11.2 Problems

11.2.1 Electric Field and Potential

11.1 **(a)** Figure 11.1 shows two point clusters of charge situated in free space placed on a line that is called the x-axis. The first, with a positive charge of $Q_1 = +8\text{e}$, is at the origin. The second, with a negative charge of $Q_2 = -4\text{e}$, is to the right at a distance equal to 0.2 m.

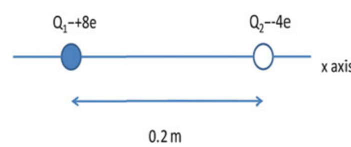
(i) What is the magnitude of the force between them?
(ii) Where would you expect to find the position of zero electric field: to the left of Q_1, between Q_1 and Q_2 or to the right of Q_2? Briefly explain your choice and then work out the exact position.

(b) The electron in a hydrogen atom orbits the proton at a radius of 5.3×10^{-11} m.

(i) What is the proton's electric field strength at the position of the electron?
(ii) What is the magnitude of the electric force on the electron?

[University of Aberystwyth, Wales]

Fig. 11.1



11.2 **(a)** A tiny ball of mass 0.6 g carries a charge of magnitude 8 μC. It is suspended by a thread in a downward electric field of intensity 300 N/C. What is the tension in the thread if the charge on the ball is

(i) positive?
(ii) negative?

(b) A uniform electric field is in the negative x-direction. Points a and b are on the x-axis, a at $x = 2$ m and b at $x = 6$ m.

(i) Is the potential difference $V_b - V_a$ positive or negative?
(ii) If the magnitude of $V_b - V_a$ is 10^5 V, what is the magnitude E of the electric field?

[University of Aberystwyth, Wales 2005]

11.3 Show that the electric potential a distance z above the centre of a horizontal circular loop of radius R, which carries a uniform charge density per unit length λ, is given by

$$V = \frac{\lambda R}{2\varepsilon_0} \frac{1}{(z^2 + R^2)^{1/2}}$$

Obtain an expression for the electrostatic field strength as a function of z.

[University of Aberystwyth, Wales 2007]

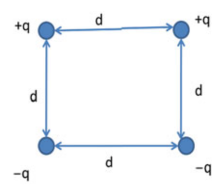
11.4 **(a)** Starting from Coulomb's law, show that the electric potential a distance r from a point charge q is given by

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

(b) Four point charges are assembled as shown in Fig. 11.2. Calculate the potential energy of this configuration (you may assume that the charges are isolated and in a perfect vacuum). Does the potential energy depend upon the order in which the charges are assembled?

(c) Is the charge configuration in (b) stable?

Fig. 11.2



11.5 A spherical liquid drop has a diameter of 2 mm and is given a charge of 2×10^{-15} C.

(i) What is the potential at the surface of the drop?
(ii) If two such drops coalesce to form a single drop, what is the potential at the surface of the drop so formed?

[Indian Institute of Technology 1973]

11.6 A pendulum bob of mass 80 mg carries a charge of 2×10^{-8} C at rest in a horizontal uniform electric field of 20,000 V/m. Find the tension in the thread of the pendulum and the angle it makes with the vertical.

[Indian Institute of Technology 1979]

11.7 An infinite number of charges, each equal to q, are placed along the x-axis at $x = 1$, $x = 2$, $x = 4$, $x = 8$, etc. Find the potential and the electric field at the point $x = 0$ due to the set of charges.

[Indian Institute of Technology 1974]

11.8 In prob. (11.7) what will be the potential and electric field in the above set-up if the consecutive charges have opposite sign?

[Indian Institute of Technology 1974]

11.9 A thin fixed ring of radius 1 m has a positive charge 1×10^{-5} C uniformly distributed over it. A particle of mass 0.9 g and having negative charge of 1×10^{-6} C is placed on the axis, at a distance of 1 cm from the centre of the ring. Show that the motion of the negative charge is approximately simple harmonic. Calculate the time period of oscillation.

[Indian Institute of Technology 1982]

11.10 Three charges, each of value q, are placed at the corners of an equilateral triangle. A fourth charge Q is placed at the centre of the triangle.

(i) If $Q = -q$ will the charges at the corners move towards the centre or fly away from it?
(ii) For what value of Q will the charges remain stationary?

[Indian Institute of technology 1978]

11.11 Two identically charged spheres are suspended by strings of equal length. The strings make an angle 30° with each other. When suspended in a liquid of density 0.8 g/cm^3, the angle remains the same. What is the dielectric constant of the liquid? The density of the material of the sphere is 1.6 g/cm^3.

