

10.2.7 Surface Tension

- 10.62** If the surface tension of the liquid–gas interface is 0.072 N/m, the density is 1 kg/L and the radius of the capillary is 1 mm, to what height will the liquid rise up the capillary?

[University of Manchester 2007]

- 10.63** A mole of gaseous molecules in a bubble obeys the ideal gas law. What is the volume of the bubble at a 100 m depth of water if the temperature is 293 K, the atmospheric pressure is 101 kPa, density of water is 1000 kg/m³ and the ideal gas constant is 8.314 J/mol/K.

[University of Manchester 2008]

- 10.64** Let n droplets each of radius r coalesce to form a large drop of radius R . Assuming that the droplets are incompressible and S is the surface tension calculate the rise in temperature if c is the specific heat and ρ is the density.

- 10.65** A soap bubble of surface tension 0.03 N/m is blown from 1 cm radius to 5 cm radius. Find the work done.

- 10.66** A small hollow vessel which has a small hole in it is immersed in water to a depth of 45 cm before any water penetrates into the vessel. If the surface tension of water is 0.073 N/m, what should be the radius of the hole?

- 10.67** What will be the depth of water at which an air bubble of radius 0.3×10^{-3} m may remain in equilibrium (surface tension of water = 0.072 N/m and $g = 9.8 \text{ m/s}^2$)?

- 10.68** A capillary tube of radius 0.2 mm and of length 6 cm is barely dipped in water. Will the water overflow through the capillary? If not what happens to the meniscus (surface tension of water = 0.073 N/m and angle of contact = 0°)?

- 10.69** A soap bubble of radius 2.0 cm is charged so that the excess of pressure due to surface tension is neutralized. If the surface tension is 0.03 N/m, what is the charge on the bubble?

- 10.70** Two soap bubbles with radii r_1 and r_2 coalesce to form a bigger bubble of radius r . Show that $r = \sqrt{r_1^2 + r_2^2}$.

10.3 Solutions

10.3.1 Kinetic Theory of Gases

- 10.1** The mean free path λ of a gas molecule is the average distance travelled by the molecule between successive collisions.

$$\lambda = x/N \quad (1)$$

where x is the total distance travelled and N is the number of collisions.
In terms of frequency f , average time of collision T and the mean molecular velocity v ,

$$\lambda = vT = \frac{v}{f} = \frac{1000}{2 \times 10^{10}} = 5 \times 10^{-8} \text{ m}$$

$$\mathbf{10.2 (a)} \quad \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 300}{28 \times 10^{-3}}} = 516.8 \text{ m/s}$$

$$\mathbf{(b)} \quad R = kn_0 = 1.38 \times 10^{-23} \times 6.02 \times 10^{23} = 8.3 \text{ J/mol K}$$

$$N = \frac{PV}{RT} = \frac{2 \times 1.013 \times 10^5 \times 1}{8.3 \times 300} = 81.365 \text{ mol/m}^3$$

$$n = \text{number/m}^3 = \frac{6.02 \times 10^{23} \times 81.365}{28 \times 10^{-3}} = 2.15 \times 10^{25} / \text{m}^3$$

$$\lambda = \frac{1}{\sigma n} = \frac{1}{0.43 \times 10^{-18} \times 2.15 \times 10^{25}} = 1.082 \times 10^{-7} \text{ m}$$

$$f = \frac{v}{\lambda} = \frac{516.8}{1.082 \times 10^{-7}} = 4.78 \times 10^9 / \text{s}$$

10.3 (a) Collision cross-section between two molecules each of radius r is equivalent to collision of one molecule of radius $2r$ with another point size molecule. Therefore the cross-section will be

$$\sigma = \pi(2r)^2 = 4\pi r^2 \quad (1)$$

Consider a rectangular box of face area 1 m^2 and length λ metres. Then the volume of the box $V = \lambda \text{ m}^3$. If n is the number of molecules per unit volume then λ is such that the total projected area arising from n molecules will just fill up an area of 1 m^2 .

$$\therefore \quad n\lambda\sigma = 1$$

$$\text{or} \quad \lambda = \frac{1}{n\sigma} = \frac{V}{4\pi r^2 N} \quad (2)$$

where we have used (1) and set $n = N/V$.

Equation (2) is based on the assumption that the target molecules are stationary. In practice, the molecule hits moving targets. This leads to an increase of collision frequency by a factor of $\sqrt{2}$ and therefore a decrease in the cross-section by a factor $\sqrt{2}$. The corrected expression for mean free path is then

$$\lambda = \frac{V}{4\pi\sqrt{2}r^2 N} \quad (3)$$

$$(b) R = N_0 k = 6.02 \times 10^{23} \times 1.38 \times 10^{-23} = 8.3 \text{ J/mol K} \quad (1)$$

$$PV = NRT \quad (2)$$

$$1.01 \times 10^5 \text{ V} = 8.13 \times 373 \text{ N} \quad (3)$$

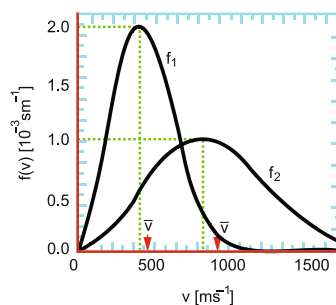
whence $N = 33.3 \text{ mol/m}^3$

$$n = \text{number/m}^3 = 33.3 \times 6.02 \times 10^{23} = 2 \times 10^{25} \text{ m}^{-3}$$

$$\lambda = \frac{1}{n\sigma\sqrt{2}} = \frac{1}{2 \times 10^{25} \times 4\pi (10^{-10})^2 \sqrt{2}} = 10^{-7} \text{ m.}$$

10.4 (a) For curve f_1 , $v_{\text{mp}} = 425 \text{ m/s}$; for curve f_2 , $v_{\text{mp}} = 850 \text{ m/s}$ (Fig. 10.5).

Fig. 10.5 Maxwell–Boltzmann velocity distribution



$$(b) v_{\text{mp}} \propto \sqrt{T}$$

$$\therefore T_2 = \frac{T_1 v_{\text{mp}}^2(1)}{v_{\text{mp}}^2(2)} = 300 \left(\frac{850}{425} \right)^2 = 1200 \text{ K}$$

$$(c) \bar{v} = \sqrt{4/\pi} v_{\text{mp}}$$

$$\bar{v}_1 = 1.1287 \times 425 = 480 \text{ m/s}$$

$$\bar{v}_2 = 1.1287 \times 850 = 959 \text{ m/s}$$

$$\bar{v}_1/\bar{v}_2 = 1/2$$

(d) and (e)

$$v_{\text{mp}} = \sqrt{\frac{2kT}{m}}$$

$$\begin{aligned}\therefore m &= \frac{2kT}{v_{\text{mp}}^2} = \frac{2 \times 1.38 \times 10^{-23} \times 1200}{(850)^2} = 4.584 \times 10^{-26} \text{ kg} \\ &= \frac{4.584 \times 10^{-26}}{1.66 \times 10^{-27}} \text{ amu} = 27.6 \text{ amu}\end{aligned}$$

Therefore the gas is N_2 .

In 5 mol of gas total number (N) of gas molecules will be

$$N = \frac{5}{28} \times 6.02 \times 10^{23} = 1.075 \times 10^{23}$$

Number of molecules $N(v) dv$ in the interval v and $v + dv$ will be

$$N(v)dv = 4\pi N \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 \exp \left[-\frac{mv^2}{2kT} \right] dv \text{ (Maxwellian distribution)} \quad (2)$$

The mean value of the interval is $v = \frac{800 + 900}{2} = 850 \text{ m/s}$
which happens to be identical with v_{mp} found in (a). In this case (2) is reduced to a simpler form

$$N(v)dv = \frac{4N dv}{\sqrt{\pi} e v_{\text{mp}}}$$

Number of N_2 molecules in 5 mol will be

$$N = \frac{5}{28} \times 6.02 \times 10^{23} = 1.075 \times 10^{23}$$

The speed interval

$$dv = 900 - 800 = 100 \text{ m/s}$$

Thus the required number of molecules is

$$N(v)dv = \frac{4 \times 1.075 \times 10^{23} \times 100}{\sqrt{\pi} \times 2.718 \times 850} = 1.05 \times 10^{22}$$

$$10.5 \quad f(v) = 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-mv^2/2kT}$$

$$\frac{df(v)}{dv} = \text{const.} \frac{d}{dv} \left[v^2 e^{-mv^2/2kT} \right] = 0$$

$$\therefore e^{-mv^2/2kT} \left[2v - \frac{m}{kT} v^3 \right] = 0$$

$$\text{whence } v = 0, \infty, \sqrt{\frac{2kT}{m}}$$

$$\text{The most probable speed is } v_{\text{mp}} = \sqrt{\frac{2kT}{m}}$$

$$10.6 \quad (i) \quad \bar{E} = \frac{3}{2} kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.21 \times 10^{-21} \text{ J}$$

$$(ii) \quad \sqrt{v^2} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 300}{44 \times 10^{-3}}} = 412.3 \text{ m/s}$$

$$(iii) \quad v_{\text{mp}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2 \times 8.31 \times 300}{44 \times 10^{-3}}} = 336.6 \text{ m/s}$$

$$(iv) \quad v_{\text{av}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8 \times 8.31 \times 300}{44 \times 10^{-3} \pi}} = 380.0 \text{ m/s}$$

