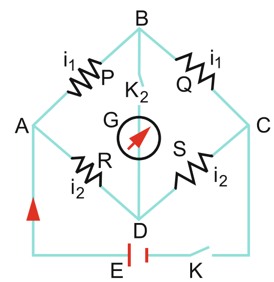
Fig. 12.2



Kirchhoff's Laws

1. *Junction theorem*: At any junction of an electric network (branched circuit), the algebraic sum of the currents flowing towards that junction is zero, i.e. the total current flowing towards the junction is equal to the total current flowing away from it:

$$\sum i = 0 \tag{12.16}$$

2. *The loop theorem*: The sum of the changes in the potential, encountered in traversing a loop (closed circuit) in a particular direction (clockwise or counterclockwise), is zero.

   (i) If a resistor is traversed in the direction of current, the change in the potential is $-iR$, while in the opposite direction it is $+iR$.
   (ii) If a seat of emf is traversed in the direction of the emf, the change in potential is $+\xi$, while in the opposite direction it is $-\xi$.

12.2 Problems

12.2.1 Resistance, EMF, Current, Power

12.1 All resistors in Fig. 12.3 are in ohms. Find the effective resistance between the points A and B.

[Indian Institute of Technology 1979]

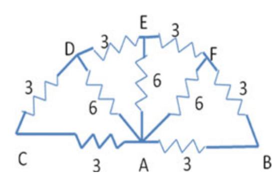


Fig. 12.3

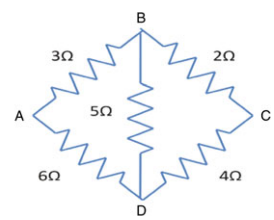
12.2 If a copper wire is stretched to make it 0.1% longer, what is the percentage change in its resistance?

[Indian Institute of Technology 1978]

12.3 The equivalent resistance of the series combination of two resistors is p. When they are joined in parallel, the equivalent resistance is q. If $p = nq$, find the minimum possible value of n.

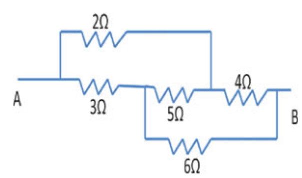
12.4 Five resistors are connected as in Fig. 12.4. Find the equivalent resistance between A and C.

Fig. 12.4



12.5 Five resistors are arranged as in Fig. 12.5. Find the effective resistance between A and B.

Fig. 12.5



12.6 Each of the resistances in the network, Fig. 12.6, is equal to R. Find the resistance between the terminals A and B.

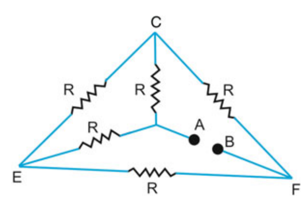
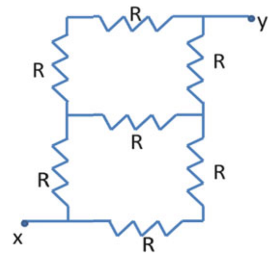


Fig. 12.6

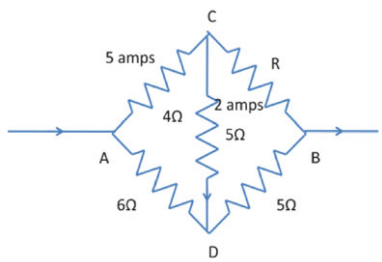
12.7 Find the equivalent resistance between the terminals x and y of the network shown in Fig. 12.7.

Fig. 12.7



12.8 A circuit is set up as shown in Fig. 12.8, in which certain resistors are known; the current in some of the branches has been measured by ammeter.
Calculate

Fig. 12.8



(i) The resistance R in CB
(ii) The potential difference between A and B
(iii) The heat developed per second between A and B.

[Northern Universities of U.K.]

12.9 Five resistances are connected as shown in Fig. 12.9. Find the equivalent resistance between the points A and B.

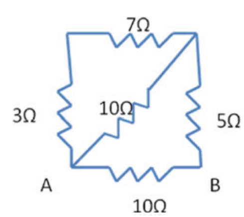


Fig. 12.9

12.10 A network of infinite resistors is shown in Fig. 12.10. Find the effective resistance of the network between terminal points A and B.

