Lenz law: The reason for the minus sign in (13.21) is given by Lenz law which states 'the induced current will appear in such a direction that it opposes the change that produced it'.

If the coil forms a closed circuit then only the induced current can be present, otherwise in the case of an open circuit one can only speak of induced emf and its direction.

In a coil of N turns

$$\xi = -N \frac{\Delta\varphi}{\Delta t} \tag{13.22}$$

If the circuit is complete current will appear and will be given by

$$i = \frac{\xi}{R} = -\frac{N}{R}\frac{\Delta\varphi}{\Delta t} \tag{13.23}$$

where R is the resistance of the circuit. The corresponding charge flowing is given by

$$\Delta q = i\Delta t = -\frac{N}{R}\Delta\varphi \tag{13.24}$$

Consider a conducting rod of length l moving sideways in the plane of paper over a U-shaped metal frame at constant speed v at right angles to a uniform magnetic field of flux density B into the paper. Then the emf induced across the ends of the rod is given by

$$\xi = -Blv \tag{13.25}$$

13.2 Problems

13.2.1 Motion of Charged Particles in Electric and Magnetic Fields

13.1 Calculate the cyclotron frequency to accelerate alpha particles in a magnetic field of 10^4 G. The mass of $^4He_2$ is 4.002603 u.

13.2 If the pole pieces of a cyclotron are 50 cm in diameter, a flux density of 15,000 G, find approximate values for the energies to which (a) protons and (b) α-particles could be accelerated. What oscillator frequency would be required in each case?

[University of London]

13.3 In a mass spectrometer, the velocity filter employs electric field E and a perpendicular magnetic field B. The deflection magnetic field, perpendicular to a beam is B'. Ions with similar charges q and mass numbers m_1 and

m_2 pass through the filter. Show that the separation between them will be $\dfrac{2E(m_2 - m_1)}{qB\,B'}$

[Indian Administrative Services]

13.4 A singly charged particle of known velocity 2.5×10^7 m/s but unknown mass moves in a bubble chamber in a circular path of radius 0.2 m in a field of 0.2 T acting perpendicular to the path. Determine the mass of the particle and identify it.

13.5 A particle of mass m and charge q travelling with a velocity v along the x-axis enters a uniform electric field $\boldsymbol{E}$ directed along the y-axis. Show that the trajectory will be a parabola.

13.6 Find the radius of a circular orbit of an electron of energy 5 keV in a field of 10^{-2} T.

[Osmania University 1992]

13.7 An electric field of 1500 V/m and a magnetic field act on an electron moving with a speed of 3000 m/s. If the resultant field is to be zero what should be the strength of the magnetic field (in Wb/m^2).

[Osmania University 1987]

13.8 An electron moves in a circle of radius 1.9 m in a magnetic field of 3×10^{-5} T. Calculate (a) the speed of electrons and (b) time taken to move round the circle.

13.9 A cyclotron is powered by a 50,000 V 5 Mc/s radio frequency source. If its diameter is 1.524 m, what magnetic field satisfies the resonance condition for deuterons?. Also what energies will they attain? Take the mass of deuteron as 2.0141 u.

13.10 Deuterons are accelerated in a conventional cyclotron. Given the resonance frequency was 11.5 Mc/s and radius of the dee 30″, calculate the resonance frequency of protons and the maximum energy of protons that is obtainable using the same magnetic field. (In a cyclotron the vacuum chamber is partitioned into two D-shaped components)

13.11 A cyclotron has a magnetic field of 15,000 G. The extraction radius is 50 cm. Calculate (a) the frequency of the *rf* necessary for accelerating deuterons and (b) the energy of the extracted beam.

[University of Liverpool]

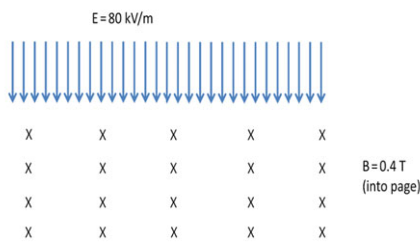
13.12 In the Bohr model of hydrogen atom the electron revolves in a circular orbit of radius 0.53 Å with a time period of 1.5×10^{-16} s. Find the corresponding current.