[Indian Institute of Technology 1976]

11.12 At the corner A of square ABCD of side 10 cm a charge 6×10^{-8} C is placed. Another charge of -3×10^{-8} C is located at the centre of the square. Find the work done in carrying a charge 5×10^{-9} C from the corner C to the corner B of the square.

[Indian Institute of Technology 1972]

11.13 A pith ball carrying a charge of 3×10^{-10} C is suspended by an insulated thread of length 50 cm. When a uniform electric field is applied in a horizontal direction, the ball is found to deflect by 2 cm from the vertical. If the mass of the ball is 0.5 g what is the magnitude and direction of the electric field?

[Indian Institute of Technology 1973]

11.14 A positively charged oil droplet remains in the electric field between two horizontal plates, separated by a distance 1 cm. If the charge on the drop is 3.2×10^{-19} C and the mass of the droplet is 10^{-14} kg what is the potential difference between the plates? Now if the polarity of the plates is reversed what is the instantaneous acceleration of the droplet?

[Indian Institute of Technology 1974]

11.15 Suppose equal amount of charge of the same sign is placed on the earth and the moon, what would be its magnitude if the gravitational attraction between the two bodies may be nullified? Take mass of the earth and moon to be 6×10^{24} and 7.4×10^{22} kg, respectively.

11.16 A spark is produced between two insulated surfaces, maintained at a constant difference of 5×10^6 V. If the energy output is 10^{-5} J, calculate the charge transferred. How many electrons have flowed?

[Indian Institute of Technology 1974]

11.17 A rod 25 cm long has a uniform linear charge density (charge per unit length) $\lambda = 200\,\mu\mathrm{C/m}$. Calculate the electric field (in N/C) at 10 cm from one end along the axis of the rod.

11.18 A disc of radius R is uniformly charged to Q and placed in the xy-plane with its centre at the origin. Find the electric field along the z-axis.

11.19 Electronic charge e may be determined by Millikan's oil drop method. Oil drops of radius r acquire a terminal speed v_1 with downward electric field E and a speed v_2 with the upward electric field. Derive an expression for e in terms of E, v_1, v_2, r and η, the viscosity of oil in air.

11.20 A circular wire of radius r has a uniform linear charge density $\lambda = \lambda_0 \cos^2 \theta$. Show that the total charge on the wire is $\pi \lambda_0 r$.

11.21 The distance between the electron and the proton in the hydrogen atom is about 0.53 Å. By what factor is the electrical force stronger than the gravitational force? Does the distance matter?

11.22 The combined charge on two small spheres is $+15\,\mu$C. If each sphere is repelled by the other by a force of 5.4 N when the spheres are 30 cm apart, find the charges on the spheres.

11.23 Charges are placed at the four corners of a square of side a, as in Fig. 11.3. Find the magnitude and direction of the electric field at the centre of the square.

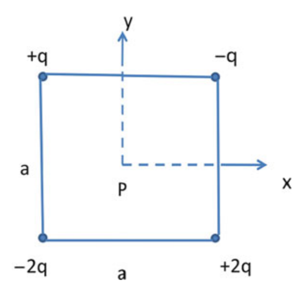


Fig. 11.3

11.24 A thin, non-conducting rod of length L carries a total charge $+Q$ spread uniformly along it. Find the electric field at point p distant y from the axis of the rod on the perpendicular bisector.

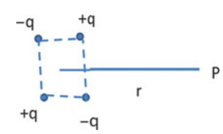
11.25 A thin non-conducting rod is bent to form an arc of a circle of radius r and subtends an angle θ_0 at the centre of the circle. If a total charge q is spread uniformly along the rod, find the electric field at the centre of the circle.

11.26 A ring of radius r located in the xy-plane is given a total charge $Q = 2\pi R\lambda$. Show that E is maximum when the distance $z = r/\sqrt{2}$.

11.27 A total charge q is spread uniformly over the inner surface of a non-conducting hemispherical cup of inner radius a. Calculate (a) the electric field and (b) the electric potential at the centre of the hemisphere.

11.28 The quadrupole consists of four charges $q, q, -q, -q$ located at the corners of a square on side a (Fig. 11.4). Show that at a point p, distant r from the centre of the charges and in the same plane, the electric field varies inversely as the fourth power of r, where $r >> a$.

Fig. 11.4



11.29 Show that the electrical and gravitational force between two bodies each of mass m and charge q will be equal at any distance r if the ratio $q/m = 8.6 \times 10^{-10}$ C/kg.