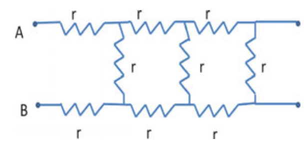


Fig. 12.10

12.11 What equal length of an iron wire and a constantan wire, each 1 mm diameter, must be joined in parallel to give an equivalent resistance of 2 Ω? (resistivity of iron and constantan are 10 and 49 μ Ω cm, respectively).

[University of London]

12.12 A coil of wire has a resistance of 20 Ω at 25°C and 25.7 Ω at 100°C. Calculate the temperature coefficient.

[University of London]

12.13 A wire of resistance 0.1 Ω/cm is bent to form a square ABCD of side 10 cm. A similar wire is connected between the corners B and D to form the diagonal BD. Find the effective resistance of this combination between A and C. A battery of negligible internal resistance is connected across A and C. Calculate the total power dissipated.

[Indian Institute of Technology 1971]

12.14 A 60 W-100 V tungsten lamp has a resistance of 20 Ω at air temperature (0°C). What is the rise in temperature of the filament under normal working conditions? The temperature coefficient of resistance of tungsten is 0.0052/°C.

[University of London]

12.15 A skeleton cube is made of wires soldered together at the corners of the cube, the resistance of each wire being 10 Ω. A current of 6 A enters at one corner

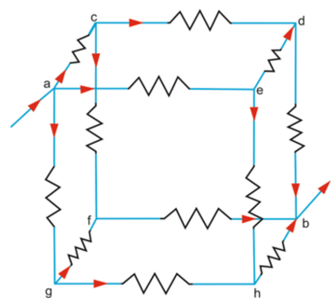


Fig. 12.11

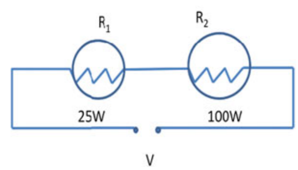
and leaves the diagonally opposite corner. Calculate the equivalent resistance of the network and the fall of potential across it (Fig. 12.11).

[University of London]

12.16 A 25 W bulb and a 100 W bulb are joined in series and connected to the mains (Fig. 12.12). Which bulb will glow brighter?

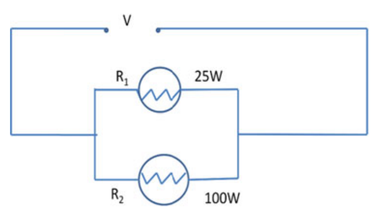
[Indian Institute of Technology 1979]

Fig. 12.12



12.17 A 25 W bulb and a 100 W bulb are joined in parallel and connected to the mains (Fig. 12.13). Which bulb will glow brighter?

Fig. 12.13

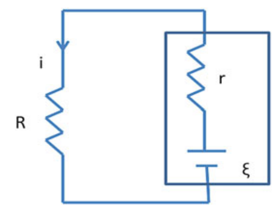


12.18 Three resistors of 4, 6 and 12 Ω are connected together in parallel. This parallel arrangement is then connected in series with a 1 and 2 Ω resistors. If a potential difference of 120 V is applied across the end of the circuit, what will be the potential drop across the part of the circuit connected in parallel?

[University of Newcastle]

12.19 In the given circuit, Fig. 12.14, show that the maximum power delivered to the external resistor R is $P = \xi^2/4r$ where r is the internal resistance of the battery of emf ξ.

Fig. 12.14



12.20 Two heater coils of power P_1 and P_2 (resistance R_1 and R_2, respectively) take individually time t_1 and t_2 to boil a fixed quantity of water. Find the time t in terms of t_1 and t_2, when they are connected to the mains in **(a)** series and **(b)** parallel to boil the same quantity of water.

12.21 A battery having an emf 24 V and a resistance 2 Ω is connected to two resistances arranged (a) in series and (b) in parallel. If the resistances are 4 and 6 Ω, respectively, calculate the watts expended in each resistance, in each of the two cases.