10.7 (a) Assumptions:

- (i) The molecules of a gas behave like hard, smooth spheres and of negligible size compared to that of the container.
- (ii) The molecules are in random motion undergoing collisions with one another and with the walls of the container for negligible duration.
- (iii) Newton's laws of motion are applicable and the number of molecules is large so that statistics may be applied.

$$(b) \quad \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad (\text{from the gas equation})$$

$$\therefore V_2 = \frac{P_1}{P_2} \cdot \frac{T_2}{T_1} V_1 = \frac{1}{0.5} \times \frac{(273 - 40)}{(273 + 27)} \times 2 = 3.1 \text{ m}^3$$

$$10.8 \quad (a) \quad \frac{n}{V} = \frac{P}{RT} = \frac{9.8 \times 10^5}{8.31 \times 400} = 294.8 \text{ mol/m}^3$$

$$\begin{aligned} (b) \quad \frac{n}{V} &= \left[\frac{P + a \frac{n^2}{V^2}}{RT} \right] \left(1 - \frac{bn}{V} \right) \\ &= \left[\frac{9.8 \times 10^5 + 0.448 \times (294.8)^2}{8.31 \times 400} \right] (1 - 4.29 \times 10^{-5} \times 294.8) \\ &= 302.66 \text{ mol/m}^3 \end{aligned}$$

10.3.2 Thermal Expansion

10.9 Let the length of the bars be L_0 , each at 0°C when they are straight. With the rise of temperature ΔT , their lengths will be (Fig. 10.2)

$$L_1 = \theta R_1 = L_0(1 + \alpha_1 \Delta T) \quad (1)$$

$$L_2 = \theta R_2 = L_0(1 + \alpha_2 \Delta T) \quad (2)$$

Subtracting (2) from (1)

$$\theta(R_1 - R_2) = \theta d = (\alpha_1 - \alpha_2)L_0 \Delta T \quad (3)$$

$$\therefore R_1 - R_2 = d$$

Adding (1) and (2)

$$\theta(R_1 + R_2) = 2L_0 + (\alpha_1 + \alpha_2)L_0 \Delta T \simeq 2L_0 \quad (4)$$

$$\therefore (\alpha_1 + \alpha_2)\Delta T \ll 2$$

$$R = \frac{R_1 + R_2}{2} = \frac{L_0}{\theta} = \frac{d}{(\alpha_1 - \alpha_2)\Delta T} \quad (5)$$

where we have used (3).

10.10 Let the initial length of the rod be $2x$ and the final total length be $2(x + \Delta x)$. Let the centre of the buckled rod be raised by y , then

$$\Delta x = \alpha x \Delta T$$

From the geometry of Fig. 10.6,

$$\begin{aligned} y &= [(x + \Delta x)^2 - x^2]^{1/2} = [2x \Delta x + (\Delta x)^2]^{1/2} \\ &\simeq \sqrt{2x \Delta x} \quad (\because \Delta x \ll 2x) \\ &= \sqrt{2\alpha x^2 \Delta T} \\ &= \sqrt{2 \times 12 \times 10^{-6} \times 20^2 \times 30} = 0.5367 \text{ m} \\ &= 53.67 \text{ cm} \end{aligned}$$

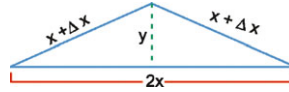


Fig. 10.6 Buckling of rail

10.11 Let $L_0(\text{S})$ and $L_0(\text{Cu})$ be the lengths of steel and copper rod at 0°C , respectively. Let the respective lengths be $L(\text{S})$ and $L(\text{Cu})$ at temperature $T^\circ\text{C}$. Then

$$L(S) = L_0(S)(1 + \alpha_s T) \quad (1)$$

$$L(\text{Cu}) = L_0(\text{Cu})(1 + \alpha_{\text{Cu}} T) \quad (2)$$

Subtracting (2) from (1)

$$L(S) - L(\text{Cu}) = L_0(S) - L_0(\text{Cu}) + [L_0(S)\alpha_s - L_0(\text{Cu})\alpha_{\text{Cu}}]T \quad (3)$$

Now in the RHS, $L_0(S) - L_0(\text{Cu})$ is constant. If $L(S) - L(\text{Cu})$ is to remain constant, then $[L_0(S)\alpha_s - L_0(\text{Cu})\alpha_{\text{Cu}}] = 0$, at any temperature T . This gives us

$$\frac{L_0(S)}{L_0(\text{Cu})} = \frac{\alpha_{\text{Cu}}}{\alpha_s} = \frac{1.7 \times 10^{-5}}{1.1 \times 10^{-5}} = \frac{17}{11} \quad (4)$$

$$\text{Furthermore, } L_0(S) - L_0(\text{Cu}) = 4 \text{ cm.} \quad (5)$$

Solving (4) and (5) we obtain

$$L_0(S) = 11.33 \text{ cm; } L_0(\text{Cu}) = 7.33 \text{ cm}$$

10.12 Let the volume of mercury in the flask be $V_0 \text{ cm}^3$ and that of glass flask 1000 cm^3 at initial temperature T_1 . At a higher temperature T_2 the volume of glass will be

$$V_g = 1000(1 + \gamma_g \Delta T) \quad (1)$$

where $\Delta T = T_2 - T_1$. The volume of mercury will be

$$V = V_0(1 + \gamma \Delta T) \quad (2)$$

The volume of air inside the flask at temperature T_2 will be

$$\begin{aligned} V_g - V &= 1000(1 + \gamma_g \Delta T) - V_0(1 + \gamma \Delta T) \\ &= 1000 - V_0 + (1000\gamma_g - V_0\gamma)\Delta T \end{aligned} \quad (3)$$

The RHS will be constant if

$$1000\gamma_g - V_0\gamma = 0 \quad (4)$$

for any value of ΔT . Therefore,

$$V_0 = 1000 \frac{\gamma_g}{\gamma} = \frac{1000 \times 27 \times 10^{-6}}{1.8 \times 10^{-4}} = 150 \text{ cm}^3$$

where we have used $\gamma_g = 3\alpha_g$.

$$\text{10.13 } Y = \frac{F/A}{\Delta L/L} \quad (1)$$

$$\text{or } F = \frac{Y A \Delta L}{L} = \frac{Y A \Delta L}{L_0(1 + \alpha \Delta T)} \simeq \frac{Y A \Delta L}{L_0} \quad (2)$$

$$(\because \alpha \Delta T \ll 1)$$

$$\text{But } \Delta L = \alpha L_0 \Delta T$$

$$\therefore F = Y A \alpha \Delta T = 2.1 \times 10^{10} \times 0.5 \times 10^{-6} \times 12 \times 10^{-6} \times 20 = 2.52 \text{ N}$$

where we have used SI units.