11.30 Two small, equally charged spheres, each of mass m, are suspended from the same point by silk threads of length L. Initially, the spheres are separated by distance $x << L$. As the charge leaks out at the rate $\mathrm{d}q/\mathrm{d}t$, the spheres approach each other with relative velocity $v = a/\sqrt{x}$, where a is a constant. Find the rate at which charge leaks out.

Show that

$$\mathrm{d}q/\mathrm{d}t = \frac{3}{2}a\sqrt{\frac{2\pi\varepsilon_0 mg}{L}}$$

11.31 A charge q is uniformly distributed over a thin ring of radius R. A very long uniformly charged thread with linear charge density λ is placed on the axis of the ring with one end coinciding with the centre of the ring. Show that the force of interaction F will be equal to $\dfrac{q\lambda}{4\pi\varepsilon_0 R}$.

11.32 A very long wire with uniform charge density λ is placed along the x-axis with one end of the thread coinciding with the origin. Show that the electric field is given by $E = \dfrac{\sqrt{2}\lambda}{4\pi\varepsilon_0 y}$ at 45° with the x-axis at a distance y from the end of the thread.

11.33 If the electric potential is given by $\varphi = cxy$, calculate the electric field.

11.34 Charges Q and $-2Q$ are placed at a fixed distance of separation. Show that the locus of points in the plane of charges, where the potential is zero, will be a circle.

[Indian Administrative Services]

11.35 Two identical thin rings, each of radius R, are coaxially placed a distance R apart. If Q_1 and Q_2 are, respectively, the charges uniformly spread on the two rings, find the work done in moving a charge q from the centre of one ring to that of the other.

[Indian Institute of Technology 1992]

11.36 A thin rod of length $2a$ is placed along the y-axis in the xy-plane. The rod carries a charge density λ (Fig. 11.5). The point P_1 is located at $(0, 2a)$ and P_2 at $(x, 0)$.

(a) Find x if the potentials at P_1 and P_2 are equal.
(b) Find the corresponding potential.

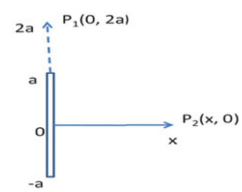


Fig. 11.5

11.37 Three charges $+q$, $+q$ and Q are located at the vertices of a right-angled isosceles triangle, Fig. 11.6. If the total interaction energy is zero what should be the value of Q?

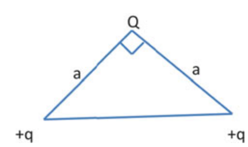


Fig. 11.6

11.38 Four charges each of magnitude q are located at the four corners of a square of side a such that like charges occupy the corners across the diagonals (Fig. 11.7). Calculate the work done in assembling these charges.

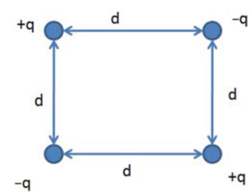


Fig. 11.7

11.39 Calculate the total potential energy of sphere of radius R carrying a uniformly distributed charge q.

11.40 A linear quadrupole (Fig. 11.8) consists of charge $+2Q$ at the origin and two charges $-Q$ at $(-d, 0)$ and $(+d, 0)$.

(i) Write down the magnitude of the electric field at P on the x-axis where $x > d$.
(ii) If $x >> d$ show that the field varies inversely as the fourth power of distance from the origin.
(iii) If $Q = 2\,\mu\text{C}$ and $d = 0.01$ mm, calculate the field at $x = 20$ cm.

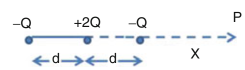


Fig. 11.8 Linear quadrupole

11.41 **(a)** If the breakdown field strength of air is 5×10^6 V/m how much charge can be placed on a sphere of radius 1 mm? **(b)** What would be the corresponding electrical potential?

11.42 An electron is released from a distance 120 cm from a stationary point charge $+2 \times 10^{-9}$ C. Calculate the speed of the electron when it is 18 cm from the point charge.

11.43 Figure 11.8 shows the linear quadrupole. Show that the electric potential $V(r)$ at a distance $r >> d$ from the central charge and in a direction normal to the axis of the quadrupole varies inversely as the third power of r.

11.44 An electron of mass $m_e = 9.1 \times 10^{-31}$ kg is accelerated in the uniform electric field $\boldsymbol{E}$ between two parallel charged plates, as shown in Fig. 11.9. There is no electric field outside of the plates. The electric field has a magnitude $E = 2.0 \times 10^3$ N/C and electron charge $e = -1.6 \times 10^{-19}$ C. The separation of the plates is 1.5 cm and the electron is accelerated from rest near the negative plate and passes through a tiny hole in the positive plate. Assume the hole is so small that it does not affect the uniform field between the plates.

