9.2 Problems

9.2.1 Bernoulli's Equation

9.1 The radius of a water pipe decreases from 10 to 5 cm. If the average velocity in the wider portion is 4 m/s, find the average velocity in the narrower region.

9.2 Verify if the continuity equation for steady incompressible flow is satisfied for the following velocity components:

$$v_x = 3x^2 - xy + 2z^2,\ v_y = 2x^2 - 6xy + y^2,\ v_z = -2xy - yz + 2y^2$$

9.3 Air streams horizontally across an aeroplane wing of area 4 m^2, weighing 300 kg. The air speed is 70 and 55 m/s over the top surface and the bottom surface, respectively. Find **(a)** the lift on the wing; **(b)** the net force on it.

9.4 A venturi meter has a pipe diameter of 20 cm and a throat diameter of 10 cm. If the water pressure in the pipe is 60,000 Pa and in the throat is 45,000 Pa, calculate the rate of flow of water in m^3/s.

9.5 A pitot tube which is used to determine the speed of an aircraft relative to air is mounted on the wing of a plane. The tube contains alcohol of density 810 kg/m^3 and registers a level difference of 15.0 cm. Assuming that the density of air at NTP is 1.293 kg/m^3, find the plane's speed in km/h relative to the air.

9.6 A garden sprinkler has 80 small holes each 2.5 mm^2 in area. If water is supplied at the rate of 2×10^{-3} m^3/s, find the average velocity of the spray.

9.7 For steady, incompressible flow which of the following values of velocity components are possible?

(a) $v_x = 3xy + y^2,\ v_y = 5xy + 2x$

(b) $v_x = 3x^2 + y^2,\ v_y = -6xy$

9.8 If the speed of flow past the lower surface of the wing of an aeroplane is 100 m/s, what speed of flow over the upper surface would give a pressure difference of 1000 Pa? Assume an air density of 1.293 kg/m^3.

9.9 A venturi meter has a pipe diameter of 4 cm and a throat diameter of 2 cm. The velocity of water in the pipe section is 10 cm/s. Find **(a)** the pressure drop; **(b)** the velocity in the throat.

9.10 Water is observed to flow through a capillary of diameter 1.0 mm with a speed of 3 m/s. Viscosity of water in CGS units is

(a) 0.018 at 0°C

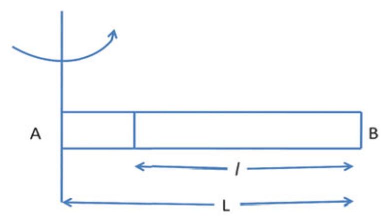
(b) 0.008 at 30°C

(c) 0.004 at 70°C

Calculate the Reynold's number and test at which of these three temperatures is the flow likely to be streamlined. Assume that for Reynold's number $R < 2200$ flow is steady.

9.11 A horizontal tube AB of length L, open at A and closed at B, is filled with an ideal fluid. The end B has a small orifice. The tube is set in rotation in the horizontal plane with angular velocity ω about a vertical axis passing through A, Fig. 9.2. Show that the efflux velocity of the fluid is given by $v = \omega l\sqrt{\frac{2L}{l} - 1}$ where l is the length of the fluid.

Fig. 9.2



9.12 A pitot tube, Fig. 9.3, is mounted along the axis of a gas pipeline of cross-sectional area A. Calculate the rate of flow of the gas across the section of the pipe if h is the difference in the liquid column and ρ_L and ρ_g are the densities of the liquid and the gas, respectively.

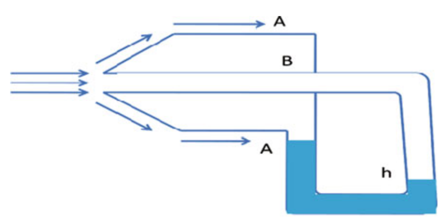


Fig. 9.3 Pitot tube

9.13 Water flows in a horizontal pipe of varying cross-section. Two manometer tubes fixed on the pipe, Fig. 9.4, at sections A_1 and A_2 indicate a difference Δh in the water columns. Calculate the rate of flow of water in the pipe.

9.14 A cylinder filled with water of volume V is fitted with a piston and is placed horizontally. There is a small hole of cross-sectional area s at the other end of

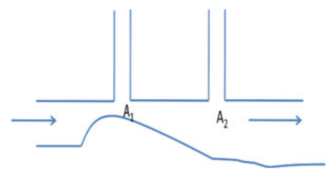


Fig. 9.4

the cylinder, s being much smaller than the cross-sectional area of the piston (Fig. 9.5). Show that the work to be done by a constant force acting on the piston to squeeze all water from the cylinder in time t is given by

$$W = \frac{1}{2}\frac{\rho V^3}{s^2 t^2}$$

where ρ is the density of water. Neglect friction and viscosity.

Fig. 9.5

9.15 A cylindrical vessel with water is rotated about its vertical axis with a constant angular velocity ω. Show that

(a) the water pressure distribution along its radius is given by $P = P_0 + \frac{1}{2}\rho\omega^2 r^2$, where ρ is the density of water and P_0 is the pressure at the central point.

