9.29 In prob. (9.28) if the tubes are connected in series then what quantity will flow through the composite tube?

9.3 Solutions

9.3.1 Bernoulli's Equation

9.1 From continuity equation

$$A_1 v_1 = A_2 v_2$$

$$\therefore \quad v_2 = \frac{A_1 v_1}{A_2} = \frac{\pi r_1^2 v_1}{\pi r_2^2} = \frac{10^2 \times 4}{5^2} = 16\,\text{m/s}$$

9.2 $v_x = 3x^2 - xy + 2z^2$

$$v_y = 2x^2 - 6xy + y^2$$

$$v_z = -2xy - yz + 2y^2$$

$$\therefore \quad \frac{\partial v_x}{\partial x} = 6x - y; \ \frac{\partial v_y}{\partial y} = -6x + 2y; \ \frac{\partial v_z}{\partial z} = -y$$

$$\nabla \cdot \boldsymbol{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = (6x - y) + (-6x + 2y) - y = 0$$

Thus the continuity equation for steady incompressible flow is satisfied.

9.3 Pressure difference across the wing

$$\Delta p = \frac{1}{2}\rho(v_1^2 - v_2^2)$$
$$= \frac{1}{2} \times 1.293 \times (70^2 - 55^2) = 1212\,\text{Pa}$$

(a) Lift = (pressure difference) (area)
= 1212 × 4 = 4848 N

(b) Net force = Lift − Weight of plane

= 4848 − (300 × 9.8)
= 1908 N in the upward direction

9.4 $\Delta P = \frac{1}{2}\rho v^2 \left(\frac{A^2}{a^2} - 1\right)$

$$\frac{A}{a} = \frac{\pi R^2}{\pi r^2} = \left(\frac{10}{5}\right)^2 = 4$$

$$60{,}000 - 45{,}000 = \frac{1}{2} \times 1000 \times 15v^2$$

or $\quad v = 1.414\,\text{m / s (throat)}$

Rate of flow of water

$$Q = vA = (1.414)(\pi \times 0.01^2) = 0.0444\,\text{m}^3/\text{s}$$

9.5 $v = \sqrt{\dfrac{2gh\rho'}{\rho}} = \sqrt{\dfrac{2 \times 9.8 \times 0.15 \times 810}{1.293}} = 42.9\,\text{m/s} = 154.5\,\text{km/h}$

9.6 Total area of the holes

$$A = 80 \times 2.5 \times 10^{-6}\,\text{m}^2 = 2 \times 10^{-4}\,\text{m}^2$$

$$Q = Av$$

$$v = \frac{Q}{A} = \frac{2 \times 10^{-3}}{2 \times 10^{-4}} = 10\,\text{m/s}$$

9.7 **(a)** $v_x = 3xy + y^2 \quad v_y = 5xy + 2x$

$$\frac{\partial v_x}{\partial x} = 3y;\ \frac{\partial v_y}{\partial y} = 5x$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 3y + 5x \neq 0$$

Therefore, steady incompressible flow is not possible.

(b) $v_x = 3x^2 + y^2 \quad v_y = -6xy$

$$\frac{\partial v_x}{\partial x} = 6x;\ \ \frac{\partial v_y}{\partial y} = -6x$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 6x - 6x = 0$$

Thus, steady incompressible flow is possible.

9.8 $\Delta P = \dfrac{1}{2}\rho(v_1^2 - v_2^2)$

$$v_1 = \sqrt{\frac{2\Delta P}{\rho} + v_2^2} = \sqrt{\frac{2 \times 1000}{1.293} + 100^2} = 107.45\,\text{m/s}$$

9.9 **(a)** $Q = v_{\text{throat}} a = v_{\text{pipe}} A$

$$\therefore \quad v_{\text{throat}} = v_{\text{pipe}} \frac{A}{a} = v_{\text{pipe}} \frac{D^2}{d^2} = 10 \times \frac{4^2}{2^2} = 40\,\text{m/s}$$

(b) $$\Delta p = \frac{1}{2}\rho v^2 \left(\frac{A^2}{a^2} - 1\right)$$

$$= \frac{1}{2} \times 1000 \times 40^2 \left(\frac{4^4}{2^4} - 1\right) = 12 \times 10^6\,\text{Pa}$$

9.10 Reynold's number $R = \dfrac{\rho D v}{\eta}$, where ρ is density, D diameter, v velocity and η coefficient of viscosity.

