Energy released in coalescing n droplets each of radius r into a large drop of radius R

$$\Delta W = 4\pi r^2 (n - n^{2/3}) S \tag{10.36}$$

$$\Delta W = 4\pi R^2 (n^{\frac{1}{3}} - 1) S \tag{10.37}$$

Capillary rise:

$$2S \cos\theta = (h + r/3) r\rho g \tag{10.38}$$

where θ is the angle of contact, r is the radius of the bore and h is the height of the liquid column.

10.2 Problems

10.2.1 Kinetic Theory of Gases

10.1 Define the mean free path of a gas both mathematically and in words. Calculate the mean free path of a molecule in a gas if the number of collisions is 2×10^{10}/s and the mean molecular velocity is 1000 m/s.

[University of Manchester 2008]

10.2 **(a)** Consider a gas that has a molecular weight of 28 and a temperature of 27°C. What is the rms speed of molecule of the gas if it has a Maxwellian velocity distribution? The ideal gas constant is 8.31 J/mol K.

(b) What is the mean free path of a molecule if the pressure is 2 atm (1 atm = 101.3 kPa), the temperature is 27°C, and the cross-section is 0.43 nm^2? Using the average velocity from part **(a)** calculate the collision frequency for the molecule. Boltzmann's constant is 1.38×10^{-23} J/K.

10.3 **(a)** By considering a volume, V, of ideal gas, containing N spherical molecules with radius r, show that the mean free path of the molecules can be defined by the equation

$$\lambda = \frac{V}{4\pi\sqrt{2} r^2 N}$$

(b) Hence, or otherwise, calculate the mean free path of air at 100°C and 1.01×10^5 Pa. Assume that the molecules of air are spheres of radius $r = 2.0 \times 10^{-10}$ m.
($k = 1.38 \times 10^{-23}$ J/K)

[University of Aberystwyth, Wales]

10.4 Figure 10.1 shows the Maxwell–Boltzmann velocity distribution functions of a gas for two different temperatures, of which first (curve f_1) is for $T_1 = 300$ K.

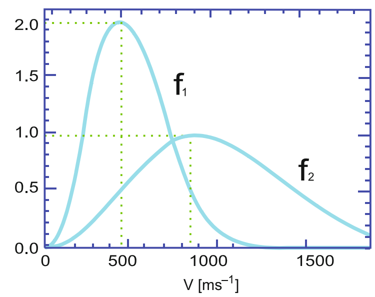

Fig. 10.1 Maxwell–Boltzmann velocity distributions

(a) Read the approximate value for the most probable speed of the molecules from the diagram for each of the two cases.
(b) What is the temperature, T_2, when the velocity distribution is given by f_2?
(c) Indicate the average speeds in the diagram for each of the two temperatures and give the ratio between the two average speeds.
(d) The gas consists of 5 mol of molecules. If the molecular velocity distribution is given by f_2, estimate the number of those molecules in the gas which have a speed between $v_A = 800\,\text{m/s}$ and $v_B = 900\,\text{m/s}$.

10.5 If the Maxwell–Boltzmann distribution of speeds is given by

$$f(v) = 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{\frac{-mv^2}{2kT}}$$

show that the most probable speed is defined by the equation

$$v_{\text{mp}} = \left(\frac{2kT}{m}\right)^{1/2}$$

10.6 For carbon dioxide gas (CO_2, molar mass $= 44.0$ g/mol) at $T = 300$ K, calculate

(i) the mean kinetic energy of one molecule
(ii) the root mean square speed, v_{rms}
(iii) the most probable speed, v_{mp}
(iv) the average speed, v_{av}

($R = 8.31$ J/K/mol, $k = 1.38 \times 10^{-23}$ J/K, $N_A = 6.02 \times 10^{23}$/mol)

10.7 **(a)** Give three assumptions that are made when deriving the properties of an ideal gas using a molecular model.

(b) A weather balloon is loosely filled with $2\,\text{m}^3$ of helium at 1 atm. and 27°C. The balloon is then released, and by the time it has reached an elevation of 7000 m the pressure has dropped to 0.5 atm. and the balloon has expanded. If the temperature at this elevation is −48°C, what is the new volume of the balloon?