13.13 As shown in Fig. 13.1, a beam of particles of charge q enters a region where an electric field is uniform and directed downwards. Its value is 80 kV/m. Perpendicular to $\boldsymbol{E}$ and directed into the page is a magnetic field $\boldsymbol{B} = 0.4\,\mathrm{T}$.

(i) If the speed of the particles is properly chosen, the particles will not be deflected by these crossed electric and magnetic fields. What speed is selected in this case?

(ii) If the electric field is cut off and the same magnetic field is maintained, the charged particles move in the magnetic field in a circular path of radius 1.14 cm. Determine the ratio of the electric charge to the mass of the particles.

Fig. 13.1



13.14 **(a)** Write an expression for the force acting on a charge q moving with velocity v in an electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{B}$.

(b) A charged particle of mass m and charge q is accelerated through a potential difference of V and then injected into a region with a magnetic field B perpendicular to the plane in which the charge moves. Derive an expression for the radius of curvature, r, of the path of the particle when in the magnetic field.

[University of Durham 2004]

13.15 In a certain mass spectrometer the magnetic field has a magnitude of 0.2 T. It is intended that this spectrometer be used to separate two isotopes of uranium, $^{235}_{92}$U(mass 3.90×10^{-25} kg) and $^{238}_{92}$U(mass 3.95×10^{-25} kg). In order to be separated the radii of curvature described by singly charged (charge $+e$) ions must differ by 2 mm. Calculate the electric potential through which the ions must be accelerated in order to achieve this.

13.16 **(a)** Write down an expression for the force experienced by a particle with charge q moving with velocity v in a magnetic field B. Under what circumstances does the particle mass m, of describe a circle of radius r?

(b) A coil of cross-sectional area A composed of N turns is placed perpendicular to a magnetic field which is uniform in space, with a strength that varies in time according to $B = B_0 \cos(15t)$. Calculate the electromotive force induced in the coil.

[University of Manchester 2007]

13.17 A water droplet of radius 1 μm is charged such that the electric field on its surface is 5.8 mV/m. (a) How many electrons does the droplet carry? (b) How strong a vertical electric field is required to prevent it from falling?

13.18 An electron of energy 1 eV enters an infinitely large region containing only a homogeneous magnetic field of 10^{-3} T, at an angle of 60° to the direction of the field. Calculate its subsequent motion assuming no energy losses. What type of energy losses will occur even in complete vacuum?

[University of Manchester 1972]

13.19 A uniform electric field is established between the plates of a parallel plate capacitor by holding one plate at ground and the other at a positive potential V as shown. A uniform magnetic field B is established perpendicular to the electric field (Fig. 13.2).
A charge $-q$ is released from rest from the lower plate.

(i) Write down the equations of motion for the velocity components of the charge.

(ii) Show that, at some time t later, the velocity of the electron in the x-direction is related to the distance y moved along the y-axis by

$$v_x = \omega y$$

(iii) By applying the conservation of energy or otherwise to determine the square of the velocity in the x–y-plane, show that

$$v_y^2 = \left(\frac{2qv}{md}y - \omega^2 y^2\right)$$

[University of Aberystwyth, Wales]

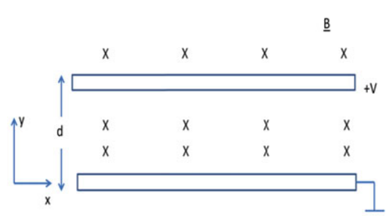


Fig. 13.2

13.20 Electrons are liberated with zero velocity from the negative plate of a parallel plate condenser, in which there is a constant magnetic field B parallel to the plates. If the separation of the plates is d and the potential across them is V, show that the electrons only arrive at the positive plate if $d^2 < \dfrac{2mV}{eB^2}$

[University of Durham 1962]

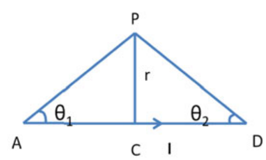
13.2.2 Magnetic Induction

13.21 The magnetic field at 40 cm from a long straight wire is 10^{-6} T. What current is carried by the wire?

13.22 A current I flows through a straight wire AB of finite length.

(a) Find the magnetic field B at distance r from the wire, the ends of the wire making inner angles θ_1 and θ_2 with P, Fig. 13.3.
(b) Obtain the limit value for B for a very long wire.