(a) $R = \dfrac{1 \times 0.1 \times 300}{0.018} = 1667$

Flow is steady because $R < 2200$

(b) $R = \dfrac{1 \times 0.1 \times 300}{0.008} = 3750$

Flow is turbulent because $R > 2200$

(c) $R = \dfrac{1 \times 0.1 \times 300}{0.004} = 7500$

Flow is turbulent because $R > 2200$

9.11 Consider a mass element dm of the fluid at distance x from the vertical axis. The centrifugal force on dm is

$$\mathrm{d}F = \mathrm{d}m\,\omega^2 x = \mathrm{d}m \frac{\mathrm{d}v}{\mathrm{d}t} = \mathrm{d}m \frac{\mathrm{d}v}{\mathrm{d}x} v$$

$$v\mathrm{d}v = \omega^2 x\,\mathrm{d}x$$

$$\int v\,\mathrm{d}v = \omega^2 \int x\,\mathrm{d}x$$

$$\frac{v^2}{2} = \frac{\omega^2}{2} x^2 \Big|_{L-l}^{L}$$

$$\therefore \quad v = \omega l \sqrt{\frac{2L}{l} - 1}$$

9.12 Applying Bernoulli's equation to points A and B,

$$p_{\text{A}} + \frac{1}{2}\rho_{\text{g}} v^2 = P_{\text{B}} \tag{1}$$

$$p_{\text{A}} + \rho_{\text{L}} g h = P_{\text{B}} \tag{2}$$

Comparing (1) and (2)

$$v = \sqrt{\frac{2gh\rho_{\mathrm{L}}}{\rho_{\mathrm{g}}}}$$

9.13 Apply Bernoulli's equation at the sections A_1 and A_2:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \tag{1}$$

$$\therefore \quad p_2 - p_1 = \Delta p = \Delta h \rho g = \frac{1}{2}\rho(v_1^2 - v_2^2)$$

$$\therefore \quad 2g\Delta h = v_1^2 - v_2^2 \tag{2}$$

$$Q = v_1 A_1 = v_2 A_2 \tag{3}$$

$$v_2 = \frac{v_1 A_1}{A_2} \tag{4}$$

Using (4) in (2)

$$2g\Delta h = v_1^2 \frac{(A_2^2 - A_1^2)}{A_2^2}$$

$$v_1 = A_2 \sqrt{\frac{2g\Delta h}{A_2^2 - A_1^2}}$$

$$Q = A_1 v_1 = A_1 A_2 \sqrt{\frac{2g\Delta h}{A_2^2 - A_1^2}}$$

9.14 Volume of water flowing out per second

$$Q = sv \tag{1}$$

where v is the speed and s is the cross-sectional area.
Volume flowing out

$$V = Qt = svt \tag{2}$$

$$\frac{1}{2}\rho v^2 = P = \frac{F}{A} = \frac{FL}{AL} = \frac{W}{V} \tag{3}$$

where L is the length of the cylinder and W is the work done.

$$\therefore \quad W = \frac{1}{2}\frac{\rho V^3}{s^2 t^2} \tag{4}$$

where we have used (2).