(i) What is the force on the electron while it is between the plates?
(ii) What is its acceleration and with what speed does it leave the hole?
(iii) What is the force on the electron outside of the plates?

[University of Aberystwyth, Wales 2005]

11.45 What is the electric potential V at a distance r from a point charge Q? Write down an expression describing the electric potential due to a continuous charge distribution.

Consider a disk of radius R which carries a uniform surface charge distribution.

(a) Find the total charge on the disc.
(b) Find the potential at a point on the axis of the disc lying at a distance x from the disc.
(c) What is the form of the potential when x becomes much larger than R? Comment on your result.

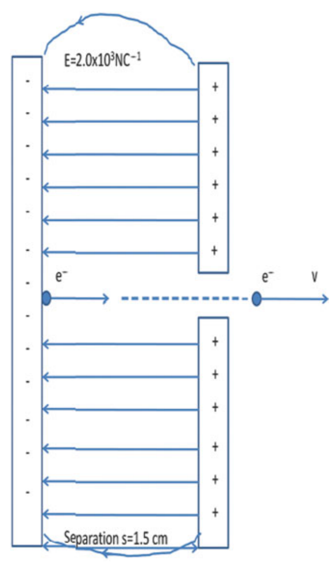


Fig. 11.9

11.46 In the Bohr's hydrogen atom model, show that the orbital motion of the electron obeys Kepler's third law of motion, that is, $T^2 \propto r^3$.

11.47 Equal charges (Q) are placed at the four corners of a square of side a. Show that the force on any charge due to the other three charges is given by 1.914 $Q^2/4\pi\varepsilon_0 a^2$.

11.48 **(a)** Calculate the electric field due to a dipole on its perpendicular bisector.
(b) Show that for the distance $x >> d/2$, where d is the distance between the charges, the field varies inversely as the cube of distance.
(c) A molecule has a dipole moment of 6×10^{-30} cm. Calculate the difference in potential energy when the dipole is placed parallel to the electric field of 3×10^6 V/m and then antiparallel to the field.

11.2.2 Gauss' Law

11.49 **(a)** State Gauss' law of electrostatics in mathematical form.
(b) Use Gauss' law to show that the electric field magnitude due to an infinite sheet of charge, carrying a surface density σ, is given by

$$E = \frac{\sigma}{2\varepsilon_0}$$

(c) A small sphere of mass 2 mg carries a charge of 5×10^{-8} C. It hangs by a silk thread attached to a vertical uniformly charged sheet such that, under the influence of both gravity and the electric force, it makes an angle of 10° with the sheet. Calculate the surface charge density of the sheet.

11.50 **(a)** State Gauss' law in differential and integral form.
(b) Show that the electric field outside a charged sphere is $Q/4\pi\varepsilon_0 r^2$, where r is the distance from the centre of the ball.
(c) Show that the electric field inside a uniformly charged solid sphere, with total charge Q and radius R, is $Qr/4\pi\varepsilon_0 R^3$.

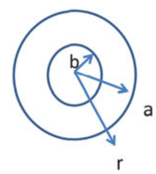
11.51 In prob. (11.50) the central part of the sphere is hollowed by creating a cavity of radius $\frac{1}{2}R$ concentric with the original sphere. If the charge density of the hollowed sphere remains unchanged show that the electric field at the surface is now 7/8 of the original value on the surface.

11.52 In prob. (11.50) show that the electric potential (a) varies as that to simple harmonic motion for $r < R$. (b) $V(0) = \dfrac{3}{2}V(R)$ where $V(\infty) = 0$.

11.53 Figure 11.10 shows a non-conducting hollow sphere with inner radius b and outer radius a. A total charge Q is uniformly distributed in the material $b < r < a$. Find the electric field for (a) $r < b$; (b) $b < r < a$; (c) $r > a$.

11.54 A charge Q is uniformly distributed in a long cylinder of radius R and charge density ρ. Find the electric field for the regions (a) $r > R$; (b) $r < R$.

Fig. 11.10



11.55 **(a)** The electric field on the surface of a thin spherical shell of radius 0.5 m is measured to be 800 N/C and points radially towards the centre of the sphere. What is the net charge within the sphere's surface?

(b) An electric field of 120 N/C points down over a football field. Calculate the surface charge density on the field.

(c) What would be the total electric flux if the field is $100 \times 75\,\text{m}^2$.

11.56 **(a)** Using Gauss' law derive Coulomb's formula for the electric field due to an isolated point charge q.

(b) A positive charge Q is uniformly distributed in a non-conducting sphere of radius R. Calculate the electric flux passing through the spherical surface of radius r concentric with the sphere for (i) $r < R$; (ii) $r > R$.