10.8 van der Waal's equation can be written in terms of moles per volume as

$$\frac{n}{V} = \left(\frac{p + a\dfrac{n^2}{V^2}}{RT}\right)\left(1 - b\frac{n}{V}\right)$$

The van der Waal's parameters for hydrogen sulphide gas (H_2S) are $a = 0.448\,\text{J}\,\text{m}^3/\text{mol}^2$ and $b = 4.29 \times 10^{-5}\,\text{m}^3/\text{mol}$. Determine an estimate of the number of moles per volume of H_2S gas at 127°C and a pressure of 9.80×10^5 Pa as follows:

(a) Calculate as a first approximation using the ideal gas equation, $\dfrac{n}{V} = \dfrac{p}{RT}$.

(b) Substitute this first approximation into the right-hand side of the equation derived in part **(a)** to find a new approximation of $\dfrac{n}{V}$ (on the left-hand side) that takes into account real gas effects.

10.2.2 Thermal Expansion

10.9 Two parallel bars of different material with linear coefficient of expansion α_1 and α_2, respectively, are riveted together at a distance d apart. An increase in temperature ΔT will cause them to bend into circular arcs with a common centre subtending an angle θ at the centre (Fig. 10.2). Find the mean radius of curvature.

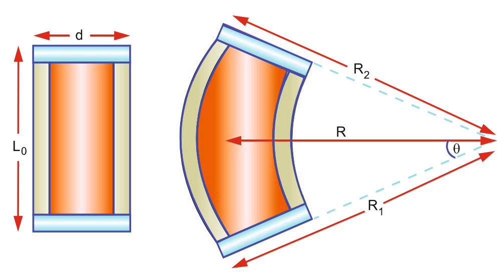

Fig. 10.2 Expansion of a bimetal strip of bars

10.10 A 20 m long steel rail is firmly attached to the road bed only at its ends. The sun raises the temperature of the rail by 30°C, causing the rail to buckle. Assuming that the buckled rail consists of two straight parts meeting in the centre, calculate how much the centre of the rail rises? For steel $\alpha = 12 \times 10^{-6}/°\text{C}$.

10.11 What should be the lengths of a steel and copper rod if the steel rod is 4 cm longer than the copper rod at any temperature. $\alpha(\text{steel}) = 1.1 \times 10^{-5}/°\text{C}$; α (copper) $= 1.7 \times 10^{-5}/°\text{C}$.

10.12 A 1 l glass flask contains some mercury. It is found that at different temperatures the volume of air inside that flask remains the same. What is the volume of mercury in this flask? Coefficient of linear expansion of glass $= 9 \times 10^{-6}/°\text{C}$; coefficient of volume expansion of mercury $= 1.8 \times 10^{-4}/°\text{C}$.
[Indian Institute of Technology 1973]

10.13 A steel wire of cross-sectional area 0.5 mm^2 is held between two fixed supports. If the tension in the wire is negligible and it is just taut at a temperature of 20°C, determine the tension when the temperature falls to 0°C (assume that the distance between the supports remains the same). Young's modulus of steel $= 2.1 \times 10^{11}$ dynes/cm^2; $\alpha = 12 \times 10^{-6}/°\text{C}$.
[Indian Institute of Technology 1973]

10.14 A glass vessel just holds 50 g of toluene at 0°C. What mass of toluene will it hold at 80°C if between 0 and 80°C the expansion coefficients are constant. The coefficient of linear expansion of glass is $8 \times 10^{-6}/°\text{C}$ and the absolute expansion of toluene is $11 \times 10^{-4°}\text{C}$.
[University of Dublin]

Gas Laws

10.15 Determine the constant in the gas equation given that a gram molecule of a gas occupies a volume of 22.4 l at NTP. [University of Durham]

10.16 A bubble of gas rises from the bottom of a lake 30 m deep. At what depth will the volume be thrice as great as it was originally (atmospheric pressure = 0.76 m of mercury; specific gravity of mercury = 13.6)?

10.17 A balloon will carry a total load of 175 kg when the temperature and pressure are normal. What load will the balloon carry on rising to a height at which the barometric pressure is 50 cm of mercury and the temperature is −10°C, assuming the envelope maintains a constant volume?
[University of London]

10.18 Two glass bulbs of volume 500 and 100 cc are connected by a narrow tube whose volume is negligible. When the apparatus is sealed off, the pressure of the air inside is 70 cm of Hg and the temperature 20°C. What does the pressure become if the 100 cc bulb is kept at 20°C and the other is heated to 100°C?