Fig. 13.3



13.23 The magnetic field at the centre of a circular current loop is 10^{-5} T. If the radius of the loop is 50 cm, find the current.

13.24 A square conducting loop, of side a carries a current I. Calculate the magnetic field at the centre of the loop.

13.25 Two wires are bent into semicircles of radius a, as in Fig. 13.4. The upper half has resistance R Ω and the lower half resistance $4R$ Ω. Find the magnetic induction at the centre of the circle.

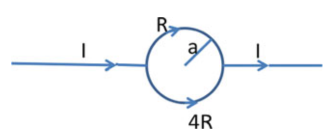
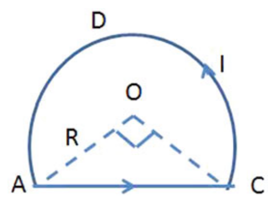


Fig. 13.4

13.26 A current I is sent through a thin wire as in Fig. 13.5. The radius of the curved part of the wire is R. Show that the magnetic induction at the point O will be $B = \frac{\mu_0 i}{2\pi R}\left(1 + \frac{3\pi}{4}\right)$

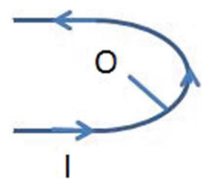
Fig. 13.5



13.27 **(a)** A current I is sent through a thin wire as in Fig. 13.6. The straight wires are very long and the radius of the curved part of the wire is R. Show that the magnetic induction at the point O will be

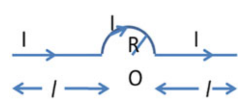
$$B = \frac{\mu_0 I(\pi + 2)}{4\pi R}$$

Fig. 13.6



(b) A wire shown in Fig. 13.7 carries current I. Find the field of induction B at the centre O.

Fig. 13.7



13.28 **(a)** A wire in the form of a polygon of n sides is circumscribed by a circle of radius a. If the current through the wire is i, show that the magnetic induction at the centre of the circle is given by

$$B = \frac{\mu_0 n i}{2\pi a}\tan\left(\frac{\pi}{n}\right)$$

(b) Show that in the limit $n \to \infty$ you get the expected result.

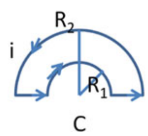
13.29 A wire of length l can form a circle or a square. A current i is set up in both the structures. Show that the ratio of magnetic induction at the centres of these structures will be approximately 0.87.

13.30 **(a)** C is the common centre of the circular arcs of the circuit-carrying current i, its arcs cutting a sector of angle, θ. Show that the magnetic induction at C is

$$B = \frac{\mu_0 i\theta}{4\pi}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

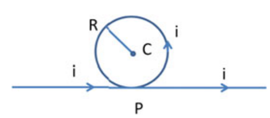
(b) C is the common centre of the semicircular arc of radii R_1 and R_2, Fig. 13.8, carrying current i. Show that the magnetic induction at C is $B = \dfrac{\mu_0 i}{4}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

Fig. 13.8



13.31 A long wire is bent into the shape as in Fig. 13.9, without any cross-contact at P. The current flows as indicated with the circular portion having radius R. Show that the magnetic induction at C, the centre of the circle is $B = \dfrac{\mu_0 i}{2R}\left(1 + \dfrac{1}{\pi}\right)$

Fig. 13.9



13.32 A current I flows through a ring of radius r placed in the xy-plane. Show that the magnetic induction at a point along the z-axis passing through the centre of the ring is given by

$$B(z) = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}}$$

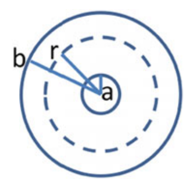
13.33 Five hundred turns of a wire are wound on a thin tube 1 m long. If the wire carries a current of 5 A, determine the field in the tube.

13.34 Two parallel wires, a distance d apart, carry equal currents I in opposite directions. Calculate the magnetic induction B for points between the wires at a distance x from one wire.

[Adapted from Hyderabad Central University 1993]

13.35 A long hollow copper cylinder with inner radius a and outer radius b carries a current I. Calculate the magnitude of the magnetic field at a point P ($a < r < b$) (Fig. 13.10).