11.57 How is electric flux related to the electric field **E**? How is the total electric flux over a closed surface related to the charge enclosed within the surface?

A thin spherical shell of radius R_1 carries a total charge Q_1 that is uniformly distributed on its surface. A second, larger concentric thin shell of radius R_2 carries a charge Q_2 also uniformly distributed over the surface of the shell. Use Gauss' law to find the electric field in the regions.

(a) $r < R_1$,
(b) $R_1 < r < R_2$, and
(c) $R_2 < r$.

The electric charges are such that $Q_1 > 0$ and the electric field is zero for $r > R_2$. Find the ratio Q_1/Q_2.

11.58 Two insulated spheres of radii 1 and 3 cm at a considerable distance apart are each charged positively with 3×10^{-8} C. They are brought into contact and separated by the same distance as before. Compare the forces of repulsion before and after contact.

[Northern Universities of UK]

11.59 What is the maximum charge that can be given to a sphere of diameter 10 cm if the breakdown voltage of air is 2×10^4 V/cm.

11.60 **(a)** Show that the capacitance, C, of a conducting sphere of radius a is given by $C = 4\pi\varepsilon_0 a$.

(b) Two isolated conducting spheres, both of radius a, initially carry charges of q_1 and q_2 and are held far apart. The spheres are connected together by a conducting wire until equilibrium is reached, whereupon the wire is removed. Show that the total electrostatic energy stored in the spheres decreases by an amount ΔU, given by

$$\Delta U = \frac{1}{16\pi\varepsilon_0 Q}(q_1 - q_2)^2$$

What happens to this energy?

11.61 Two spherical conductors of radii R_1 and R_2 and charges Q_1 and Q_2, respectively, are brought in contact and separated. Show that their charge densities will be inversely proportional to their radii.

11.62 A light spherical balloon is made of conducting material. It is suggested that it could be kept spherical simply by connecting it to a high-voltage source. The balloon has a diameter of 100 mm.

(a) What is the voltage of the source if the electric field on the balloon surface is 5×10^6 V/m?
(b) What gas pressure inside the balloon would produce the same effect?
(c) The voltage source is removed and the balloon remains at the same voltage. Calculate the total electrostatic energy of the balloon.

11.63 A soap bubble of radius R_1 is given a charge q. Due to mutual repulsion of the surface charges the radius is increased to R_2, the pressure remaining constant. Show that the charge is given by

$$q = \left[\frac{32}{3}\pi^2\varepsilon_0 p R_1 R_2 \left({R_1}^2 + R_1 R_2 + {R_2}^2\right)\right]^{1/2}$$

11.64 An insulating spherical shell of inner radius r_1 and outer radius r_2 is charged so that its volume charge density is given by

$$\rho(r) = 0 \text{ for } 0 \leq r \leq r_1$$
$$\rho(r) = \frac{A}{r} \text{ for } r_1 \leq r \leq r_2$$
$$\rho(r) = 0 \text{ for } r > r_2$$

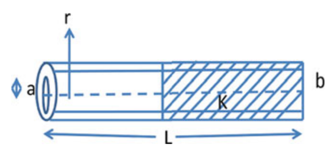
where A is a constant and r is the radial distance from the centre of the shell. Find the electric field due to the shell throughout all space.

11.65 (**a**) Show that electrostatic field is conservative.
(**b**) An isolated soap bubble of radius 1 cm is at a potential of 10 V (assume the bubble material is a conductor); calculate the total charge on the bubble. If it collapses to a drop of radius 1 mm (no charge loss or gain during the process), what is the change of the electrostatic energy of the system?

[University of Aberystwyth, Wales 2008]

11.66 A long cylinder of charge q has a radius a. The charge density within its volume, ρ, is uniform (Fig. 11.11). Describe the form of the electric field generated by the cylinder. Find the electric field strength at a distance r from the axis of the cylinder in the regions (i) $r > a$ and (ii) $0 < r < a$.
If a non-relativistic electron moves in a circle at a constant distance R from the axis of the cylinder, where $R > a$, find an expression for its speed.
[University of Manchester 2006]

Fig. 11.11



11.67 Consider an isolated non-conducting sheet with charge density σ. The electric field at 25 cm from the sheet is found to be 200 V/m, directed towards the sheet. Calculate σ on the sheet. What electric field is expected at 2 cm from its surface? How are the values of σ and E changed if a conducting sheet is substituted.

11.2.3 Capacitors

11.68 Calculate the capacitance of a parallel plate capacitor of area A and thickness d if a dielectric slab of thickness t, area A and dielectric constant k is inserted. How is the capacitance modified if a metal of thickness t is introduced?