[University of Durham]

10.2.3 Heat Transfer

10.19 Two slabs of cross-sectional area A and of thickness d_1 and d_2 and thermal conductivities k_1 and k_2 are arranged in contact face to face. The outer face of the first slab is maintained at temperature T_1°C, that of the second one at T_2°C and the interface at T°C. Calculate

(a) Rate of flow of heat through the composite slab
(b) The interface temperature
(c) The equivalent conductivity

10.20 n slabs of the same thickness, the cross-sectional area $A_1, A_2, \ldots, A_n$ and thermal conductivities $k_1, k_2, \ldots, k_n$ are placed in contact in parallel and maintained at temperatures T_1 and T_2. Calculate

(a) the rate of flow of heat through the composite slab
(b) the equivalent conductivity

10.21 A bar of copper and a bar of iron of equal length are welded together end to end and are lagged. Determine the temperature of the interface when the free end of the copper bar is at 100°C and the free end of the iron is at 0°C and the conditions are steady. Thermal conductivities: copper $= 92$, iron $= 16$ cal/m/s/°C.

[University of Durham]

10.22 A block of ice is kept pressed against one end of a circular copper bar of diameter 2 cm, length 20 cm, thermal conductivity 90 SI units and the other end is kept at 100°C by means of a steam chamber. How long will it take to melt 50 g of ice assuming heat is only supplied to the ice along the bar, $L = 8 \times 10^4$ cal/kg.

[University of Dublin]

10.23 At low temperatures, say below 50 K, the thermal conductivity of a metal is proportional to the absolute temperature, that is, $k = aT$, where a is a constant with a numerical value that depends on the particular metal. Show that the rate of heat flow through a rod of length L and cross-sectional area A and whose ends are at temperatures T_1 and T_2 is given by $Q = \frac{aA}{2L}(T_1^2 - T_2^2)$.

10.24 Find the radial flow of heat in a material of thermal conductivity placed between two concentric spheres of radii r_1 and r_2 $(r_1 < r_2)$ which are maintained at temperatures T_1 and T_2 $(T_1 > T_2)$.

10.25 Find the radial rate of flow of heat in a material of thermal conductivity k placed between a co-axial cylinder of length L and radii r_1 and r_2, respectively ($r_1 < r_2$), maintained at temperatures T_1 and T_2, respectively ($T_1 > T_2$).

10.26 A small pond has a layer of ice on the surface that is 1 cm thick. If the air temperature is $-10°$C, find the rate (in m/h) at which ice is added to the bottom of the layer. The density of ice is 917 $\mathrm{kg/m^3}$, the thermal conductivity of ice is 0.59 W/m/K, and the latent heat of fusion is 333 kJ/kg. Assume that the underlying water is at 0°C.

10.27 An object is cooled from 85 to 75°C in 2 min in a room at 30°C. What time will be taken for the object to cool from 55 to 45°C.

10.28 A calorimeter containing first 40 g and then 100 g of water is heated and suspended in the same constant temperature enclosure. It is found that the time to cool from 50 to 40°C in the two cases was 15 and 33 min, respectively. Calculate the water equivalent of the calorimeter.

10.29 Two steel balls of identical material and surface quality have their radii in the ratio 1:2. When heated to 100°C and left to cool, they lose their heat by radiation. Find the rate of cooling $\mathrm{d}\theta/\mathrm{d}t$ for the balls.

10.30 A resistance thermometer gives readings of 24.9 Ω at the ice point, 29.6 Ω at the steam point and 26.3 Ω at some unknown temperature. What is the unknown temperature on the Celsius scale?

[The University of Wales, Aberystwyth 2004]

10.31 Solar constant (S) is defined as the average power received from the sun's radiation per square metre of earth's surface. Calculate S assuming sun's radius (R) as 6.95×10^8 m, the mean earth–sun distance (r) as 1.49×10^{11} m, sun's surface temperature $T = 5740$ K and Boltzmann's constant $\sigma = 5.67 \times 10^{-8}\ \mathrm{W/m^2/K^4}$.

10.32 Calculate the temperature of the solar surface if the radiant intensity at the sun's surface is 63 $\mathrm{MW/m^2}$. Stefan–Boltzmann constant $\sigma = 5.67 \times 10^{-8}\ \mathrm{W/m^2/K^4}$.

