.

15.23 In Young's double-slit experiment the bandwidth β is given by the expression: $\beta = \dfrac{\lambda L}{d}$, where L is the slit-screen distance, d is the slit separation and λ is the wavelength of light used:

- **(a)** Obtain an expression for the intensity distribution in the fringe system of Young's double-slit experiment.
- **(b)** Hence show that the average intensity of the fringes is equal to $2I$, where I is the intensity of each beam.

15.24 In a Fresnel's biprism experiment, the bandwidth of 0.195 mm is observed at a distance of 1 m from the slit. The image of the coherent sources is then produced at the same distance from the slit by placing a convex lens a 30 cm from the slit. Two images are found to be separated by 0.7 cm. Calculate the wavelength of light used.

[Kakatiya University 2002]

15.25 The inclined faces of a biprism of refractive index 1.50 make an angle of 2° with the base. A slit illuminated by monochromatic light is placed at a distance of 10 cm from the biprism. If the distance between two dark fringes observed at a distance of 1 m from the prism is 0.18 mm, find the wavelength of light used.

[University of Delhi]

15.26 A beam of monochromatic light of wavelength 5.82×10^{-7} m falls normally on a glass wedge with the wedge angle of 20 s of an arc. If the refractive index of glass is 1.5, find the number of interference fringes per centimetre of the wedge length.

[Indian Administration Services]

15.27 Light of wavelength 6000 Å falls normally on a thin wedge film of refractive index 1.4, forming fringes that are 2 mm apart. Find the angle of the wedge.

[Delhi University]

15.28 In Newton's rings apparatus, the radii of the nth and $(n + 20)$th dark rings are found to be 0.162 and 0.368 cm, respectively, when light of wavelength 546 nm is used. Calculate the radius of curvature, R, of the lower surface of the lens.

[University of Manchester 2007]

15.29 The radius of the 10th dark ring in Newton's rings apparatus changes from 60 to 50 mm when a liquid is introduced between the lens and the plate. Calculate the refraction index of the liquid.

[Nagarjuna University 2003]

15.30 Newton's rings may be formed in the reflective light by two curved surfaces as in Fig. 15.12a,b with the monochromatic light of wavelength λ incident from the top.

Show that the radius of the nth ring is given by the expression for the two situations:

$$r_n^2\left(\frac{1}{R_1} \pm \frac{1}{R_2}\right) = n\lambda \qquad \text{(dark rings)}$$

$$= (n + \frac{1}{2})\lambda \qquad \text{(bright rings)}$$

Minus sign in the bracket of left side for situation (a) and plus sign for situation (b).

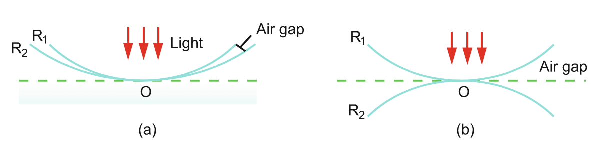


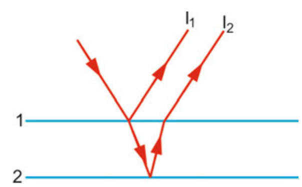
Fig. 15.12

15.31 In Young's experiment for what order does the band of wavelength of red light ($\lambda = 780\,\text{nm}$) coincide with $(m + 1)$th order in the band of blue light ($\lambda = 520\,\text{nm}$)?

15.32 Each of the parallel glass plates 1 and 2 reflects 25% of narrow monochromatic beam of light incident on it and transmits the remainder. Find the ratio of the minimum and maximum intensities in the interference pattern formed by the two beams I_1 and I_2 (Fig. 15.13).

[adapted from Hyderabad Central University 1991]

Fig. 15.13



15.33 A thin 4×10^{-5} cm thick film of refractive index 1.5 is illuminated by white light normal to its surface. Which colour will be intensified in the visible spectrum?

15.34 A parallel beam of light ($\lambda = 5890\,\text{A}^\circ$) is incident on a thin glass plate of refractive index 1.5 such that the angle of refraction in the plate is 60°. Calculate the smallest thickness of the plate which will appear dark by reflection.

[Srivenkateswara University 2000]

15.35 A beam of waves of wavelength ranging from 5800 to 3500 Å is allowed to fall normaly on a thin air film of thickness 0.2945 μm. What is the colour shown in reflection by the film?