11.69 Two capacitors C_1 and C_2 are connected in parallel and their combined capacitance is measured as 9 μF. When they are combined in series their capacitance is 2 μF. What are the individual capacitances?

11.70 Find the energy which may be stored in capacitors of 2 and 4 μF when taken (a) singly, (b) in series and (c) in parallel when a potential difference of 100 V is available.
[University of New Castle]

11.71 An air capacitor with plates 1 m^2 and 0.01 m apart is charged with 10^{-6} C of electricity. Calculate the change in energy which results when the capacitor is submerged in oil of relative permittivity 2.0.
[University of Manchester]

11.72 Two parallel plates, each of area $1\,\mathrm{m}^2$, are separated by a distance 0.001 m and have a capacity of $0.1\,\mu\mathrm{F}$. What must then the dielectric constant of the material separating the plates be?

[University of Newcastle]

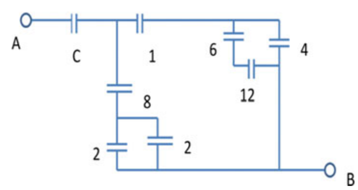
11.73 A capacitor of capacitance $5\,\mu\mathrm{F}$ is charged up to a PD of 250 V. Its terminals are then connected to those of an uncharged capacitor of capacitance $20\,\mu\mathrm{F}$. What would be the resulting voltage?

[Northern Universities of UK]

11.74 From Fig. 11.12 find the value of capacitance C if the equivalent capacitance between points A and B is $1\,\mu\mathrm{F}$. All the capacitances are in microfarads.

[Indian Institute of Technology 1977]

Fig. 11.12



11.75 Three capacitors of capacitance 4, 3 and $2\,\mu\mathrm{F}$, respectively, are connected in series to a battery of 240 V. Calculate (a) the charge, (b) the potential and (c) the electrostatic energy associated with each of the three capacitors, stating in each case the units in which the results are expressed.

[Northern Universities of U.K.]

11.76 Each of the two capacitors A and B of capacitances 1.0 and $2.0\,\mu\mathrm{F}$, respectively, are charged initially by connecting them in turn to a 12 V battery. What is the final potential difference of the combination if the capacitors are later connected in parallel such that

(a) the positive plate of one is connected to the positive plate of the other;
(b) the positive plate of one is connected to the negative plate of the other.

[Indian Institute of Technology 1971]

11.77 Two capacitors of capacitances C_1 and C_2 charged to potential difference V_1 and V_2 are connected in parallel. Calculate the energy loss when (a) the positive ends are joined and (b) the positive end of one is joined to the negative end of the other.

11.78 A capacitor of capacitance C is charged to potential V by connecting it to a battery. Let q be the charge on it, E the electric field within the plates and U the energy stored. When a dielectric of constant K is introduced filling completely the space between the plates, how will the following quantities change (i) V, (ii) E, (iii) q, (iv) C and (v) U, when (a) the battery remains connected and (b) the battery is disconnected?

11.79 In prob. (11.78) if the plate separation is increased, how would the following quantities change (i) V, (ii) E, (iii) C, (iv) q and (v) U when (a) the battery remains connected and (b) the battery is disconnected?

11.80 Show that the force of attraction between the plates of a parallel plate capacitor is given by $F = \dfrac{1}{2}\dfrac{\varepsilon_0 A V^2}{d}$, where A is the area, d the distance of separation, V the voltage to which the plates are charged and ε_0 the permittivity.

11.81 Let n identical droplets, each of radius r and charge q, coalesce to form a large drop of radius R and charge Q. Assuming that the droplets are incompressible, show that (a) the radius $R = n^{1/3} r$; (b) the capacitance C' of the large drop is $C' = n^{1/3} C$, where C is the capacitance of the droplet; (c) the potential V' of the large drop is given by $V' = n^{2/3} V$, where V is the potential of the droplet; (d) the surface charge density $\sigma' = n^{1/3}\sigma$; (e) the energy U' stored in the large drop is given by $U' = n^{5/3} U$ where U is the energy stored in the droplet.

11.82 A cylindrical capacitor has radii a and b. Show that half of the stored electrical potential energy lies within a cylinder whose radius is $\sqrt{ab}$.

11.83 A capacitor of capacitance $C_1 = 3.0\,\mu\text{F}$ withstands the maximum voltage $V_1 = 4.0\,\text{kV}$, while a capacitor of capacitance $C_2 = 6.0\,\mu\text{F}$ the maximum voltage $V_2 = 3.0\,\text{kV}$. If they are connected in series what maximum voltage can the system withstand?