[University of London]

12.22 Power at the rate of 10^4 kW has to be supplied through 30 km of cable of resistance 0.7 Ω/km. Find the rate of energy loss, if the power is transmitted at 100 kV.

[University of Dublin]

12.23 Three equal resistors, connected in series across a source of emf together, dissipate 10 W of power. What would be the power dissipated, if the same resistors are connected in parallel across the same source of emf?

[Indian Institute of Technology 1972]

12.24 A heater is designed to operate with a power of 1000 W in a 100 V line. It is connected in combination with a resistance of 100 Ω and a resistance R to a 100 V mains as shown in Fig. 12.15. What should be the value of R so that the heater operates with a power of 62.5 W?

[Indian Institute of Technology 1978]

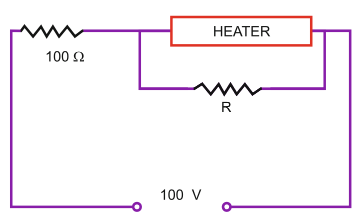


Fig. 12.15

12.2.2 Cells

12.25 Twelve cells having the same emf are connected in series and are kept in a closed box. Some of the cells are wrongly connected. This battery is connected in series with an ammeter and two cells identical with others. The current is 3 A when the cells and the battery aid each other and is 2 A when the cells and the battery oppose each other. How many cells in the battery are wrongly connected?

[Indian Institute of Technology 1976]

12.26 Let there be m rows of cells with each row containing n cells in series, each cell having internal resistance r. Show that maximum current in the external resistance R will be available when $R = nr/m$.

12.27 Two cells with the same emf and internal resistances r_1 and r_2 are connected in series to an external resistance R. Find the value of R so that the potential difference across the first cell is zero.

12.28 A certain circuit consists of three resistors connected in parallel across 200 V mains. The rate of production of heat in them is in the ratio of 5:3:2 and together they generate heat at the rate of 1 kW h in 2 h. Find the power used if the three resistances are connected in series across 248 V mains.

[Northern Universities of UK]

12.29 A battery of emf 2 V and internal resistance 0.1 Ω is being charged with a current of 5 A. In what direction will the current flow inside the battery? What is the potential difference between the two terminals of the battery?

[Indian Institute of Technology 1980]

12.30 A 6 V battery of negligible internal resistance is connected in series with a 3 Ω and a 5 Ω resistance. A further resistance of 2 Ω is connected in parallel with the 5 Ω resistance.

(a) Find the current flowing in each resistance.
(b) Find the power dissipated in each resistance.
(c) Compare the total value for the power dissipated in the resistances with the value for the power supplied by the battery.

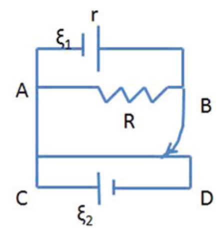
[University of Newcastle]

12.2.3 Instruments

12.31 The terminals of a cell are connected to a resistance and the fall of potential across R is balanced against the fall across the potentiometer wire. When R is 20 and 10 Ω, respectively, the corresponding lengths on the potentiometer are 150 and 120 cm. Calculate the internal resistance of the cell (Fig. 12.16).

[University of London]

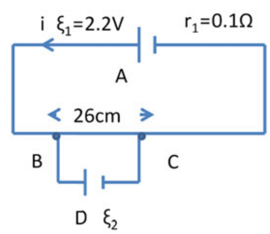
Fig. 12.16



12.32 A thin uniform wire 50 cm long and of 1 Ω resistance is connected to the terminals of an accumulator of emf 2.2 V and the internal resistance 0.1 Ω (Fig. 12.17). If the terminals of another cell can be connected to two points 26 cm apart on the wire without altering the current in the wire, what is the emf of the cell?