[Osmania University]

15.36 Show that the minimum thickness of non-reflecting film is $\lambda/4\mu$.

[Kakatiya University 2001]

15.37 A beam of parallel rays is incident at an angle of 30° with the normal on a plane-paralleled film of thickness 4×10^{-5} cm and refractive index 1.50. Show that the reflected light whose wavelength is 7542 Å will reinforce.

[Mumbai University]

15.38 **(a)** Explain with necessary theory how a Michelson interferometer may be employed to find the difference in wavelength of D_1 and D_2 lines in the sodium spectrum.

(b) The distance through which the mirror of the Michelson interferometer has to be displaced between two consecutive positions of maximum distinctness of D_1 and D_2 lines of sodium is 2.89×10^{-5} cm. Calculate $\Delta\lambda$ assuming that $\lambda_1 \approx \lambda_2 = 5.89 \times 10^{-5}$ cm.

15.39 In Michelson's interferometer 100 fringes cross the field of view when the movable mirror is displaced through 0.02948 mm. Calculate the wavelength of the monochromatic light used.

[Delhi University]

15.40 The plates of Fabry – Perot interferometer have a reflectance amplitude of $r = 0.90$. Calculate the resolving power of wavelengths near 600 nm when the plates are separated by 2 mm.

[University of Wales, Aberystwyth 2005]

15.2.5 Diffraction

15.41 In Fraunhofer diffraction due to a narrow slit a screen is placed 2 m away from the lens to obtain the pattern. If the slit width is 0.2 mm and the first minima are 5 mm on either side of the central maximum, find the wavelength of light.

[Delhi University]

15.42 The intensity I_θ for the single-slit diffraction pattern is given by $I_\theta = I_{\mathrm{m}} (\sin\alpha/\alpha)^2$, where $\alpha = \dfrac{\pi a}{\lambda}\sin\theta$, and I_{m} is the intensity of the central maximum. Show that the intensity maxima can be found out from the condition, $\tan\alpha = \alpha$.

15.43 **(a)** Obtain the expression for $\Delta\theta$, the half-width at half central maximum of single-slit Fraunhofer diffraction.
(b) Calculate $\Delta\theta$ for $\dfrac{a}{\lambda} = 4$.

[Osmania University]

15.44 A beam of light contains a mixture of wavelengths λ_1 and λ_2. When the light is incident on a single slit the first diffraction minimum of λ_1 coincides with the second minimum of λ_2. How are the two wavelengths related?

15.45 A single slit is illuminated normally by a monochromatic light of wavelength of 5600 Å and diffraction bands are observed on a screen 2 m away. If the centre of the second dark band is 1.6 cm from the central bright band, deduce the slit width.

15.46 **(a)** What are missing orders in the double-slit diffraction pattern? Explain.
(b) Deduce the missing order for a double-slit diffraction pattern, if the slit widths are 0.16 mm and they are 0.8 mm apart.

[Brahampur University]

15.47 What conditions must be satisfied for the central maximum of the envelope of the double-slit diffraction pattern to contain exactly n interference fringes? Find n given $d = 0.20$ mm and $a = 0.0120$ mm.

15.48 In a grating spectrum which spectral line in the fourth order will overlap with the third order of 5400 Å?

[Osmania University]

15.49 What is the highest order spectrum which may be seen with monochromatic light of wavelength 6000 Å by means of a diffraction grating with 5000 lines/cm.

[Delhi University]

15.50 A grating has slits that are each 0.1 mm wide. The distance between the centres of any two adjacent slits is 0.3 mm. Which of the higher order maxima are missing?

[Andhra University 1999]

15.51 A grating has 5×10^3 lines/cm. The opaque spaces are twice the transparent spaces. Find the orders of the spectrum that will be absent.

[Osmania University 2004]

15.52 How many orders will be observed by a grating having 4000 lines/cm, if a visible light in the range 4000–7000 Å is incident normally?

[Kanpur University]

15.53 Show that in a grating if the opaque and the transparent strips are of equal width then all the even orders, except $m = 0$, will be missing.

15.54 Show that the intensity of the first secondary maxima relative to that of central maxima in the single-slit diffraction is about 4.5%.

15.55 A plane diffraction grating in the first order shows an angle of minimum deviation of 20° at the mercury blue line of wavelength 4358 Å. Calculate the number of lines per centimetre.