9.15 **(a)** The components of $m\omega^2 r$ parallel to the x-axis and z-axis are $m\omega^2 x$ and $m\omega^2 z$, respectively. Taking y in the upward direction

$$\mathrm{d}p = \rho(\omega^2 x\mathrm{d}x + \omega^2 z\mathrm{d}z - g\mathrm{d}y) \tag{1}$$

In the $x - z$-plane, $y =$ constant. Hence $\mathrm{d}y = 0$.
Integrating (1)

$$p = \frac{\rho\omega^2 x^2}{2} + \frac{\rho\omega^2 z^2}{2} + C$$

where C is the constant of integration.

$$\begin{aligned} p &= \frac{\rho\omega^2}{2}(x^2 + z^2) + C \\ &= \frac{1}{2}\omega^2 r^2 + C \end{aligned}$$

$p = p_0$ at $r = 0$, then $C = p_0$

$$\therefore \quad p = p_0 + \frac{1}{2}\rho\omega^2 r^2$$

(b) Particle at P is in equilibrium under centrifugal force and gravity, Fig. 9.6. Let PM be tangent at $P(r, y)$ making an angle θ with the r-axis. PN is normal at P. If N is the normal reaction

$$N\cos\theta = mg$$

$$N\sin\theta = m\omega^2 r$$

$$\therefore \quad \tan\theta = \frac{\omega^2 r}{g}$$

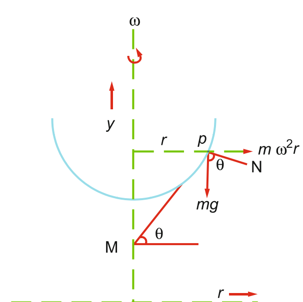


Fig. 9.6

$$\therefore \quad \frac{\mathrm{d}y}{\mathrm{d}r} = \frac{\omega^2 r}{g}$$

$$y = \int \mathrm{d}y = \frac{\omega^2}{g} \int r\,\mathrm{d}r + c$$

$$y = \frac{\omega^2 r^2}{2g} + c$$

$$y = 0, r = 0, c = 0$$

$$y = \frac{1}{2}\frac{\omega^2 r^2}{g}$$

Figure of revolution of the curve is a paraboloid.

9.3.2 Torricelli's Theorem

9.16 Using Bernoulli's equation

$$P_2 + \frac{1}{2}\rho v_2^2 = P_1 + \frac{1}{2}\rho v_1^2$$
$$3.1 \times 10^5 + \frac{1}{2} \times 1000 v_2^2 = 3.5 \times 10^5 + 0$$
$$v_2 = 8.94\,\mathrm{m/s}$$

9.17 **(a)** Use Bernoulli's equation at two points A and B at height h_A and h_B, respectively, Fig. 9.7.

$$P + \rho g h_\mathrm{A} = P + \rho g h_\mathrm{B} + \frac{\rho v^2}{2} \tag{1}$$

where P is the atmospheric pressure, ρ is the density of water and v is the efflux velocity.

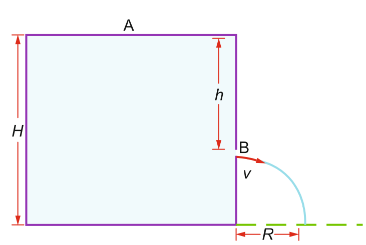


Fig. 9.7

Calling $h_A - h_B = h$ (2)

$$v = \sqrt{2gh} \tag{3}$$

Using simple kinematics, the range

$$R = vt = \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}}$$
$$R = 2\sqrt{h(H-h)} \tag{4}$$

(b) In (4) R is unchanged if we replace h by $H - h$. Therefore, the second hole must be punched at a depth $H - h$ to get the same range.

9.18 From prob. (9.17)

$$R = 2\sqrt{h(H-h)} \tag{1}$$

Maximum range is obtained by setting $dR/dh = 0$ and holding H as constant. This gives $h = H/2$ and substituting this value in (1), we get $R_{max} = H$.