11.84 A Geiger–Muller tube consists of a thin uniform wire of radius 'a' of length L surrounded by a concentric hollow metal cylinder of radius b with a gas of dielectric constant K between them. Apply Gauss' law to calculate the capacitance of the tube.

11.85 Two spherical metallic shells of radii a and b ($b > a$) constitute a capacitor with the outer shell grounded and contact is made with the inner one through a hole in the outer one. Show that the capacitance is given by $C = \dfrac{4\pi\varepsilon_0 ab}{b - a}$.

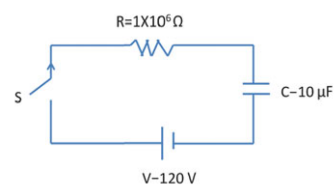
11.86 Show that for two concentric shells of radii a and b ($b \approx a$), the capacitance reduces to that of a parallel plate capacitor

11.87 In an R–C circuit the emf supplied by the battery is 120 V, $R = 1 \times 10^6\,\Omega$ and $C = 10\,\mu\text{F}$. The switch S is closed at $t = 0$. Find

(i) the time taken for the charge to reach 90% of its final value;
(ii) the energy stored in the capacitor at one time constant;
(iii) the Joule heating in the resistor at one time constant.

11.88 After how many time constants will the energy stored in the capacitor in Fig. 11.13 reach one-half of its equilibrium value?

Fig. 11.13



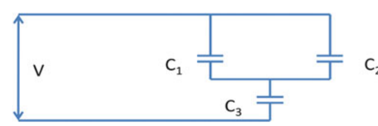
11.89 Two square metal plates measuring 'a' on the side are used as a parallel plate capacitor with the plates slightly inclined at an angle θ. If the smaller gap between the plates is D, then show that the capacitance is given by

$$C = \frac{\varepsilon_0 a^2}{D}\left(1 - \frac{a\theta}{2D}\right)$$

11.90 Capacitors $C_1 = 8\,\mu\text{F}$, $C_2 = 4\,\mu\text{F}$ and $C_3 = 3\,\mu\text{F}$ are arranged as in Fig. 11.14. A voltage of $V = 100\,\text{V}$ is applied. Determine

(a) the potential difference across C_1, C_2 and C_3.
(b) the charge q_1, q_2 and q_3 on C_1, C_2 and C_3.
(c) the energy U_1, U_2 and U_3 stored in the capacitors.

Fig. 11.14



11.91 Capacitors $C_1 = 8\,\mu\text{F}$, $C_2 = 4\,\mu\text{F}$ and $C_3 = 3\,\mu\text{F}$ are arranged as in Fig. 11.15. A voltage of $V = 100\,\text{V}$ is applied. Determine

(a) the charges q_1, q_2 and q_3 on C_1, C_2 and C_3, respectively.
(b) the potential difference across C_1, C_2 and C_3.
(c) the energy U_1, U_2 and U_3 stored in the capacitors.

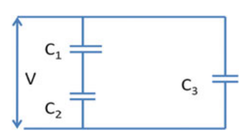


Fig. 11.15

11.92 Find the effective capacitance between points a and b in Fig. 11.16. Assume that $C_1 = C_2 = C_3 = C_4 = 2\,\mu\text{F}$ and $C_5 = 1\,\mu\text{F}$.

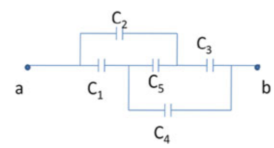


Fig. 11.16

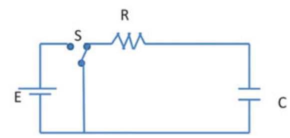


Fig. 11.17

11.93 Consider a circuit consisting of a resistor R and a capacitor C in series with a battery of emf ξ and a switch (Fig. 11.17). The capacitor is initially uncharged and the switch is closed at time $t = 0$. By considering the potential drop across each of the components of the circuit, verify that the charge Q on the capacitor has the form

$$Q = C\xi\left(1 - \mathrm{e}^{-\frac{r}{RC}}\right)$$

(a) What is the current flowing in the circuit?
(b) What is the power supplied by the battery as a function of t?
(c) What is the power dissipated in the resistor as a function of t?
(d) What is the rate at which energy is stored in the capacitor as a function of t?

[University of Durham 2000]

11.94 For the circuit shown in Fig. 11.18,

(i) What is the initial battery current immediately after switch S is closed?
(ii) What is the battery current a long time after switch S is closed?
(iii) If the switch has been closed for a long time and is then opened, find the current through the 600 kΩ resistor as a function of time.