(b) Show that the figure of revolution of water is a paraboloid.

9.16 A manometer is fixed to a water tap. When the valve is closed the manometer shows the reading of 3.5×10^5 Pa. When the valve is open the reading becomes 3.1×10^5 Pa. Find the speed of water.

9.2.2 Torricelli's Theorem

9.17 A water container is filled up to a height H. A small hole is punched at the side wall at a depth h below the water surface. Show **(a)** that the distance from the foot of the wall at which the stream strikes the floor is $2\sqrt{h(H-h)}$; **(b)** the second hole through which the second stream has the same range must be punched at a depth $H - h$.

9.18 In prob. (9.17) show that the hole must be punched at a depth $h = H/2$ for maximum range and that this maximum distance is H.

9.19 A large tank is filled with water at the rate of $70\,\text{cm}^3/\text{s}$. A hole of cross-section $0.25\,\text{cm}^2$ is punched at the bottom of the tank. Find the maximum height to which the tank can be filled.

9.20 A tank of cross-sectional area A is filled with water up to a height h_1. Water leaks out from a small hole of area 'a' at the bottom. Find the time taken for the water level to decrease from h_1 to h_2.

9.21 A large tank is filled with water. The total pressure at the bottom is 3.0 atm. If a small hole is punched at the bottom what is the velocity of efflux?

9.22 Two tanks with a large opening are filled with a liquid. A hole of cross-sectional area A_1 is punched in tank 1 and another of cross-sectional area A_2 in tank 2 at depths h_1 and h_2, respectively. If $A_1 = 2A_2$ and the volume flux is identical, then what should be the ratio h_1/h_2?

9.23 A wide container with a small orifice in the bottom is filled with water and kerosene. If the water column measures 60 cm and kerosene column 40 cm, calculate the efflux velocity of water. Take the specific gravity of water as 1.0 and kerosene as 0.8 and neglect viscosity.

9.24 A wide vessel filled with water is punched with two holes on the opposite side each with cross-sectional area of $1.0\,\text{cm}^2$. If the difference in height of the holes is 51 cm, calculate the resultant force of reaction of the water flowing out of the vessel.

9.2.3 Viscosity

9.25 Water is conveyed through a tube 8 cm in diameter and 4 km in length at the rate of 120 l/min. Calculate the pressure required to maintain the flow. Coefficient of viscosity of water, $\eta = 0.001$ SI units. 1 atm $= 1.013 \times 10^5$ Pa.

9.26 Two capillary tubes AB and BC are joined end to end at B. AB is 16 cm long and of diameter 0.4 cm. BC is 4 cm long and of diameter 0.2 cm. The composite tube is held horizontally as in Poiseuille's experiment, with A connected to a vessel of water giving a constant head of 3 cm and C open to air. Calculate the pressure difference between B and C.

9.27 Two raindrops fall through air with terminal velocity of v_T cm/s. If the drops coalesce what will be the new terminal velocity?

9.28 $Q\,\text{cm}^3$ of water flows per second through a horizontal tube of uniform bore of radius r and of length l. Another tube of half the length but radius $2r$ is connected in parallel to the same pressure head. What will be the total quantity of water flowing per second through these two tubes?

9.29 In prob. (9.28) if the tubes are connected in series then what quantity will flow through the composite tube?

9.3 Solutions

9.3.1 Bernoulli's Equation

9.1 From continuity equation

$$A_1 v_1 = A_2 v_2$$

$$\therefore \quad v_2 = \frac{A_1 v_1}{A_2} = \frac{\pi r_1^2 v_1}{\pi r_2^2} = \frac{10^2 \times 4}{5^2} = 16\,\mathrm{m/s}$$

9.2 $v_x = 3x^2 - xy + 2z^2$

$$v_y = 2x^2 - 6xy + y^2$$

$$v_z = -2xy - yz + 2y^2$$

$$\therefore \quad \frac{\partial v_x}{\partial x} = 6x - y; \ \frac{\partial v_y}{\partial y} = -6x + 2y; \ \frac{\partial v_z}{\partial z} = -y$$

$$\nabla \cdot \boldsymbol{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = (6x - y) + (-6x + 2y) - y = 0$$

Thus the continuity equation for steady incompressible flow is satisfied.

9.3 Pressure difference across the wing

$$\Delta p = \frac{1}{2}\rho(v_1^2 - v_2^2)$$
$$= \frac{1}{2} \times 1.293 \times (70^2 - 55^2) = 1212\,\mathrm{Pa}$$

(a) Lift = (pressure difference) (area)
= 1212 × 4 = 4848 N

(b) Net force = Lift − Weight of plane

= 4848 − (300 × 9.8)
= 1908 N in the upward direction

9.4 $\Delta P = \frac{1}{2}\rho v^2 \left(\frac{A^2}{a^2} - 1\right)$

$$\frac{A}{a} = \frac{\pi R^2}{\pi r^2} = \left(\frac{10}{5}\right)^2 = 4$$