9.19 For the water level to remain stationary volume efflux = rate of filling = x

$$vA = \left(\sqrt{2gh}\right) A = x = 70\,\text{cm}^3/\text{s}$$
$$h = \frac{x^2}{2gA^2} = \frac{(70)^2}{2 \times 980 \times (0.25)^2} = 40\,\text{cm}$$

9.20 Let the water level be at a height x at any instant. The efflux velocity will be $v = \sqrt{2gx}$. As the water flows out, the level of water comes down, Fig. 9.8.

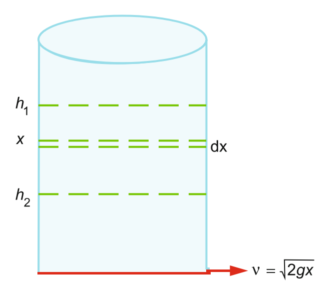


Fig. 9.8

Volume flux, $Q = av = a\sqrt{2gx}$

Volume flux is also equal to $Q = A\dfrac{\mathrm{d}x}{\mathrm{d}t}$

We then have $a\sqrt{2gx} = A\dfrac{\mathrm{d}x}{\mathrm{d}t}$

$$t = \int \mathrm{d}t = \frac{A}{a\sqrt{2g}} \int_{h_2}^{h_1} \frac{\mathrm{d}x}{\sqrt{x}} = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{h_1} - \sqrt{h_2}\right]$$

9.21 Pressure at the bottom due to water column $= (3 - 1)$ atm $= 2$ atm $= 2 \times 10^5$ Pa.

$$P = h\rho g$$

$$\therefore \quad h = \frac{P}{\rho g} = \frac{2 \times 10^5}{1000g} = \frac{200}{g}$$

$$v = \sqrt{2gh} = \sqrt{2g\frac{200}{g}} = 20\,\mathrm{m/s}$$

Second method

Apply Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

where the left side refers to the point inside the tank and right side to a point outside the tank.

$$3 \times 10^5 + 0 = 1 \times 10^5 + \frac{1}{2} \times 1000 v_2^2$$

$$\therefore \quad v_2 = 20\,\mathrm{m/s}$$

9.22 $Q = v_1 A_1 = v_2 A_2$

$$\left(\sqrt{2gh_1}\right)\,(2A_2) = \left(\sqrt{2gh_2}\right) A_2$$

$$\therefore \quad \frac{h_1}{h_2} = \frac{1}{4}$$

9.23 Apply Bernoulli's equation to a point just outside the hole and a point at the top of the kerosene surface. If P is the atmospheric pressure, h_1 and h_2 the heights of water and kerosene columns, respectively, ρ_1 and ρ_2 the respective densities,

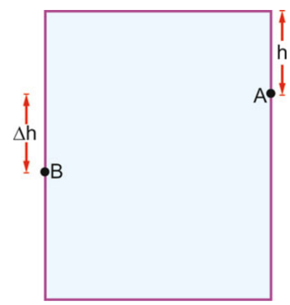
$$P + \frac{1}{2}\rho_1 v_1^2 = P + h_1\rho_1 g + h_2\rho_2 g$$

$$\therefore \quad v_1 = \sqrt{2g\left(h_1 + \frac{h_2\rho_2}{\rho_1}\right)}$$

Substituting $h_1 = 60\,\text{cm}$, $h_2 = 40\,\text{cm}$, $\rho_1 = 1$, $\rho_2 = 0.8$ and $g = 980$, we find $v_1 = 425\,\text{cm/s}$ or 4.25 m/s.