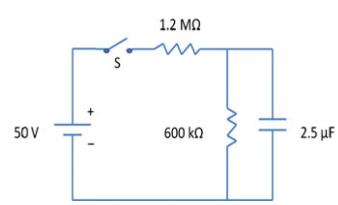


Fig. 11.18

11.95 A capacitor of capacitance $C = 500\,\mu\text{F}$ is charged to a voltage of 900 V and is then discharged through a resistance $R = 200\,\text{k}\Omega$ when a switch is closed.

(i) Find the initial charge stored in the capacitor.
(ii) Find the initial discharge current when the switch is closed.
(iii) Find the voltage across the capacitor in a time $t = 25\,\text{s}$ after the start of discharge.
(iv) Find the time constant of this capacitor resistor network combination.
(v) Work out an equation to show the time it takes for the charge in the capacitor to drop by one-half of its starting value and find this time.

[University of Aberystwyth, Wales 2008]

11.96 Charge $q = 10^{-9}\,\text{C}$ is uniformly distributed in a sphere of radius $R = 1\,\text{m}$.

(i) Find the divergence of the electric field inside the sphere.
(ii) A proton is moved from infinity to $r = 0.8\,\text{m}$ from the centre of the sphere. Find the electric force experienced by the proton at $r = 0.8\,\text{m}$.
(iii) Find the work done by the electric field of the charged sphere when the proton is moved from infinity to its current position ($r = 0.8\,\text{m}$).

11.97 **(a)** Write down the integral and differential forms of Gauss' law in a dielectric, defining all quantities used.
(b) A parallel plate capacitor is completely filled with a non-conducting dielectric. Show that the electric displacement, D, is uniform between the plates and calculate its value. (You may assume that the plates each have area A and are separated by a small distance d. Each plate carries a surface charge density σ C/m^2.)
(c) The dielectric has a non-uniform relative permittivity

$$K(x) = ax + b$$

where a and b are constants and x is the perpendicular distance from one plate. Using Gauss' law, show that the electric field between the plates satisfies

$$E(x) = \frac{E_0}{K(x)}$$

where E_0 is a constant. Find the value of E_0.
(d) Show that the voltage across the capacitor is given by

$$V = \frac{E_0}{\varepsilon_0 a} \ln\left(1 + \frac{Qd}{b}\right)$$

and calculate the capacitance.
(e) Find the volume polarization charge density in the dielectric.

11.98 Both gravitational field and electric field obey inverse square law. Using this analogy show that the differential Gauss' law for gravitation is given by $\nabla \cdot g = -\rho/G$, where ρ is the mass density.

11.3 Solutions

11.3.1 Electric Field and Potential

11.1 **(a)**

(i) $F = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \dfrac{(8 \times 1.6 \times 10^{-19})(-4 \times 1.6 \times 10^{-19})}{4\pi \times 8.85 \times 10^{-12} \times (0.2)^2}$
$= -1.8432 \times 10^{-25}\,\text{N}$

(ii) For the position of zero electric field the forces due to the two charges must be equal in magnitude but oppositely directed. Clearly the neutral point must be on the x-axis. On the left of Q_1, the forces will be oppositely directed but cannot be equal as $|Q_1| > |Q_2|$. Between Q_1 and Q_2, the forces are exerted in the same direction. On the right of Q_2 conditions are favourable for a null point. Let the zero electric field be situated at a distance x from Q_2 on the right.

$$\frac{8e}{(x+0.2)^2} - \frac{4e}{x^2} = 0$$

whence $x = 0.4828$ on the right of Q_2.

(b) $E = \dfrac{e}{4\pi\varepsilon_0 r^2} = \dfrac{1.6 \times 10^{-19}}{4\pi \times 8.85 \times 10^{-12} \times (5.3 \times 10^{-11})^2}$
$= 5.12 \times 10^{18}\,\text{N/C}$
Force $F = Ee = 5.12 \times 10^{18} \times 1.6 \times 10^{-19} = 81.92\,\text{N}$

11.2 **(a)** As the electric field is downwards, the force on the positive charge will be downwards and the force on the negative charge will be upwards.

(i) $q = +8\,\mu\text{C}$

$F_q = qE = +8 \times 10^{-6} \times 300 = 2.4 \times 10^{-3}\,\text{N}$

$F_g = mg = 0.6 \times 10^{-3} \times 9.8 = 5.88 \times 10^{-3}\,\text{N}$

$\therefore$ Tension in the thread

$T = F_g + F_q = 5.88 \times 10^{-3} + 2.4 \times 10^{-3} = 8.28 \times 10^{-3}\,\text{N}$