[Andhra University 2003]

15.56 A diffraction grating used at normal incidence gives a green line, $\lambda = 5400$ Å in a certain order superimposed on the violet line, $\lambda = 4050$ Å of the next higher order. If the angle of diffraction is 30°, how many lines are there per centimetre in the grating?

[Delhi University]

15.57 Calculate the least width that a grating must have to resolve the components of D lines (5890 and 5896 Å) in the second order. The grating has 800 lines/cm.

[Osmania University 2002]

15.58 A grating of width 3″ is ruled with 10,000 lines/in. Find the smallest wavelength separation that can be resolved in the first-order spectrum at a mean wavelength of 6000 Å.

[Kakatiya University2002]

15.59 Examine two spectral lines of wavelengths 5890 and 5896 Å which can be clearly resolved in the (i) first order and (ii) second order by diffraction grating 2 cm wide and having 425 lines/cm.

[Delhi University]

15.60 The refractive indices of a glass prism for the C and F lines are 1.6545 and 1.6635, respectively. The wavelength of these two lines in the solar spectrum are 6563 and 5270 Å, respectively. Calculate the length of the base of 60° prism which is capable of resolving sodium lines of wavelengths 5890 and 5896 Å.

[Vikram University]

15.61 Find the separation of two points on the moon that can be resolved by a 500 cm telescope. The distance of the moon is 3.8×10^5 km from the earth. The eye is most sensitive to light of wavelength 5500 Å.

[Nagpur University]

15.62 Lycopodium particles that have an average diameter of 30 μm are dusted on a glass plate. If a parallel beam of light of wavelength 589 nm is passed through the plate, what is the angular radius of the first diffraction maximum?

[Kakatiya University 2004]

15.63 The intensity distribution for Fraunhofer diffraction of a circular spectrum of radius R is of the form

$$I = I_0 \left[\frac{2J_1(\rho)}{\rho} \right]^2$$

where $\rho = \dfrac{2\pi a}{\lambda} \sin\theta$ and $J_1(x)$ is the Bessel function of the first kind. Show that by Rayleigh's criterion the minimum angular resolution of a telescope is given by $\theta \approx 1.22 \dfrac{\lambda}{D}$, where D is the diameter of the circular aperture.

15.64 Calculate the radii of the 1st and 25th circles on a zone plate behaving like a convex lens of focal length 50 cm for $\lambda = 5000$ Å.

[Kakatiya University]

15.2.6 Polarization

15.65 The values of refractive indices for ordinary and extraordinary rays n_o and n_e for calcite are 1.642 and 1.478, respectively. Calculate the phase retardation for $\lambda = 6000$ Å with the plate thickness 0.04 mm.

[Kakatiya University 2003]

15.66 Two polarizing sheets have their polarizing directions parallel. Determine the angle by which either sheet must be turned so that the intensity falls to half of its value?

15.67 Sun rays incident obliquely on a pond are completely polarized by reflection. Find the elevation of the sun (in degrees) above the horizon.

15.68 Light is incident from water ($\mu = 1.33$) on the glass ($\mu = 1.5$). Find the polarizing angle for the boundary separating water and glass.

15.69 What is the minimum thickness of a quarter wave plate if the material has $\mu_0 = 1.553$ and $\mu_e = 1.544$ at a wavelength of 6000 Å.

[Andhra University 2003]

15.70 A tube 20 cm long containing sugar solution rotates the plane of polarization through an angle of 13.2° If the specific rotation is 66°, find the amount of sugar present in a litre of the solution.

[Osmania University 2003]

15.71 A system of three polarizing sheets intercept a beam of initially unpolarized light. The polarizing direction of the first sheet is parallel to the y-axis, that of the second sheet is at an angle of θ counterclockwise from the y-axis and that of the third sheet is parallel to the x-axis. The intensity of light emerging from the three-sheet system is 11.52% of the original intensity I_0. Determine the angle θ. In which direction is the emerging light polarized?

15.72 Describe briefly how a linear polarizer produces polarized light from an incident unpolarized beam. Is the transmission axis of a pair of polaroid sunglasses usually oriented horizontally or vertically for an observer standing upright and why is this?

What is a Brewster window? Calculate the inclination of a Brewster window with refractive index $n = 1.5$ in a laser cavity in which the gaseous medium has a refractive index $n = 1.0$.

[Durham University]