[Northern Universities of UK]

Fig. 12.17



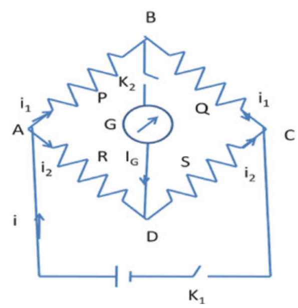
12.33 In a Wheatstone bridge, four resistors P, Q, R and S are arranged as in Fig. 12.18. Show that

(a) condition for null deflection in the galvanometer G is $\dfrac{P}{Q} = \dfrac{R}{S}$

(b) if a non-zero current i_g flows through the galvanometer then

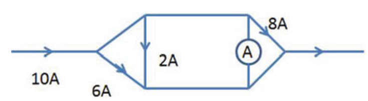
$$\frac{i_g}{i} = \frac{QR - PS}{G(Q+S) + (P+R)(G+Q+S)}$$

Fig. 12.18



12.34 Figure 12.19 shows a network carrying various currents. Find the current through the ammeter A.

Fig. 12.19



12.35 A galvanometer together with an unknown resistance in series is connected across two identical batteries, each of 1.5 V. When the batteries are connected in series, the galvanometer records a current of 1 A and when the batteries are in parallel the current is 0.6 A. What is the internal resistance of the battery? [Indian Institute of Technology 1973]

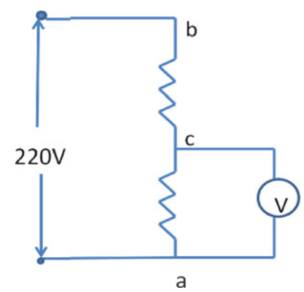
12.36 In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

12.37 A potentiometer wire of length 100 cm has a resistance of 10 Ω. It is connected in series with a resistance and cell of emf 2 V and of negligible internal resistance. A source of emf 10 mV is balanced against a length of 40 cm of the potentiometer wire. What is the value of the external resistance? [Indian Institute of Technology 1976]

12.38 A potential difference of 220 V is maintained across a 12,000 Ω rheostat ab (see Fig. 12.20). The voltmeter V has a resistance of 6000 Ω and point c is at one-fourth the distance from a to b. What is the reading in the voltmeter? [Indian Institute of Technology 1977]

12.39 The balance point in a meter bridge experiment is obtained at 30 cm from the left. If the right-hand gap contains resistance of 3.5 Ω, what is the value of the resistance in the left-hand gap?

Fig. 12.20



12.2.4 Kirchhoff's Laws

12.40 A moving coil meter has a full scale reading of 1 mA and a resistance of 80 Ω. How could the meter be used to measure (a) 100 mA full scale and (b) 80 V full scale?

[University of Manchester]

12.41 A pocket voltmeter has a resistance of 120 Ω. What will it read when connected to a battery of emf 9 V and an internal resistance 15 Ω?

[University of Oxford]

12.42 When a galvanometer is shunted with a 1 Ω resistance, only 1% of the main current passes through it. What is the resistance of the galvanometer?

12.43 A 10 V battery, having an internal resistance of 1.0 Ω, is joined in parallel with another of 20 V and internal resistance of 2 Ω. Calculate the current flowing through each battery, and the rates of expenditure of energy in the two batteries and the 30 Ω resistance (Fig. 12.21).

[University of Cambridge]

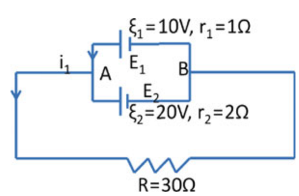


Fig. 12.21

12.44 A battery of emf 1 V and internal resistance 2 Ω is connected to another battery of emf 2 V and internal resistance 1 Ω in parallel with an external resistance of 10 Ω. Find the currents?

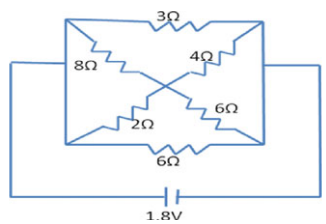
12.45 The electric current of 5 A is divided into three branches, forming a parallel combination. The lengths of the wire in the three branches are in the ratio 2, 3 and 4; their diameters are in the ratio 3, 4 and 5. Find the current in each branch, if the wires are of the same material.

[Indian Institute of Technology 1975]

12.46 Calculate the current through the 3 Ω resistor and the power dissipated in the entire circuit shown in Fig. 12.22. The emf of the battery is 1.8 V and its internal resistance is 2/3 Ω.