10.14 The coefficient of apparent expansion of a liquid

$$A = \gamma - g = \gamma - 3\alpha = 11 \times 10^{-4} - 3 \times 8 \times 10^{-6} = 1.076 \times 10^{-3}$$

$$\text{Apparent expansion} = \frac{\text{mass expelled}}{(\text{mass left}) (\text{temperature rise})}$$

$$A = \frac{W}{W_0 \Delta T} = \frac{50 - W_0}{W_0 \Delta T}$$

$$\therefore W_0 = \frac{50}{1 + A \Delta T} = \frac{50}{1 + 1.076 \times 10^{-3} \times 80} = 46 \text{ g}$$

Gas Laws

10.15 $PV = nRT$ (gas equation)

$$\begin{aligned} \therefore R &= \frac{PV}{nT} = \frac{(1.0129 \times 10^5) (22.4 \times 10^{-3})}{(1.0) (273)} \\ &= 8.31 \text{ J/mol/K} \end{aligned}$$

$$\text{10.16 } P_0 = 0.76 \times 13,600 \times 9.8 = 1.0129 \times 10^5 \text{ Pa}$$

$$\text{Pressure at depth 30 m, } P = 30 \times 1000 \times 9.8 = 2.94 \times 10^5 \text{ Pa.}$$

$$\therefore \text{Total pressure inside the bubble, } P_1 = P_0 + P$$

$$= (1.0129 + 2.94) \times 10^5 = 3.9529 \times 10^5 \text{ Pa}$$

$$P_2 V_2 = P_1 V_1 \quad (\text{Boyle's law})$$

$$\therefore P_2 (3V_1) = P_1 V_1$$

$$\therefore P_2 = P_1/3 = 1.3176 \times 10^5 \text{ Pa}$$

This corresponds to a water depth equivalent of $1.3176 \times 10^5 - 1.0129 \times 10^5 = 0.3047 \times 10^5 \text{ Pa}$.

$$\text{Therefore water depth} = \frac{30 \times 0.3047 \times 10^5}{2.94 \times 10^5} = 3.11 \text{ m}$$

$$10.17 \quad \frac{\rho_1 T_1}{P_1} = \frac{\rho_2 T_2}{P_2}$$

$$M_1 = V\rho_1, \quad M_2 = V\rho_2$$

$$\therefore \frac{M_1}{M_2} = \frac{\rho_1}{\rho_2} = \frac{P_1 T_2}{P_2 T_1}$$

$$\begin{aligned} \therefore M_2 &= \frac{P_2}{P_1} \cdot \frac{T_1}{T_2} M_1 = \frac{50}{76} \times \frac{273}{263} \times 175 \\ &= 119.5 \text{ kg} \end{aligned}$$

10.18 Let n moles be total mass of air in the two bulbs.

Initially, $T = 273 + 20 = 293 \text{ K}$, $P = 76 \text{ cm of Hg}$, $V = V_1 + V_2 = 100 + 500 = 600 \text{ cc}$.

$$n = \frac{PV}{RT} = \frac{70 \times 600}{293R}$$

Finally, let n_1 and n_2 moles be the mass of air in the small and large bulb, respectively. Under new conditions

$$n_1 = \frac{P_1 V_1}{RT_1} = \frac{100P_1}{293R}$$

$$n_2 = \frac{P_2 V_2}{RT_2} = \frac{500P_2}{293R} \quad (\because P_2 = P_1)$$

$$\text{But } n = n_1 + n_2 = \frac{100P_1}{293R} + \frac{500P_1}{373R} = \frac{70 \times 600}{293R}$$

Cancelling off R , we find $P_1 = 85.23 \text{ cm of Hg}$.

10.3.3 Heat Transfer

10.19 (a) In the first slab, heat flow is given by

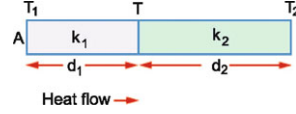
$$-\frac{dQ_1}{dt} = \frac{k_1 A (T_1 - T)}{d_1} \quad (1)$$

In the second slab, heat flow is given by

$$-\frac{dQ_2}{dt} = \frac{k_2 A (T - T_2)}{d_2} \quad (2)$$

Now the continuity of heat flow requires that heat flow must be the same in both the slabs (Fig. 10.7). Thus

Fig. 10.7 Heat flow in the composite slab made of two slabs in series



$$\frac{dQ_1}{dt} = \frac{dQ_2}{dt} = \frac{dQ}{dt} \quad (3)$$

Using (3) in (1) and (2)

$$T_1 - T = \frac{-d_1}{k_1 A} \frac{dQ}{dt} \quad (4)$$

$$T - T_2 = \frac{-d_2}{k_2 A} \frac{dQ}{dt} \quad (5)$$

Adding (4) and (5)

$$T_1 - T_2 = \frac{-1}{A} \left[\frac{d_1}{k_1} + \frac{d_2}{k_2} \right] \frac{dQ}{dt} \quad (6)$$

$$\text{or} \quad -\frac{dQ}{dt} = \frac{A(T_1 - T_2)}{\frac{d_1}{k_1} + \frac{d_2}{k_2}} \quad (7)$$

(b) Rewriting (7)

$$-\frac{dQ}{dt} = \frac{A(T_1 - T_2)}{d} \frac{d}{\frac{d_1}{k_1} + \frac{d_2}{k_2}} = \frac{A(T_1 - T_2)k}{d} \quad (8)$$

with $d = d_1 + d_2$, and the equivalent conductivity

$$k_{\text{eq}} = \frac{d_1 + d_2}{\frac{d_1}{k_1} + \frac{d_2}{k_2}} \quad (9)$$

Formula (9) can be generalized to any number of slabs in series.

$$k = \frac{\sum d_i}{\sum \frac{d_i}{k_i}} \quad (10)$$

(c) Eliminating dQ/dt between (4) and (5)

$$T = \frac{(k_1 T_1 / d_1) + (k_2 T_2 / d_2)}{(k_1 / d_1) + (k_2 / d_2)} \quad (11)$$

10.20 (a) The rate of flow of heat through the composite slab (Fig. 10.8) is given by

$$-\frac{dQ}{dt} = \frac{(T_1 - T_2)}{d} \sum_l^n k_l A_l \quad (1)$$

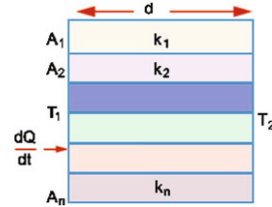
(b) Rewriting (1)

$$-\frac{dQ}{dt} = \frac{(T_1 - T_2)}{d} \left(\sum_1^n A_i \right) \left(\frac{\sum k_i A_i}{\sum A_i} \right) \quad (2)$$

The equivalent conductivity of the system is

$$k_{eq} = \frac{\sum k_i A_i}{\sum A_i} \quad (3)$$

Fig. 10.8 Heat flow in a composite slab made of n slabs in parallel



10.21 $T = \frac{(k_1 T_1/d_1) + (k_2 T_2/d_2)}{(k_1/d_1) + (k_2/d_2)}$

as $d_1 = d_2$

$$T = \frac{k_1 T_1 + k_2 T_2}{k_1 + k_2} = \frac{92 \times 100 + 16 \times 0}{92 + 16} = 85.18^\circ\text{C}$$

10.22 Heat transferred/second

$$\begin{aligned} -\frac{dQ}{dt} &= kA \frac{(\theta_2 - \theta_1)}{d} = k\pi r^2 \frac{(\theta_2 - \theta_1)}{d} \\ &= 90\pi (0.01)^2 \frac{(100 - 0)}{0.2} = 14.137 \text{ J/s} \end{aligned}$$

Heat required to melt 0.05 kg of ice

$$= 0.05 \times 8 \times 10^4 = 4000 \text{ cal} = 16720 \text{ J}$$

$$\text{Time required} = 16720/14.137 = 1183 \text{ s.}$$

10.23 Heat flow

$$dQ = A \frac{dT}{dx} k = A a T \frac{dT}{dx}$$

$$\text{or } A a T dT = dQ dx$$

$$\text{Integrating } A a \int_{T_2}^{T_1} T dT = \int_0^Q dQ \int_0^L dx$$

$$\therefore \frac{Aa}{2} (T_1^2 - T_2^2) = QL$$

$$\text{or } Q = \frac{aA}{2L} (T_1^2 - T_2^2)$$

$$\mathbf{10.24} \quad \frac{dQ}{dt} = -kA \frac{dT}{dr}$$

$$\therefore dT = -\frac{1}{k} \frac{dQ}{dt} \frac{dr}{4\pi r^2}$$

When steady state is reached, dQ/dt will be independent of r and is constant (Fig. 10.9). Integrating

$$\int_{T_1}^{T_2} dT = -\frac{1}{4\pi k} \frac{dQ}{dt} \int_{r_1}^{r_2} \frac{dr}{r^2} = -\frac{1}{4\pi k} \left(\frac{1}{r_2} - \frac{1}{r_1} \right) \frac{dQ}{dt}$$

$$\therefore T_2 - T_1 = \frac{1}{4\pi k} \frac{(r_1 - r_2)}{r_1 r_2} \frac{dQ}{dt}$$

$$\text{or } T_1 - T_2 = \frac{1}{4\pi k} \frac{(r_2 - r_1)}{r_1 r_2} \frac{dQ}{dt}$$