10.33 Calculate the amount of heat lost per second by radiation by a sphere 10 cm diameter at a temperature of 227°C when placed in an enclosure at 27°C ($\sigma = 5.67 \times 10^{-8}\ \mathrm{W/m^2/K^4}$)

[Nagarjuna University 2002]

10.34 A body emits most intense radiation at $\lambda_\mathrm{m} = 480$ nm. If the temperature of the body is lowered so that total radiation is now 1/16 of the previous value, what is the wavelength of the most intense radiation under new conditions? Wien's constant $b = 3 \times 10^{-3}$ m K.

10.2.4 Specific Heat and Latent Heat

10.35 The latest heat of fusion of a material is 6 kJ/mol and the heat capacity (C_p) in solid and liquid phases of the material is a linear function of temperature $C_\mathrm{p} = 30.6 + 0.0103\,T$, with units J/mol/K. How much heat is required to increase the temperature of 1 mol of the material from 20 to 200°C if the fusion phase transition occurs at 80°C?

[University of Manchester 2007]

10.36 The variation of the specific heat of a substance is given by the expression $C = A + BT^2$, where A and B are constants and T is Celsius temperature. Show that the difference between the mean specific heat and the specific heat at midpoint $T/2$ is $BT^2/12$.

10.37 The temperature of equal masses of three different liquids A, B and C is 12, 18 and 28°C, respectively. When A and B are mixed the temperature is 16°C. When B and C are mixed, it is 23°C. What would be the temperature when A and C are mixed?

[Indian Institute of Technology 1976]

10.38 A 3.0 g bullet moving at 120 m/s on striking a 50 g block of wood is arrested within the block. Calculate the rise of temperature of the bullet if (a) the block is fixed; (b) the block is free to move. The specific heat of lead is $0.031\ \mathrm{cal/g°C}$.

10.39 Calculate the difference in temperature between the water at the top and bottom of a 25 m high waterfall assuming that 15% of the energy of fall is spent in heating the water ($J = 4.18$ J/Cal).

[University of Durham]

10.40 A piece of lead falls from a height of 100 m on to a fixed non-conducting slab which brings it to rest. Show that its temperature immediately after the collision is raised by approximately 7.1 K (the specific heat of lead is 30.6 cal/kg 0°C between 0 and 100°C).

10.2.5 Thermodynamics

10.41 **(a)** Define

(i) an isobaric process
(ii) an isochoric process

(iii) an adiabatic process
(iv) an isothermal process

(b) Show that the work done on a gas during an adiabatic compression from initial conditions (P_1, V_1) to final conditions (P_2, V_2) is given by the equation

$$W = \frac{1}{\gamma - 1}(p_2 V_2 - p_1 V_1)$$

10.42 A sample containing 2 k mol of monatomic ideal gas is put through the cycle of operations as in Fig. 10.3. Find the values of T_A, T_B and V_C.

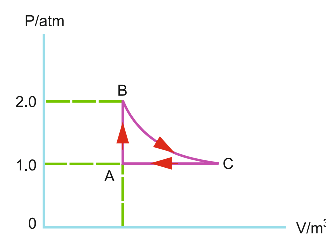

Fig. 10.3 A thermodynamic cycle

10.43 Show that for a monatomic ideal gas undergoing an adiabatic process, $PV^{5/3} = \text{constant}$.

10.44 **(a)** State the first law of thermodynamics, expressing the law in its infinitesimal form. Explain carefully each term used and note whether or not each term is path dependent.

(b) Show that the work done on a gas during an isothermal compression from an initial volume V_1 to a final volume V_2 is given by the equation

$$W = -nRT \ln\left(\frac{V_2}{V_1}\right)$$

(c) An ideal gas system, with an initial volume of $1.0\,\text{m}^3$ at standard temperature and pressure, undergoes the following three-stage cycle:
Stage 1 – an isothermal expansion to twice its original volume.
Stage 2 – a process by which its volume remains constant, its pressure returns to its original value and 10^4 J of heat is added to the system.
Stage 3 – an isobaric compression to its original volume, with 3×10^4 J of heat being removed from the system.

(i) How many moles of gas are present in the system?
(ii) Calculate the work done on the system during each of the three stages.

(iii) What is the resultant change in the internal energy over the whole three-stage cycle?
(At STP, temperature $= 0°\text{C} = 273.15\,\text{K}$ and pressure $= 1\,\text{atm} = 1.01 \times 10^5\,\text{Pa}$, $R = 8.31/\text{J/K/mol}$.)