[Indian Institute of Technology 1971]

Fig. 12.22



12.47 A series circuit is made up of two cells of emf 1.5 and 3 V, respectively, and two coils each of resistance 10 Ω, arranged in the order cell, coil, cell, coil. The centre points of the two coils are joined by a sensitive galvanometer which shows no deflection. If the cell of 1.5 V has an internal resistance of 5 Ω, calculate the internal resistance of the other cell.

[University of London]

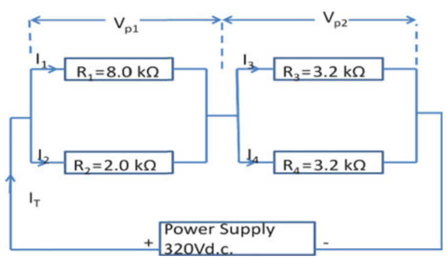
12.48 **(a)** Figure 12.23 shows a series parallel resistive circuit connected to a 320 V d.c. supply. For the circuit shown work out the following:

- **(i)** The total equivalent resistance R_T of the circuit and the total current I_T.
- **(ii)** The voltage V_{p1} across resistors R_1 and R_2.
- **(iii)** The voltage V_{p2} across resistors R_3 and R_4.
- **(iv)** The currents I_1 and I_3.
- **(v)** The total power for the whole circuit and the power dissipated in resistor R_3.

(b) Consider the case of a heavy duty battery whose emf $\xi = 24$ V and internal resistance of $r = 0.01\,\Omega$. If the terminals were accidentally short circuited by a heavy copper bar of negligible resistance what power would be dissipated within the battery?

[University of Aberystwyth, Wales 2005]

Fig. 12.23



12.49 A battery with emf $\xi = 24\,\mathrm{V}$ has internal resistance $r = 0.02\,\Omega$. A load resistor $R = 140\,\Omega$ is connected to the terminals of a battery:

(i) Find the current flowing in the circuit under load conditions.
(ii) Find the terminal voltage of the battery under load conditions.
(iii) Find the power dissipated in the resistor R and in the battery's internal resistance r.
(iv) Find the open circuit voltage of the battery under no load conditions and explain your answer.

12.50 Figure 12.24 shows a series parallel resistive circuit connected to a dc supply.

(i) Find the total equivalent resistance of the circuit.
(ii) Find currents I_T, I_1 and I_3.
(iii) Find voltages V_1, V_2 and V_3.
(iv) Find power dissipated in resistor R_5 and the total power dissipated in the circuit.

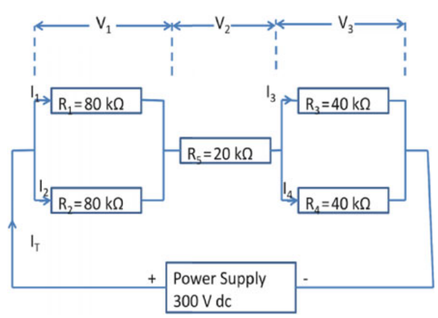
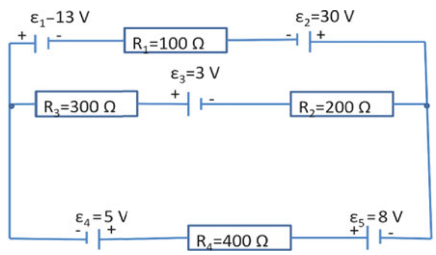


Fig. 12.24

12.51 Apply Kirchhoff's rules to the circuit shown in Fig. 12.25 to produce three equations with three unknown branch currents. You do not have to solve these equations for individual current.

[University of Aberystwyth, Wales 2008]

Fig. 12.25

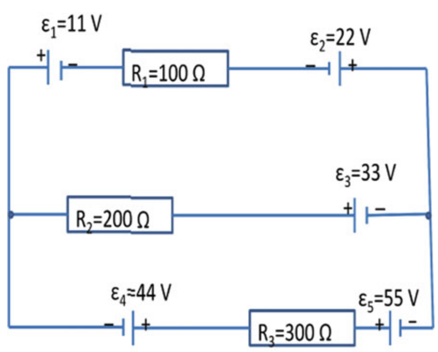


12.52 **(i)** State Kirchhoff's two rules

(ii) Apply Kirchhoff's rules to the circuit shown in Fig. 12.26 to produce three equations with three unknown branch currents. You do not have to solve these equations for individual current.