$$\therefore \frac{dQ}{dt} = 4\pi k \frac{r_1 r_2}{r_2 - r_1} (T_1 - T_2)$$

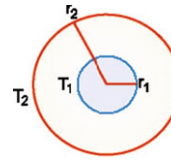


Fig. 10.9 Radial flow of heat through two concentric spheres

10.25 Rate of flow of heat

$$\frac{dQ}{dt} = -kA \frac{dT}{dr} \quad (1)$$

Neglecting the area of the faces, area of the cylinder $A = 2\pi rL$. For steady state, $dQ/dt = \text{constant}$. We can then write (1) as

$$\frac{dr}{r} = -\frac{2\pi Lk}{dQ/dt} dT$$

Integrating

$$\int_{r_1}^{r_2} \frac{dr}{r} = \ln(r_2/r_1) = -\frac{2\pi Lk}{dQ/dt} (T_2 - T_1)$$

$$\text{or } dQ/dt = \frac{2\pi Lk}{\ln(r_2/r_1)} (T_1 - T_2)$$

$$\mathbf{10.26} \quad dQ/dt = kA(T_1 - T_2)/d$$

$$= (0.59A) \times \frac{10}{0.01} = 590 \text{ A J/s} \quad (1)$$

Let x m/s ice be added at the bottom of the layer.
Mass of ice formed per second

$$M = \rho \times A \quad (2)$$

The required energy per second

$$E = \rho \times AL \quad (3)$$

Equating (1) and (3), $\rho \times AL = 590 \text{ A}$

$$\therefore x = \frac{590A}{\rho AL} = \frac{590}{917 \times 333 \times 10^3} = 1.932 \times 10^{-6} \text{ m/s} = 0.00695 \text{ m/h}$$

$$\mathbf{10.27} \quad -\frac{d\theta}{dt} = C(\theta - \theta_0) \quad (\text{Newton's law of cooling})$$

$$\frac{\theta_1 - \theta_2}{t} = C \left[\frac{\theta_1 + \theta_2}{2} - \theta_0 \right] \quad (C = \text{constant})$$

$$\frac{85 - 75}{2} = C \left[\frac{85 + 75}{2} - 30 \right]$$

$$\therefore C = 0.1$$

$$\frac{55 - 45}{t} = 0.1 \left[\frac{55 + 45}{2} - 30 \right]$$

$$\therefore t = 5 \text{ min}$$

$$10.28 \quad -\frac{d\theta}{dt} = \frac{K}{ms}(\theta - \theta_0) \quad (K = \text{constant}) \quad (1)$$

Let x be the water equivalent of the calorimeter.

$$\frac{50 - 40}{15} = \frac{K}{(x + 40 \times 1)} \left[\frac{50 + 40}{2} - \theta_0 \right] \quad (2)$$

$$\frac{50 - 40}{33} = \frac{K}{(x + 100 \times 1)} \left[\frac{50 + 40}{2} - \theta_0 \right] \quad (3)$$

Dividing (2) by (3)

$$\frac{x + 100}{x + 40} = \frac{11}{5}$$

or $x = 10 \text{ g}$

$$10.29 \quad -\frac{d\theta}{dt} = \frac{K}{ms}(\theta - \theta_0)$$

Now $K \propto \text{surface area}$ or $\propto r^2$ and $m \propto r^3$.

$$\therefore \quad d\theta/dt \propto 1/r$$

$$\therefore \quad \frac{(d\theta/dt)_1}{(d\theta/dt)_2} = \frac{r_2}{r_1} = \frac{2}{1}$$

10.30 Assuming a linear variation of resistance with temperature

$$R_T = R_0(1 + \alpha T) \quad (1)$$

$$29.6 = 24.9(1 + 100\alpha) \quad (2)$$

$$\text{whence} \quad \alpha = 1.8875 \times 10^{-3} \text{ W}/^\circ\text{C} \quad (3)$$

$$26.3 = 24.9(1 + 1.8875 \times 10^{-3} T) \quad (4)$$

whence $T = 29.79^\circ\text{C}$

10.31 If R is the radius of the sun, r the mean distance of the earth from the sun, E the energy emitted from 1 m^2 of the sun's surface per second, T the absolute temperature of the sun's surface and σ the Boltzmann–Stefan constant, then

$$S = \frac{4\pi R^2 E}{4\pi r^2} = \frac{R^2 \sigma T^4}{r^2}$$

$$= \frac{(6.95 \times 10^8)^2 (5.67 \times 10^{-8}) (5740)^4}{(1.49 \times 10^{11})^2} = 1339 \text{ W/m}^2$$

- 10.32** Radiant intensity at the sun's surface is the power emitted by 1 m^2 of sun's surface.

$$\sigma T^4 = 63 \times 10^6$$

$$T = \left(\frac{63 \times 10^6}{5.67 \times 10^{-8}} \right)^{1/4} = 5773 \text{ K}$$

- 10.33** $-\frac{dE}{dt} = \sigma A (T_1^4 - T_2^4)$ (Stefan-Boltzmann formula)
- $$= \sigma \pi r^2 (T_1^4 - T_2^4)$$
- $$= \pi (5.67 \times 10^{-8}) (0.05)^2 (500^2 - 300^2)$$
- $$= 7.12 \times 10^{-5} \text{ W}$$

- 10.34** $\lambda_m T = b = 3 \times 10^{-3} \text{ m K}$ (Wien's law) (1)

$$\frac{dE}{dt} = \sigma T^4 \quad \text{(Boltzmann law)} \quad (2)$$

If dE/dt goes down to 1/16 of its original value then by (2) the temperature $T \rightarrow T/2$. Therefore $\lambda_m \rightarrow 2\lambda_m$ by (1). Thus the wavelength under new conditions $\lambda'_m = 2\lambda_m = 2 \times 480 = 960 \text{ nm}$.

10.3.4 Specific Heat and Latent Heat

- 10.35** $Q = \int_{20}^{80} m C_p dT + mL + \int_{80}^{200} m C_p dT$
- $$= \int_{20}^{200} m C_p dT + mL \quad (\because C_p \text{ relation for solid and liquid phase is identical})$$
- $$= \int_{20}^{200} 1 \times (30.6 + 0.0103 T) dT + 1 \times 6000$$
- $$= 11710 \text{ J}$$

- 10.36** $\bar{C} = \frac{\int C dT}{\int dT} = \frac{\int_0^T (A + BT^2) dT}{T} = \frac{1}{T} \left(AT + \frac{BT^3}{3} \right) = A + \frac{BT^2}{3}$
- $$C(\text{midpoint}) = A + B(T/2)^2 = A + \frac{BT^2}{4}$$
- $$\therefore \bar{C} - C(\text{midpoint}) = A + \frac{BT^2}{3} - \left(A + \frac{BT^2}{4} \right) = \frac{BT^2}{12}$$

- 10.37** Let the specific heats of liquids A, B and C be, respectively, C_A , C_B and C_C . When A and B are mixed, equilibrium of the mixture requires that

$$MC_A(16 - 12) = MC_B(18 - 16)$$

$$\text{or } C_B = 2C_A$$

When B and C are mixed

$$MC_B(23 - 18) = MC_C(28 - 23)$$

$$\text{or } C_C = C_B = 2C_A$$

When A and C are mixed, let the equilibrium temperature be T .

$$MC_A(T - 12) = MC_C(28 - T) = 2MC_C(28 - T)$$

$$\therefore T = 22.67^\circ\text{C}$$

- 10.38 (a)** The block is fixed. The kinetic energy of the bullet is entirely converted into heat energy. Let m be the mass and v the velocity of the bullet.

$$Q = \frac{1}{2}mv^2 = \frac{1}{2}(3 \times 10^{-3})(120)^2 = 21.6 \text{ J} = 5.167 \text{ cal}$$

Rise in temperature

$$\Delta T = \frac{Q}{mc} = \frac{5.167}{3 \times 0.031} = 55.56^\circ\text{C}$$

- (b)** The block is free to move. In this case, after the collision some kinetic energy will go into the block + bullet system.

$$\frac{1}{2}mv^2 = \frac{1}{2}(M + m)v_1^2 + Q \quad (\text{energy conservation}) \quad (1)$$

$$mv = (M + m)v_1 \quad (\text{momentum conservation}) \quad (2)$$

where M is the mass of the block and v_1 the final velocity of the block + bullet system. Eliminating v_1 and simplifying

$$Q = \frac{1}{2}mv^2 \left(\frac{M}{M + m} \right) = \frac{1}{2} \times 3 \times 10^{-3} \times (120)^2 \left(\frac{50}{50 + 3} \right)$$

$$= 20.38 \text{ J} = 4.875 \text{ cal}$$

$$\Delta T = \frac{Q}{mc} = \frac{4.875}{3 \times 0.031} = 52.42^\circ\text{C}$$

- 10.39** Potential energy available from m kg of water through a fall of h metres is mgh J. $15/100 mgh$. Mechanical energy is converted into heat.

$$\therefore \frac{15}{100} mgh = mc\Delta T \times 4.18 = m \times 1000\Delta T \times 4.18$$

$$\therefore \Delta T = \frac{15 \times 9.8 \times 25}{100 \times 1000 \times 4.18} = 0.0088^\circ\text{C}$$

10.40 Mechanical energy available, $W = mgh$

Heat absorbed, $H = mc\Delta T$ J.