10.45 The initial values for the volume and pressure of a certain amount of nitrogen gas are $V_1 = 0.06\,\text{m}^3$ and $p_1 = 10^5\,\text{N/m}^2$, respectively.

First, the gas undergoes an isochoric process (process 1–2), which triples the pressure; then it is followed by an isobaric process (process 2–3), which reduces the volume by a factor of three; finally, the volume of the gas is tripled by an isothermal process (process 3–4).

(a) Give the initial and final temperatures, T_1 and T_4, of the nitrogen gas if the temperature after the first (isochoric) process is $T_2 = 1083\,\text{K}$.
(b) Find the volume, V_4, and pressure, p_4, at the final state of the gas, then sketch the three processes in a p–V diagram.
(c) How much heat is gained by the nitrogen gas during the first (isochoric) process and how much heat is given away by the nitrogen gas during the second (isobaric process)? The amount of heat required to raise the temperature of 1 mol of nitrogen by 1 K while the gas pressure is kept constant is $c_\text{p} = 29.12\,\text{J/(mol K)}$.
(d) Find the change in the internal energy of the nitrogen gas by the end of the final process compared to the initial value.

[University of Aberystwyth, Wales]

10.46 When a gas expands adiabatically, its volume is doubled while its absolute temperature is decreased by a factor 1.32. Compute the number of degrees of freedom for the gas molecules.

10.47 A heat engine absorbs heat of 10^5 k cal from a source, which is at 127°C and rejects a part of heat to sink at 27°C. Calculate the efficiency of the engine and the work done by it.

[Osmania University 2004]

10.48 A reversible engine has an efficiency of 1/6. When the temperature of the sink is reduced by 62°C its efficiency gets doubled. Find the temperatures of the source and the sink.

10.49 Assuming that air temperature remains constant at all altitudes and that the variation of g with altitude is negligible

(a) show that the pressure P at an altitude h above sea level is given by $p = p_0 \exp(-Mgh/RT)$, where M is the molecular weight of the gas.
(b) show that $n = n_0 \exp(-Mgh/RT)$ where n is the number of molecules per unit volume.
(c) taking the average molecular weight of air to be 29 g, calculate the height at which the air pressure would be half the value at sea level.

10.50 **(a)** Write down the efficiency for a Carnot cycle as a function of

(i) the heat flows to and from the reservoirs and
(ii) the temperatures of the two reservoirs.

(b) Describe the working of an Otto engine and efficiency for the air standard Otto cycle as a function of temperature as well as volume. Start by sketching this cycle in a standard P–V diagram. Explain the four steps of this cycle in terms of associated temperature and volume changes as well as the heat exchanged with external reservoirs.
(c) Compare the Carnot and the Sterling cycle using P–V diagram.

10.51 **(a)** $1 \times 10^{-3}\,\text{m}^3$ of He at normal conditions ($p_0 = 1\,\text{bar}$, $T_0 = 0°\text{C}$) is heated to a final temperature of 500 K. What is the entropy change for

(i) an isobaric and
(ii) an isochoric process?
Use $C_P{}^{\text{He}} = 21\,\text{J/(mol K)}$ and $C_V{}^{\text{He}} = 12.7/(\text{mol K})$.

(b) Calculate the change in entropy ΔS_1 for 1 kg of water being heated from 0 to 50°C. Compare this change in entropy ΔS_2 for 0.5 kg of water at 0°C being mixed with 0.5 kg of water at a temperature of 100°C. Use $C_V{}^{H_2O} = 4.13 \times 10^3\,\text{J/(kg K)}$.

10.52 Consider a reversible isothermal expansion of an ideal gas in contact with the reservoir at temperature T, from an initial volume V_1 to a final volume V_2.

(a) What is the change in the internal energy of the system?
(b) Calculate the work done by the system.
(c) What is the amount of heat absorbed by the system?
(d) Find the change of entropy of the system.
(e) Find the change of the entropy of the system plus the reservoir.

10.53 Internal energy, heat, enthalpy, work, and the Gibbs free energy (Gibbs function) are all measured in units of joules.

(a) What is the difference between these forms of energy? Write down the equations relating these forms of energy.
(b) Which of the above are state variables? What properties distinguish a state variable from other variables?