[University of Aberystwyth, Wales 2007]

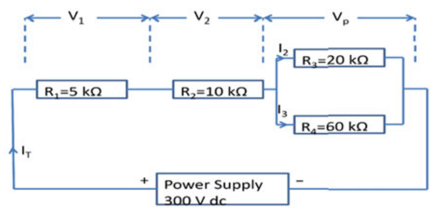
Fig. 12.26



12.53 Figure 12.27 shows a series parallel resistive circuit connected to a dc supply. For the circuit shown work out the following:

(i) The voltages V_1, V_2 across resistors R_1 and R_2.
(ii) The voltage V_p across resistors R_3 and R_4.
(iii) The currents I_T, I_2 and I_3.

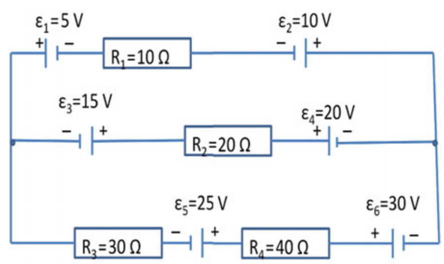
Fig. 12.27



12.54 Apply Kirchhoff's rules to the circuit shown in Fig. 12.28 to produce three equations with three unknown branch currents. You do not have to solve these equations for individual I.

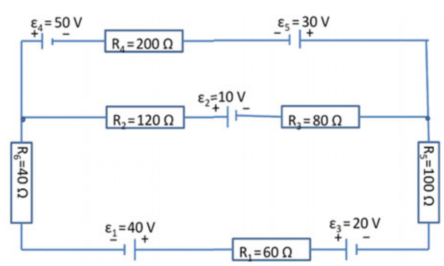
[University of Aberystwyth, Wales 2006]

Fig. 12.28



12.55 Apply Kirchhoff's rules to the circuit shown in Fig. 12.29 and present the simultaneous equations necessary to calculate the currents in each of the branches of the circuit. You do not have to solve these equations for the branch currents.

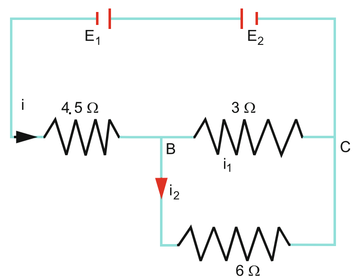
Fig. 12.29



12.56 In the circuit shown in Fig. 12.30, the cells E_1 and E_2 have emfs 4 and 8 V and internal resistances 0.5 and 1 Ω, respectively. Calculate the current in each resistor and the potential difference across each cell.

[Indian Institute of Technology 1973]

Fig. 12.30



12.3 Solutions

12.3.1 Resistance, EMF, Current, Power

12.1 In the segment ACD, the two 3 Ω resistances give 6Ω, which with 6 Ω in parallel yields $\frac{6 \times 6}{6+6} = 3\,\Omega$. This together with 3 Ω, across DE in series, gives 6 Ω which together with 6 Ω across AE in parallel gives 3 Ω. By a similar reasoning resistance along AFB is 6 Ω, which with 3 Ω, across AB in parallel yields the effective resistance across AB:

$$R_{\mathrm{AB}} = \frac{6 \times 3}{6+3} = 2\,\Omega$$

12.2 $R = \frac{\rho l}{A} = \frac{\rho l^2}{Al} = \frac{\rho l^2}{v_0}$

where v_0 is the constant volume. Change in the resistance

$$\Delta R = 2\rho l \frac{\Delta l}{v_0}$$

$$\therefore \quad \frac{\Delta R}{R} \equiv 2\frac{\Delta l}{l} \equiv 2 \times \frac{0.1}{100} = \frac{0.2}{100} \quad \text{or} \quad 0.2\%$$