Loss of mechanical energy = gain of heat energy

$$\therefore mgh = mc\Delta T \text{ J}$$

$$\therefore \Delta T = \frac{gh}{cJ} = \frac{9.8 \times 100}{30.6 \times 4.18} = 7.66^\circ\text{C} = 7.66 \text{ K}$$

10.3.5 Thermodynamics

- 10.41** (a) (i) In an isobaric process pressure remains constant.
(ii) In an isochoric process volume remains constant.
(iii) In an adiabatic process heat is neither absorbed nor evolved by the system.
(iv) In an isothermal process temperature remains constant.
(b) For adiabatic process use the relation

$$PV^\gamma = P_1 V_1^\gamma = \text{const.} \quad (1)$$

$$\text{or } P = \frac{P_1 V_1^\gamma}{V^\gamma} \quad (2)$$

Work done on the gas

$$\begin{aligned} W &= - \int_{v_1}^{v_2} P dV = - \int_{v_1}^{v_2} P_1 V_1^\gamma \frac{dV}{V^\gamma} = -P_1 V_1^\gamma \int_{v_1}^{v_2} \frac{dV}{V^\gamma} \\ \frac{P_1 V_1^\gamma}{\gamma - 1} (V_2^{1-\gamma} - V_1^{1-\gamma}) &= \frac{1}{\gamma - 1} (P_1 V_1^\gamma V_2^{1-\gamma} - P_1 V_1) \\ \therefore W &= \frac{1}{\gamma - 1} (P_2 V_2 - P_1 V_1) \end{aligned} \quad (3)$$

where we have used the relation $P_1 V_1^\gamma = P_2 V_2^\gamma$.

W is positive if $V_1 > V_2$ (compression) and negative if $V_1 < V_2$ (expansion).

10.42 Applying the gas equation

$$T_A = \frac{P_A V_A}{nR} = \frac{(1.013 \times 10^5)(44.8)}{(2000)(8.31)} = 273 \text{ K}$$

As the process AB is isometric (isochoric), $V_B = V_A$.

$$T_B = \frac{T_A P_B}{P_A} = \frac{273 \times 2}{1} = 546 \text{ K}$$

As the process CA is isobaric, Charles' first law applies. Thus

$$\frac{V_C}{T_C} = \frac{V_A}{T_A} \rightarrow V_C = \frac{V_A T_C}{T_A}$$

As BC is an isothermal process, $T_C = T_B = 546 \text{ K}$.

$$\therefore V_C = \frac{V_A T_B}{T_A} = \frac{(44.8)(546)}{273} = 89.6 \text{ m}^3$$

10.43 For adiabatic process, $dQ = 0$ so that $dU = -dW$. The energy of 1 mol of monatomic gas is given by

$$U = \frac{3}{2} RT$$

$$\therefore dU = \frac{3}{2} R dT$$

$$dW = PdV = \frac{RT}{V} dV$$

$$\therefore \frac{3}{2} R dT = -\frac{RT}{V} dV$$

$$\therefore \frac{dT}{T} + \frac{2}{3} \frac{dV}{V} = 0$$

Integrating

$$\ln T + \frac{2}{3} \ln V = \text{constant}$$

$$\text{or } TV^{2/3} = \text{constant}$$

Eliminating T from the gas equation

$$PV^{5/3} = \text{constant}$$

10.44 (a) $U_f - U_i = \Delta U = Q - W$ (First law of thermodynamics)

Let a system change from an initial equilibrium state i to a final equilibrium state f in a definite way, the heat absorbed by the system being Q and the work done by the system being W . The quantity $Q - W$ represents the change in internal energy of the system.

Both Q and W are path dependent while ΔU is path independent.

$$(b) \quad W = \int dW = - \int P dV \quad (1)$$

$$PV = nRT_1 = \text{constant}$$

$$\text{or} \quad P = \frac{nRT_1}{V} \quad (T = \text{constant for isothermal process}) \quad (2)$$

Substituting (2) in (1)

$$W = -nRT \int_{V_1}^{V_2} \frac{dV}{V} = -nRT \ln \left(\frac{V_2}{V_1} \right)$$

(c) At the beginning of the cycle, $P_1 = 1.01 \times 10^5 \text{ Pa}$, $V_1 = 1 \text{ m}^3$, $T_1 = 273.15 \text{ K}$.

At the end of stage 1, $P_2 = 5.05 \times 10^4 \text{ Pa}$, $V_2 = 2 \text{ m}^3$, $T_2 = 273.15 \text{ K}$.

At the end of stage 2, $P_3 = 1.01 \times 10^5 \text{ Pa}$, $V_3 = 2 \text{ m}^3$, $T_3 = T_2 P_3/P_2 = 546.3 \text{ K}$.

$$(i) \quad n = \frac{P_1 V_1}{RT_1} = \frac{1.01 \times 10^5 \times 1.0}{8.31 \times 273.15} = 44.5 \text{ mol}$$

$$(ii) \quad \text{Stage 1, } W_1 = -nRT \ln (V_2/V_1) \\ = -44.5 \times 8.31 \times 273.15 \ln(2/1) = -70046 \text{ J}$$

$$\text{Stage 2, } W_2 = P_2 \Delta V = P_3(V_3 - V_2) = P(V_2 - V_2) = 0$$

$$\text{Stage 3, } W_3 = P_3(V_1 - V_3) = 1.01 \times 10^5(1 - 2) \\ = -1.015 \times 10^5 \text{ J}$$

(iii) $\Delta U = 0$, overall.

$$\mathbf{10.45} \quad (a) \text{ and } (b) \quad \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\therefore \quad T_1 = \frac{P_1}{P_2} \frac{V_1}{V_2} T_2 = \frac{P_1}{3P_1} \frac{V_1}{V_1} 1083 = 361 \text{ K}$$

$$n = \frac{P_1 V_1}{RT_1} = \frac{(10^5)(0.06)}{(8.31)(361)} = 2 \text{ mol}$$

$$\frac{P_3 V_3}{T_3} = \frac{P_2 V_2}{T_2}$$

$$\therefore \quad T_3 = \frac{P_3}{P_2} \frac{V_3}{V_2} T_2 = \frac{P_2}{P_2} \frac{(V_2/3)}{V_2} 1083 = 361 \text{ K}$$

$$T_4 = T_3 = 361 \text{ K} \quad (\because \text{process } 3 \rightarrow 4 \text{ isothermal process})$$

$$\frac{P_4 V_4}{T_4} = \frac{P_3 V_3}{T_3}$$

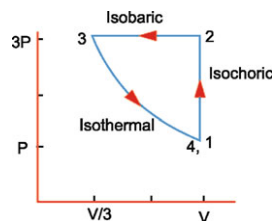
$$\therefore P_4 = \frac{V_3}{V_4} \frac{T_4}{T_3} P_3 = \frac{V_3}{3V_3} \frac{T_3}{T_3} P_2 = \frac{P_2}{3} = \frac{3P_1}{3} = P_1 = 10^5 \text{ N/m}^2$$

$$V_4 = \frac{nRT_4}{P_4} = \frac{2 \times 8.31 \times 361}{10^5} = 0.06 \text{ m}^3$$

Thus the P , V , T coordinates of the initial and final points on the indicator diagram are identical.