10.2.6 Elasticity

10.54 **(a)** A 100 MPa force is applied to the surface of a material (surface area, $1\,\text{m}^2$) that exerts a shear across the material (Fig. 10.4). The sample has a thickness of 10 cm and causes the surface to be displaced by 0.1 cm. What is the shear modulus of the material?

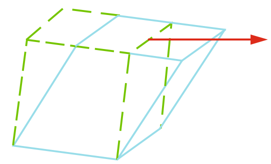

Fig. 10.4 Shear deformation

(b) What is the bulk modulus of a material if a 100 MPa increase in pressure causes a 1% reduction in its volume?

[University of Manchester 2008]

10.55 A wire has a length of 10 m and a cross-sectional area of 20 mm^2. When a 20 kg block of lead is attached to it, it stretches by 2.5 cm. Find

(i) the stress
(ii) the strain
(iii) Young's modulus for the wire

10.56 Show that the isothermal elasticity $K_T = P$ and adiabatic elasticity $K_H = \gamma P$.

10.57 For a given material, the Young's modulus is 2.5 times the rigidity modulus. Find its Poisson's ratio.

10.58 A 1.2 m long metal wire is fixed securely at both ends to two solid supports so that the wire is initially horizontal. When a 29 g mass is attached from the midpoint of the wire, the midpoint is observed to move down by 20 mm. If the diameter of the wire is 0.1 mm, estimate the Young's modulus for the wire material.

[The University of Wales, Aberystwyth 2004]

10.59 The rubber cord of a catapult has a cross-sectional area of 2 mm and an initial length of 0.2 m and is stretched to 0.25 m to fire a small object of mass 15 g. If the Young's modulus is $Y = 6 \times 10^8\ N/m^2$, what is the initial velocity of the object that is released?

10.60 A 10 kg object is whirled in a horizontal circle on the end of a wire. The wire is 0.3 m long and has a cross-section $10^{-6}\ m^2$ and has the breaking stress $4.8 \times 10^7\ N/m^2$. What is the maximum angular speed the object can have?

10.61 A steel wire is fixed at one end and hangs freely. The breaking stress for steel is equal to $7.8 \times 10^8\ N/m^2$ and its density is 7800 kg/m^2. Find the maximum length of the wire so that it does not break under its own weight.

10.2.7 Surface Tension

10.62 If the surface tension of the liquid–gas interface is 0.072 N/m, the density is 1 kg/L and the radius of the capillary is 1 mm, to what height will the liquid rise up the capillary?

[University of Manchester 2007]

10.63 A mole of gaseous molecules in a bubble obeys the ideal gas law. What is the volume of the bubble at a 100 m depth of water if the temperature is 293 K, the atmospheric pressure is 101 kPa, density of water is $1000\,\text{kg/m}^3$ and the ideal gas constant is 8.314 J/mol/K.

[University of Manchester 2008]

10.64 Let n droplets each of radius r coalesce to form a large drop of radius R. Assuming that the droplets are incompressible and S is the surface tension calculate the rise in temperature if c is the specific heat and ρ is the density.

10.65 A soap bubble of surface tension 0.03 N/m is blown from 1 cm radius to 5 cm radius. Find the work done.

10.66 A small hollow vessel which has a small hole in it is immersed in water to a depth of 45 cm before any water penetrates into the vessel. If the surface tension of water is 0.073 N/m, what should be the radius of the hole?

10.67 What will be the depth of water at which an air bubble of radius 0.3×10^{-3} m may remain in equilibrium (surface tension of water = 0.072 N/m and $g = 9.8\,\text{m/s}^2$)?

10.68 A capillary tube of radius 0.2 mm and of length 6 cm is barely dipped in water. Will the water overflow through the capillary? If not what happens to the meniscus (surface tension of water = 0.073 N/m and angle of contact = 0°)?

10.69 A soap bubble of radius 2.0 cm is charged so that the excess of pressure due to surface tension is neutralized. If the surface tension is 0.03 N/m, what is the charge on the bubble?

10.70 Two soap bubbles with radii r_1 and r_2 coalesce to form a bigger bubble of radius r. Show that $r = \sqrt{r_1^2 + r_2^2}$.

10.3 Solutions

10.3.1 Kinetic Theory of Gases

10.1 The mean free path λ of a gas molecule is the average distance travelled by the molecule between successive collisions.

$$\lambda = x/N \tag{1}$$