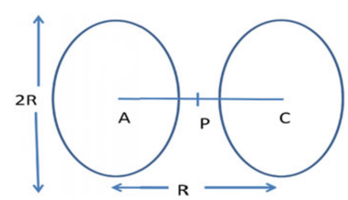
Fig. 13.10



13.36 Helmholtz coils consist of a pair of loops each with N turns and radius R. They are placed coaxially at distance R and the same current I flows through the loops but in the opposite sense. Show that the magnetic field at P, midway between the centres A and C, Fig. 13.11, is given by

$$B = \frac{8N\mu_0 I}{5^{3/2} R}$$

Fig. 13.11



13.37 A plastic of radius R has charge q distributed over its surface. If the disc rotates at an angular frequency about its axis, show that the induction B at the centre is given by

$$B = \frac{\mu_0 \omega q}{2\pi R}$$

13.38 Show that in the case of Helmholtz coils (prob. 13.36), the magnetic induction in the vicinity of the midpoint P is fairly uniform.

13.39 Consider a long straight rod of copper wire. It has a radius of 3×10^{-2} m and 100 A flowing uniformly through it. Find a value for the magnetic field (i) 1 m away and (ii) 6×10^{-3} m away from the central axis of the rod.

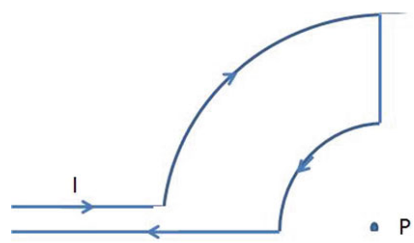
[University of Durham 2004]

13.40 Use Ampere's law to calculate the magnetic field for a long cylindrical conductor of radius R and a current I flowing through it at a distance r from the central axis of the conductor when (a) $r > R$ and (b) $r < R$

13.41 Current I flows in two concentric circular arcs of radii r and $2r$, Fig. 13.12. Both arcs are quarter of a circle with P as the centre. Determine B at P.

[University of Durham]

Fig. 13.12



13.42 **(a)** A current I flows in a straight wire of length L. Show that the magnitude of the magnetic field at a perpendicular distance x from the midpoint of the wire is given by

$$|B| = \frac{\mu_0 I}{4\pi} \frac{L}{x\sqrt{(L/2)^2 + x^2}}$$

where μ_0 is the permeability of free space. What is the direction of the B field?

(b) A loop of wire of length l carries a current I. Compare the magnetic fields at the centre of the loop when it is bent into (a) a square and (b) an equilateral triangle.

[University of Durham 2000]

13.43 Two identical, parallel co-axial coils of radius r, and having N turns, are separated by a distance r along their common axis. They both carry a current I in the same direction. Derive an expression for the magnetic field on the axis at the mid-point in between the coils. Evaluate the field when $N = 100$, $r = 20$ cm and $I = 2$ A.

13.44 State the relationship between the tangential component of the magnetic field B summed around a closed curve C and the current I_c passing through the area enclosed by the curve.

A long cylinder conductor of radius R carries a current I along its length. The current is uniformly distributed throughout the cross-section of the conductor. Calculate the magnetic field at a distance $r = R/2$ from the axis of the conductor. Find the distance $r > R$ from the axis of the conductor where the magnitude of the magnetic field is the same as at $r = R/2$.

[University of Durham]

13.45 **(a)** Show that (ignoring edge effects) the self-inductance, L, of a solenoid with n turns per unit length, length l and cross-sectional area A, is given by

$$L = \mu_0 n^2 A l$$

(b) A solenoid with 100 turns, length 10 cm and of radius 1 cm, carries a current of 5A. Calculate the magnetic energy stored in the solenoid.

(c) The current in the solenoid of part (b) is reduced to zero at a uniform rate over 5 s. Calculate the emf induced in the coil.

[University of Durham 2005]

13.46 **(a)** What is the magnetic field at a point 50 mm from a long straight wire carrying a current of 3 A?