(c) Heat gained by N_2 in the first (isochoric) process (Fig. 10.10):

Fig. 10.10 The P - V diagram



$$C_V = C_P/\gamma = 29.12/1.4 = 20.8 \text{ J/(mol K)}$$

$$Q_1 = nC_V \Delta T = 2 \times 20.8 \times (1083 - 311) = 32115 \text{ J}$$

$$W_{12} = 0 \quad (\because \text{process } 1 \rightarrow 2 \text{ is isochoric})$$

$$\therefore \Delta U_1 = Q_1 = 32115 \text{ J}$$

$$Q_2 = nC_P \Delta T = 2 \times 29.12 \times (361 - 1083) = -42049 \text{ J}$$

$$W_{23} = P \Delta V = P_2(V_3 - V_2) = -3 \times 10^5 \times \frac{2}{3} \times 0.06 = -12,000 \text{ J}$$

$$\therefore \Delta U_2 = Q_2 - W_{23} = -42049 + 12000 = -30049 \text{ J}$$

$$\Delta U_3 = 0 \quad (\because \text{process } 3 \rightarrow 4 \text{ is isothermal})$$

(d) Net change in energy

$$\Delta U = \Delta U_1 + \Delta U_2 + \Delta U_3 = 32115 - 30049 + 0 = 2066 \text{ J}$$

The expected value is zero.

10.46 Number of degrees of freedom, $f = \frac{2}{\gamma - 1}$ we can find γ from the relation

$$T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$$

$$\therefore \left(\frac{V_2}{V_1} \right)^{\gamma-1} = \frac{T_1}{T_2} = 1.32$$

$$\therefore 2^{\gamma-1} = 1.32$$

$$\text{whence } \gamma = 1 + \frac{\log 1.32}{\log 2.0} = 1.4$$

$$\therefore f = \frac{2}{1.4 - 1} = 5$$

10.47 The efficiency of the engine is

$$e = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{400} = 0.25 \quad \text{or} \quad 25\%$$

Work done by the engine

$$\begin{aligned} W &= e \times Q_1 = 0.25 \times 10^8 \text{ cal} \\ &= 0.25 \times 10^8 \times 4.18 \text{ J} \\ &= 1.05 \times 10^8 \text{ J} \end{aligned}$$

$$\mathbf{10.48} \quad e = 1 - \frac{T_2}{T_1} = \frac{1}{6} \quad (1)$$

$$\therefore 5T_1 - 6T_2 = 0 \quad (2)$$

When the temperature of the sink is reduced by 62°C , the efficiency becomes

$$e' = 2e \quad (3)$$

$$e' = 2e = 1 - \frac{(T_2 - 62)}{T_1} = \frac{1}{3} \quad (4)$$

$$\therefore 2T_1 - 3T_2 - 186 = 0 \quad (5)$$

Solving (2) and (5), $T_1 = 372 \text{ K} = 99^\circ\text{C}$, $T_2 = 310 \text{ K} = 37^\circ\text{C}$.

$$\mathbf{10.49} \quad (\mathbf{a}) \quad PV = P_0V_0 \quad (\text{isothermal conditions}) \quad (1)$$

Dividing by m , the mass of air

$$\frac{PV}{m} = \frac{P_0V_0}{m}$$

$$\text{or} \quad \frac{P}{\rho} = \frac{P_0}{\rho_0}$$

$$\therefore \rho = \frac{P\rho_0}{P_0} \quad (2)$$

where the density of air on earth's surface is ρ_0 and pressure is P_0 , the corresponding quantities at height y being ρ and P .

Now, for a small increase in height dy , the pressure decreases by dp and is given by

$$dp = -\rho g dy = -\frac{P}{P_0} \rho_0 g dy \quad (3)$$

where we have used (2). The negative sign shows that the pressure decreases as the height increases.

Integrating

$$\begin{aligned} \int_0^h dy &= -\frac{P_0}{\rho_0 g} \int_{P_0}^P \frac{dp}{P} \\ \therefore h &= -\frac{P_0}{\rho_0 g} \ln \left(\frac{P}{P_0} \right) \\ \therefore P &= P_0 \exp \left(-\frac{\rho_0 g h}{P_0} \right) \end{aligned} \quad (4)$$

$$\text{Now, } P_0 V_0 = \mu RT \quad (5)$$

since the temperature is assumed to be constant and μ is in moles. Furthermore

$$\rho_0 = \frac{\mu M}{V_0} \quad (6)$$

where M is the molecular weight. Combining (5) and (6)

$$\frac{\rho_0}{P_0} = \frac{M}{RT} \quad (7)$$

Substituting (7) in (4)

$$P = P_0 \exp \left(-\frac{Mgh}{RT} \right) \quad (8)$$

(b) If n is the number density, that is, the number of molecules per unit volume and m_0 the mass of each molecule then

$$\begin{aligned} \rho &= m_0 n \\ \text{and } \rho_0 &= m_0 n_0 \\ \therefore \frac{\rho}{\rho_0} &= \frac{n}{n_0} \end{aligned} \quad (9)$$

Combining (9) with (2)

$$\frac{p}{p_0} = \frac{n}{n_0} \quad (10)$$

Using (10) in (8)

$$n = n_0 \exp\left(-\frac{Mgh}{RT}\right) \quad (11)$$

$$(c) \quad \frac{p}{p_0} = \frac{1}{2} = \exp\left(-\frac{Mgh}{RT}\right)$$

$$\therefore h = \frac{RT}{Mg} \ln 2 = \frac{8.31 \times 273 \times 0.693}{0.029 \times 9.8} = 54224 \text{ m} \simeq 54 \text{ km}$$

10.50 (a) For a Carnot cycle

$$(i) \quad e = \frac{Q_H - Q_C}{Q_H}$$

$$(ii) \quad e = \frac{T_H - T_C}{T_H}$$

The symbols H and C are for hot and cold reservoirs.

(b) The Otto cycle, Fig. 10.11, consists of two reversible adiabatic processes (paths AB and CD) and two reversible isochoric processes (path DA and BC).

Suppose we start at the point C. The temperature T_C at C is low, slightly above atmospheric temperature. The cylinder is filled with air charged with the combustible gas or vapour. The air is compressed adiabatically to the point D. At D a spark causes combustion, heating the air at constant volume to the point A. The heated air expands adiabatically along the path AB. At B, a valve is opened and the pressure drops to that of the atmosphere. The point C is reached at constant volume. The cycle is complete.

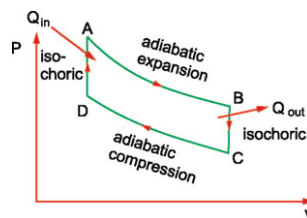


Fig. 10.11 The Otto cycle

During the isochoric heating and cooling no work can be done by or on the gas:

$$\Delta Q = \Delta U = \int_1^2 dU = C_v \int_{T_1}^{T_2} dT = C_v(T_2 - T_1) \quad (1)$$

so that

$$Q_{\text{out}} = C_v(T_B - T_C), \quad Q_{\text{in}} = C_v(T_A - T_D) \quad (2)$$

where the heats (positive) are those which are given out and put into the system, respectively. The thermodynamic efficiency

$$e = \frac{\text{work done by the gas}}{\text{heat put into the system}} = \frac{W}{Q_{\text{in}}} \quad (3)$$

Since the internal energy does not change over the entire cycle, by first law of thermodynamics, net heat added to the system equals the work done by the system, so that

$$W = Q_{\text{in}} - Q_{\text{out}} \quad (4)$$

$$e = \frac{W}{Q_{\text{in}}} = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} = \left(1 - \frac{T_B - T_C}{T_A - T_D}\right) \quad (5)$$

In an adiabatic expansion or compression

$$TV^{\gamma-1} = \text{constant} \quad (6)$$

$$\therefore T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1}, \quad T_D V_D^{\gamma-1} = T_C V_C^{\gamma-1} \quad (7)$$

$$\text{or } T_A = T_B \left(\frac{V_B}{V_A}\right)^{\gamma-1}, \quad T_D = T_C \left(\frac{V_C}{V_D}\right)^{\gamma-1} \quad (8)$$

From Fig. 10.11 we note that

$$\frac{V_B}{V_A} = \frac{V_C}{V_D} = r \quad (\text{compression ratio}) \quad (9)$$

Using (8) and (9) in (5)

$$e = (1 - r^{1-\gamma}) \quad (10)$$

Thus the higher the compression ratio the greater is the efficiency.