9.24 Volume efflux at A and B, Fig. 9.9

Fig. 9.9



$$Q_A = v_A S$$
$$Q_B = v_B S$$
$$(\text{Mass efflux})_A = \rho v_A S$$
$$(\text{Mass efflux})_B = \rho v_B S$$

Force $F_A = (\text{rate of change of momentum})_A$

$$= \rho v_A S v_A = \rho S v_A^2$$
$$= \rho S(2gh) = 2\rho S g h$$
$$F_B = 2\rho S g(h + \Delta h)$$
$$F_B - F_A = 2\,\rho\,S\,g\,\Delta\,h$$

(because the vector force is in the opposite direction)

$$= 2 \times 1000 \times 1.0 \times 10^{-4} \times 9.8 \times 0.51 = 1.0\,\text{N}$$

9.3.3 Viscosity

9.25 Volume of liquid flowing per second

$$V = \frac{\pi r^4 P}{8\eta l}$$

$$P = \frac{8\eta l V}{\pi r^4} = \frac{8 \times 0.001 \times 4000 \times 0.002}{3.14 \times (0.04)^4} = 0.0796 \times 10^5 \text{ Pa}$$

$$\text{Pressure head } h = \frac{P}{\rho g} = \frac{0.0796 \times 10^5}{1000 \times 9.8} = 0.8 \text{ m}$$

9.26 $$P_A - P_B = \frac{8\eta l_1 Q}{\pi r^4} = \frac{8\eta Q}{\pi} \frac{(0.16)}{(2 \times 10^{-3})^4} = \frac{8\eta Q\ (0.01)}{\pi \times 10^{-12}} \quad (1)$$

$$P_B - P_0 = \frac{8\eta Q \times (0.04)}{\pi \times 10^{-12}} \quad (2)$$

Adding (1) and (2)

$$P_A - P_0 = \frac{8\eta Q \times 0.05}{\pi \times 10^{-12}} \quad (3)$$

Dividing (2) by (3)

$$\frac{P_B - P_0}{P_A - P_0} = 0.8$$

$$\therefore \quad P_B - P_0 = 0.8 \times (P_A - P_0) = 0.8 \times 3 = 2.4 \text{ cm of water.}$$

9.27 The terminal velocity v_T is given by

$$v_T = \frac{2}{9} r^2 g \frac{(\rho_1 - \rho_2)}{\eta} \quad (1)$$

where r is the radius of the drop, ρ_1 and ρ_2 are the densities of the drop and air, respectively, g is the gravity and η is the coefficient of viscosity. If the new radius is r' and the new terminal velocity v'_T, then

$$\frac{v'_T}{v_T} = \frac{r'^2}{r^2} \quad (2)$$

Under the assumption that the drops are incompressible, the volume remains constant:

$$\frac{4\pi}{3}(r')^3 = 2 \times \frac{4\pi}{3} r^3$$

$$\therefore \quad r' = 2^{1/3} r \quad (3)$$

Using (3) in (2)

$$v'_T = 2^{2/3} v_T = 4^{1/3} v_T$$

9.28 For the first tube $Q_1 = \dfrac{\pi P r^4}{8\eta l} = Q$

For the second tube $Q_2 = \dfrac{\pi P (2r)^4}{8\eta l/2} = \dfrac{32\pi P r^4}{8\eta l} = 32Q$

Total quantity of water flowing is

$$Q_1 + Q_2 = Q + 32Q = 33Q$$

9.29 Let the pressure at the beginning of the first tube be P_1 and at the end P_2. Since the water flow must be continuous, the rate of flow in the two tubes must be identical, that is, $Q_1 = Q_2$. Let the atmospheric pressure be P_0.

$$P_1 - P_2 = \frac{8\eta l Q_1}{\pi r^4} \quad \text{(for the first tube)} \tag{1}$$

$$P_2 - P_0 = \frac{8\eta (l/2) Q_2}{\pi (2r)^4} = \frac{8\eta l Q_1}{32\pi r^4} \quad \text{(for the second tube)} \tag{2}$$

Adding (1) and (2)

$$P_1 - P_0 = \frac{8\eta l}{\pi r^4}\frac{33Q_1}{32} \tag{3}$$

But $P_1 - P_0 = \dfrac{8\eta l Q}{\pi r^4}$ (for single tube of length l and radius r) (4)

Comparing (3) and (4), we get $Q_1 = \dfrac{32Q}{33}$.