(b) A small current element $I\,\mathrm{d}\boldsymbol{l}$ with $\mathrm{d}\boldsymbol{l} = 2\hat{k}$ and $I = 2$ A is centred at the origin. Find the magnetic field $\mathrm{d}\boldsymbol{B}$ at the following points:

(i) on the x-axis at $x = 3$ m,
(ii) on the x-axis at $x = -6$ m,
(iii) on the z-axis at $z = 3$ m,
(iv) on the y-axis at $y = 3$ m.

[University of Aberystwyth, Wales 2005]

13.47 A thin torus, of radius 0.1 m, is wound uniformly with 100 turns of wire. If a current of 2.0 A flows through the wire, what are the magnitudes of the B and H fields generated within the torus if it contains (i) a vacuum and (ii) a material with relative permeability 500? What is the magnetization in the material (Fig. 13.13)?

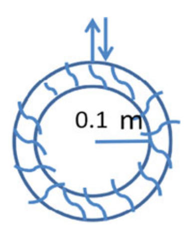


Fig. 13.13

13.48 It is believed that the earth's magnetic field is produced by circulating current in the core. If the mean radius of such currents is 1000 km, what is the order of magnitude of current required to account for the earth's dipolar magnetic field of magnitude 6×10^{-5} T at the north magnetic pole?

[University of Manchester 1972]

13.49 A pair of circular coils each having 50 turns of radius 50 cm are separated by 50 cm. A current of 10 A passes through the coils which are connected in series. Midway between the coils, a flat metal disc of radius 10 cm, is revolving at 1000 rpm What is the emf generated between the centre and the rim of the disc($\mu_0 = 4\pi \times 10^{-7}$ H/m)?

[University of Manchester 1959]

13.50 A conductor 1 m long moves with a velocity given by $(3\hat{i} + 2\hat{j} + \hat{k})$ m/s through a magnetic field given by $(\hat{i} + 2\hat{j} + 3\hat{k})$ Wb/m^2 How will the voltage developed across the ends of the conductor vary with its orientation? For what orientation will the voltage be zero?

[University of Durham 1962]

13.51 A proton travelling with velocity of $\boldsymbol{v} = (\hat{i} + 3\hat{j})10^4$ m/s is located at $x = 2$ m and $y = 3$ m at some instant t. Calculate the magnetic field at time t at the position $x = 2$ m, $y = 3$ m.

13.2.3 Magnetic Force

13.52 Two long straight wires lie parallel to each other at a distance 5 cm apart. If one carries a current of 2 A and the other a current of 3 A in the opposite direction, find the force each wire exerts on the other (per metre of wire)?

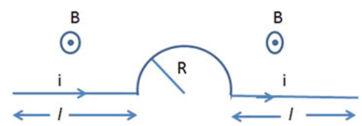
13.53 Two parallel wires 20 cm apart attract each other with a force of 10^{-5} N/m length. If the current in one wire is 10 A, find the magnitude and direction of current in the other wire?

13.54 A 2 m long wire weighs 4 g and carries a 10 A current. It is constrained to move only vertically above another wire carrying 15 A in the opposite direction. At what separation would its weight be supported by magnetic force?

13.55 Three long parallel wires, each carrying 20 A in the same direction, are placed in the same plane with the spacing of 10 cm. What is the magnitude of net force per metre on (a) an outer wire and (b) central wire?

13.56 Calculate the force acting on a bent wire (Fig. 13.14) carrying current i placed in a uniform magnetic field B, normal to the plane of paper in terms of i, B, l and R.

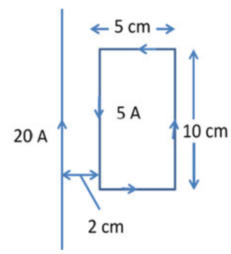
Fig. 13.14



13.57 A long straight wire carries a current of 20 A, as shown in Fig. 13.15. A rectangular coil with two sides parallel to the straight wire has sides 5 and 10 cm with the near side a distance 2 cm from the wire. The coil carries a current of 5 A.

(i) Find the force on each segment of the rectangular coil due to the current in the long straight wire.
(ii) What is the net force on the coil?

Fig. 13.15



13.58 **(a)** A very long straight wire PQ of negligible diameter carries a steady current I_1. A rigid square coil ABCD of side l and n turns is set up with sides AB and DC parallel to the coplanar with PQ as shown in Fig. 13.16. The side of the coil AB is at distance d from the wire PQ. Derive an expression for the resultant force on the coil when a steady current I_2 flows through it. What is the direction of the force?