(c)

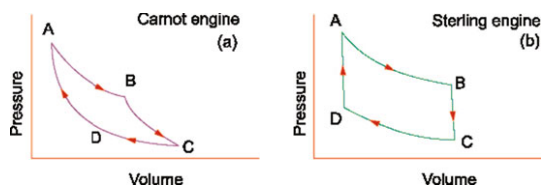


Fig. 10.12 (a) Carnot cycle consisting of two isothermal processes AB and CD; two adiabatic processes BC and DA. (b) Sterling cycle consisting of two isothermal process AB and CD, two isochoric processes BC and DA

$$\mathbf{10.51} \quad (\mathbf{a}) \quad n = \frac{PV}{RT} = \frac{1.013 \times 10^5 \times 1 \times 10^{-3}}{8.31 \times 273} = 0.04465 \text{ mol}$$

$$(i) \quad dQ = nC_p dT$$

$$\begin{aligned} \Delta S &= \int_{T_i}^{T_f} \frac{dQ}{T} = nC_p \int_{T_i}^{T_f} \frac{dT}{T} = nC_p \ln \left(\frac{T_f}{T_i} \right) \\ &= 0.04465 \times 21 \times \ln(500/273) = 0.567 \text{ J/K} \end{aligned}$$

$$(ii) \quad dQ = nC_v dT$$

$$\begin{aligned} \Delta S &= \int_{T_i}^{T_f} \frac{dQ}{T} = nC_v \int_{T_i}^{T_f} \frac{dT}{T} = nC_v \ln \left(\frac{T_f}{T_i} \right) \\ &= 0.04465 \times 12.7 \times \ln(500/273) = 0.343 \text{ J/K} \end{aligned}$$

$$(\mathbf{b}) \quad dQ_1 = mC_v dT$$

$$\begin{aligned} \Delta S_1 &= m C_v \int_{T_i}^{T_f} \frac{dT}{T} = mC_v \ln \left(\frac{T_f}{T_i} \right) \\ &= 1.0 \times 4.13 \times 10^3 \times \ln(313/273) = 695 \text{ J/K} \end{aligned}$$

When 0.5 kg water at 0°C is mixed with 0.5 kg water at 100°C, the final temperature would be 50°C.

$$\begin{aligned} \Delta S_2 &= 0.5 \times 4.13 \times 10^3 \times \ln(313/273) \\ &\quad + 0.5 \times 4.13 \times 10^3 \times \ln(313/373) \\ &= 282.35 - 362.15 = -79.8 \text{ J/K} \end{aligned}$$

- 10.52** (a) The internal energy of an ideal gas is given by $U = nC_V T$. In isothermal expansion where the temperature and the amount of gas remain constant, the internal energy does not change. Thus $\Delta U = 0$.
 (b) The work done is

$$\begin{aligned} W &= - \int_{V_1}^{V_2} P dV = - \int_{V_1}^{V_2} \left(\frac{nRT}{V} \right) dV \\ &= -nRT \int_{V_1}^{V_2} \frac{dV}{V} = -nRT \ln(V_2/V_1). \end{aligned}$$

- (c) Using the first law of thermodynamics, $\Delta Q = \Delta U + W$, and putting $\Delta U = 0$, we have

$$\Delta Q = W = nRT \ln(V_2/V_1)$$

- (d) The change in entropy is

$$\Delta S = \int ds = \int \frac{dQ}{T} = \frac{1}{T} \int dQ$$

because the temperature T does not change. Thus

$$\Delta S = nR \ln(V_2/V_1)$$

- (e) By assumption the temperature of the reservoir does not change and because it loses heat ΔQ to the gas, the entropy change of the reservoir will be

$$\Delta S_{\text{res}} = -\Delta S = -nR \ln(V_2/V_1)$$

Therefore the entropy change of the system plus the reservoir equals zero, which is the definition of a reversible process.

- 10.53** (a) The internal energy U of a system tends to increase if energy is added as heat Q and tends to decrease if energy is lost as work W done by the system.
 Heat is energy that is transferred from one body to another due to difference in temperature of the bodies.
 Enthalpy (H) is the total heat and is defined by

$$H = U + PV$$

Work (W) is energy that is transferred from one body to another body due to a force that acts between them.

The function, $G = U + PV - TS$, is known as Gibb's function or Gibb's energy.

Relations:

(i) *Enthalpy*

$$H = U + PV \quad (1)$$

$$dH = dU + PdV + VdP = Tds + Vdp \quad (2)$$

$$\therefore Tds = dU + PdV \quad (3)$$

It follows that enthalpy is a function of entropy (S) and pressure (P).

$$\therefore H = f(S, P)$$

$$dH = \left(\frac{\partial H}{\partial S} \right)_P dS + \left(\frac{\partial H}{\partial P} \right)_S dP \quad (4)$$

Comparing (4) with (2)

$$\left(\frac{\partial H}{\partial S} \right)_P = T \text{ and } \left(\frac{\partial H}{\partial P} \right)_S = V \quad (5)$$

(ii) *Gibb's function:*

$$G = U + PV - TS \quad (6)$$

$$dG = dU + PdV + VdP - TdS - SdT$$

$$\text{But } TdS = dU + PdV$$

$$\therefore dG = VdP - SdT \quad (7)$$

Thus G is a function of two independent variables P and T .

$$G = f(P, T)$$

$$dG = \left(\frac{\partial G}{\partial P} \right)_T dP + \left(\frac{\partial G}{\partial T} \right)_P dT \quad (8)$$

Comparing (8) with (7)

$$\left(\frac{\partial G}{\partial P} \right)_T = V \quad (9)$$

$$\left(\frac{\partial G}{\partial T} \right)_P = -S \quad (10)$$

(iii) *Internal energy:*

$$dQ = T dS \quad (11)$$

$$dU = dQ - dW \quad (12)$$

$$dW = P dV \quad (\text{isobaric process}) \quad (13)$$

$$\therefore dU = dQ - P dV \quad (14)$$

$$dU = T dS - P dV \quad (15)$$

$$\therefore \left(\frac{\partial U}{\partial S} \right)_V = T \quad (16)$$

$$\therefore \left(\frac{\partial U}{\partial V} \right)_S = -P \quad (17)$$

- (b) The quantities U , T , S , P and V are functions of the condition or state of the body only, in other words, all the differentials are perfect differentials and are state variables. Since the differentials which occur in (15) are perfect differentials, they are valid for all changes whatever their nature. On the other hand, dQ is not a perfect differential, but represents only an infinitesimal quantity of heat, and for a cycle $\oint dQ$ is not zero, but is equal to the work done. Similarly, dW is also not a perfect differential.

Note that the internal energy, the entropy and the volume are all proportional to the mass of the substance under consideration, while the temperature and the pressure are independent of it.

The condition of a given mass of a body (say 1 mol) can be defined by U , T , S , P , V or combinations of them, of which only two are independent. It follows that enthalpy and Gibb's function are also acceptable as state functions, apart from the internal energy but not the heat or work.

10.3.6 Elasticity

$$10.54 \quad (a) \quad \eta = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/y} = \frac{100 \times 10^6 / 1^2}{0.1/10} = 10^{10} \text{ Pa}$$

$$(b) \quad K = \frac{\Delta P}{(-\Delta V/V)} = \frac{100 \times 10^6}{1/100} = 10^{10} \text{ Pa}$$

$$10.55 \quad (i) \quad \text{Stress} = \frac{\text{force}}{\text{area}} = \frac{mg}{A} = \frac{20 \times 9.8}{20 \times 10^{-6}} = 9.8 \times 10^6 \text{ Pa}$$

$$(ii) \quad \text{Strain} = \frac{\text{elongation}}{\text{original length}} = \frac{2.5 \times 10^{-2}}{10} = 2.5 \times 10^{-3}$$

$$(iii) \quad \text{Young's modulus} = \frac{\text{stress}}{\text{strain}} = \frac{9.8 \times 10^6}{2.5 \times 10^{-3}} = 3.92 \times 10^9 \text{ Pa}$$

10.56 For a perfect gas of 1 mol

$$PV = RT \quad (1)$$

Under isothermal conditions $T = \text{constant}$. Differentiating (1)

$$P dV + V dP = 0 \quad (2)$$

The bulk modulus for the isothermal process

$$K_T = -V \left(\frac{dP}{dV} \right)_T = P \quad (3)$$

For adiabatic compression in which heat of compression remains in the gas

$$PV^\gamma = \text{constant} \quad (4)$$

where $\gamma = C_p/C_v$ is the ratio of specific heats at constant pressure and constant volume. Differentiating (4)

$$\gamma P V^{\gamma-1} dV + V^\gamma dP = 0 \quad (5)$$

Thus adiabatic elasticity K_H is given by

$$K_H = -V \left(\frac{dP}{dV} \right)_H = \gamma P \quad (6)$$

It follows that

$$K_H = \gamma K_T \quad (7)$$

The adiabatic elasticity is greater than the isothermal elasticity by a factor γ which is always greater than unity.

10.57 $Y = 2\eta(1 + \sigma)$

$$\therefore \sigma = \frac{Y}{2\eta} - 1 = \frac{1}{2} \times 2.5 - 1 = 0.25$$

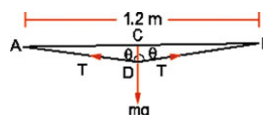
10.58 From Fig. 10.13 the new length $L' = 2AD = 2\sqrt{AC^2 + CD^2}$

$$= 2\sqrt{(0.6)^2 + (0.02)^2} = 1.200666 \text{ m}$$

Elongation of the wire, $\Delta L = L' - L = 1.200666 - 1.20 = 0.000666 \text{ m}$.

$$\text{Strain} = \Delta L/L = 0.000666/1.2 = 5.55 \times 10^{-4}$$

Fig. 10.13 Load fixed to the midpoint of a horizontal wire



For equilibrium

$$2T \cos \theta = mg$$

$$F = T = \frac{mg}{2 \cos \theta} = \frac{29 \times 10^{-3} \times 9.8}{2 \times (0.02/60)} = 426.3 \text{ N}$$

$$\text{Stress} = \frac{F}{A} = \frac{426.3}{\pi (0.05 \times 10^{-3})^2} = 5.43 \times 10^{10} \text{ Pa}$$

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{5.43 \times 10^{10}}{5.55 \times 10^{-4}} = 9.78 \times 10^{13} \text{ Pa}$$

10.59 Elastic energy $E = \frac{1}{2} Y (\text{strain})^2 (\text{volume})$

$$E = \frac{1}{2} \times 6 \times 10^8 \left(\frac{0.05}{0.20} \right)^2 (2 \times 10^{-6} \times 0.25) = 9.375 \text{ J}$$

The elastic energy is converted into kinetic energy.

$$E = \frac{1}{2} mv^2$$

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 9.375}{15 \times 10^{-3}}} = 35.3 \text{ m/s}$$

10.60 $F = m\omega^2 r$

$$\text{Breaking stress} = \frac{F}{A} = \frac{m\omega^2 r}{A} = 4.8 \times 10^7$$

$$\omega = \sqrt{\frac{4.8 \times 10^7 A}{mr}} = \sqrt{\frac{4.8 \times 10^7 \times 10^{-6}}{10 \times 0.3}} = 4 \text{ rad/s}$$

10.61 Stretching force = weight of the wire = (volume) (density) $\times g$

$$F = LA\rho g$$

where L is the length of wire, ρ the density, A the area of cross-section and g the acceleration due to gravity.

Breaking stress = maximum stretching force/area

$$\begin{aligned}
 &= \frac{LA\rho g}{A} = L\rho g \\
 7.8 \times 10^8 &= L \times 7800 \times 9.8 \\
 \therefore L &= 1.021 \times 10^4 \text{ m} = 10.2 \text{ km}
 \end{aligned}$$

Note that the result is independent of cross-sectional area of the wire.

10.3.7 Surface Tension

$$10.62 \quad S = \frac{\left(h + \frac{r}{3}\right) r \rho g}{2 \cos \theta}$$

Assuming that the contact angle $\theta = 0$

$$\begin{aligned}
 h &= \frac{2s}{r \rho g} - \frac{r}{3} = \frac{2 \times 0.072}{10^{-3} \times 10^3 \times 9.8} - \frac{10^{-3}}{3} \\
 &= 0.01436 \text{ m} \\
 &= 1.436 \text{ cm}
 \end{aligned}$$

10.63 Pressure due to water column of depth h is

$$P = hg\rho$$

Total pressure of the bubble, ignoring surface tension,

$$\begin{aligned}
 P' &= P + P_0 = hg\rho + P_0 \\
 &= 100 \times 9.8 \times 1000 + 1.01 \times 10^5 \\
 &= 10.81 \times 10^5 \\
 P'V &= nRT \\
 \therefore V &= \frac{nRT}{P'} = \frac{1 \times 8.314 \times 293}{10.81 \times 10^5} = 2.25 \times 10^{-3} \text{ m}^3
 \end{aligned}$$

10.64 As the drops are incompressible, the volume is constant.

$$\begin{aligned}
 n \frac{4}{3} \pi r^3 &= \frac{4\pi}{3} R^3 \\
 \therefore R &= rn^{1/3}
 \end{aligned}$$

$$\begin{aligned}
 \text{Decrease in surface area} &= 4\pi r^2 n - 4\pi R^2 = 4\pi r^2 (n - n^{2/3}) \\
 \text{Energy released} &= (\text{decrease in surface area}) (\text{surface tension})
 \end{aligned}$$

$$\Delta W = 4\pi r^2 \left(n - n^{2/3} \right) s$$

$$\text{Also } \Delta W = 4\pi R^2 S \left(n^{1/3} - 1 \right)$$

Then there will be a rise in temperature as energy is converted into heat. Energy conservation gives

$$mc \Delta\theta = 4\pi R^2 s \left(n^{1/3} - 1 \right)$$

$$\frac{4\pi}{3} R^3 c \rho \Delta\theta = 4\pi s R^3 \left[\frac{1}{r} - \frac{1}{R} \right]$$

$$\therefore \Delta\theta = \frac{3s}{\rho c} \left[\frac{1}{r} - \frac{1}{R} \right]$$

$$\text{10.65 } W = 2 \times 4\pi \left(r_2^2 - r_1^2 \right) S$$

The factor of 2 arises as there are two surfaces.

$$\begin{aligned} W &= 8\pi \left((0.05)^2 - (0.01)^2 \right) \times 0.03 \\ &= 0.0018 \text{ J} \end{aligned}$$

10.66 The excess pressure must be equal to the pressure due to the water column of depth h before the water leaks into the vessel.

$$\frac{2S}{r} = \rho gh$$

$$\therefore r = \frac{2s}{\rho gh} = \frac{2 \times 0.073}{1000 \times 9.8 \times 0.45} = 0.033 \times 10^{-3} \text{ m} = 0.033 \text{ mm}$$

10.67 Balancing the excess pressure in the bubble with the pressure due to a water column of depth h

$$\frac{2S}{r} = \rho gh$$

$$\therefore h = \frac{2s}{r\rho g} = \frac{2 \times 0.072}{0.3 \times 10^{-3} \times 1000 \times 9.8} = 0.049 \text{ m} = 4.9 \text{ cm}$$

$$\text{10.68 } h = \frac{2s}{r\rho g} = \frac{2 \times 0.073}{0.2 \times 10^{-3} \times 1000 \times 9.8} = 0.07449 \text{ m} = 7.45 \text{ cm}$$

The tube is inadequate as it is only 6 cm long. Water will not overflow. But the radius of meniscus r_1 would now increase such that the following condition is satisfied:

$h_1 r_1 = hr$, where r_1 is the radius of the meniscus.

$$r_1 = \frac{hr}{h_1} = \frac{7.45 \times 0.2}{6.0} = 0.248 \text{ mm}$$

- 10.69** When a bubble is charged, the charges stick to the bubble's surface and due to mutual repulsion tend to expand the surface while the surface tension tends to decrease the surface. An equilibrium is reached with a smaller excess of pressure.

Pressure due to electric charge is

$$P = \frac{\sigma^2}{2\epsilon_0}$$

where σ is the charge density and ϵ_0 is the permittivity.

If the excess pressure due to surface tension is neutralized by the electric charges then

$$\frac{\sigma^2}{2\epsilon_0} = \frac{4S}{r} \quad \text{or} \quad \sigma = \sqrt{\frac{8\epsilon_0 S}{r}}$$

The charge

$$\begin{aligned} q &= 4\pi r^2 \sigma = 8\pi \sqrt{2\epsilon_0 S r^3} \\ &= 8\pi \sqrt{2 \times 8.85 \times 10^{-12} \times 0.03 \times (0.02)^3} = 2.06 \times 10^{-9} \text{ C} \end{aligned}$$

- 10.70** Under isothermal conditions

$$P_1 V_1 + P_2 V_2 = PV \quad (1)$$

$$P_1 = \frac{4S}{r_1}, P_2 = \frac{4S}{r_2}, P = \frac{4S}{r} \quad (2)$$

$$V_1 = \frac{4\pi}{3} r_1^3, V_2 = \frac{4\pi}{3} r_2^3, V = \frac{4\pi}{3} r^3 \quad (3)$$

Using (2) and (3) in (1) and simplifying we get

$$r = \sqrt{r_1^2 + r_2^2} \quad (4)$$