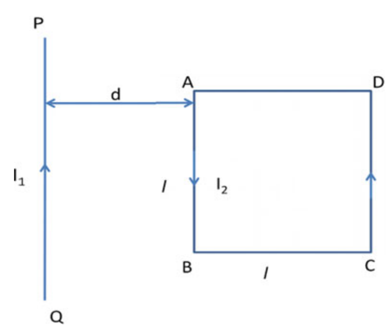


Fig. 13.16

(b) Calculate the magnitude of the force when

$$I_1 = 1\,\text{A}, I_2 = 4\,\text{A}, d = 0.10\,\text{m}, n = 10, \text{ and } l = 0.05\,\text{m}.$$

[University of Durham 2004]

13.59 Two parallel rails of negligible resistance are at distance d apart and are connected by a resistor of resistance R. A conducting rod lies perpendicular to the two rails and is free to slide on the rails. A constant magnetic field B is perpendicular to the loop formed by the rails, rod and resistor. An external agent drags the rod at velocity v along the rails. Find (a) the current flowing in the resistor, (b) the total power delivered to the resistor and (c) the magnitude of the applied force that is needed to move the rod with this velocity.
[University of Durham]

13.2.4 Magnetic Energy, Magnetic Dipole Moment

13.60 In prob. (13.37) show that the magnetic moment of the disc will be $\mu = \dfrac{\omega q R^2}{4}$.

13.61 The earth has a magnetic dipole moment of $6.4 \times 10^{21}\,\text{A/m}^2$. Show that this dipole moment can be produced by passing a current of 5×10^7 A in a single wire going around the magnetic equator.

13.62 Calculate the energy density at the centre of a circular loop of wire 10 cm radius carrying a current of 100 A.

13.63 Given that the magnetic field at the centre of hydrogen atom is 13.5 Wb/m^2, calculate the magnetic energy density at the centre of hydrogen atom due to the circulating electron.

13.64 A wire of length l forms a circular coil. If a current i is set up in the coil show that when the coil has one turn the maximum torque in a given magnetic field developed will be $\dfrac{1}{4\pi} l^2 i B$

13.65 A charge q is uniformly distributed over the volume of a uniform sphere of mass m and radius R, which rotates with an angular velocity ω about the axis passing through its centre. Show that the ratio of the magnetic moment and the angular momentum will be $\dfrac{\mu}{L} = \dfrac{q}{2m}$.

13.66 An electric dipole, whose dipole moment has magnitude 1.6×10^{-29} Cm is placed in a electric field of 1000 V/m. The direction of the dipole moment makes an angle of 30° to the direction of electric field. What is the potential energy of the dipole?

13.2.5 Faraday's Law

13.67 A flexible circular wire expands such that its radius increases linearly with time. It is located in a magnetic field perpendicular to the loop. Show that the emf induced in the wire varies linearly with time.

13.68 An aeroplane, with a wing span of 30 m, is flying horizontally at a speed of 720 km/h, at a point where the vertical component of the earth's field is 0.4 Oe. What is the emf developed between its wing tips.

[University of Durham]

13.69 A metal disc of radius 0.1 m spins about a horizontal axis lying in the magnetic meridian at a speed of 5 rev/s. If the horizontal component of the earth's field is $B = 2 \times 10^{-5}\ \mathrm{Wb/m^2}$, calculate the potential difference between the centre and the outer edge of the disc.

[University of Durham]

13.70 A coil is 30 turns of wire, each of area 10 cm^2, is placed with its plane at right angles to a magnetic field of 0.1 T. When the coil is suddenly withdrawn from the field, a galvanometer in series with the coil indicates that 10^{-5} C passes around the circuit. What is the combined resistance of the coil and the galvanometer?

[University of Cambridge]

13.71 A wire loop of area 0.2 m^2 has a resistance of 20 Ω. A magnetic field, normal to the loop, initially has a magnitude of 0.25 T and is reduced to zero at a uniform rate in 10^{-4} s. Estimate the induced emf and the resulting current.

13.72 A square wire with a loop of resistance 4 Ω, with sides 25 cm rotates 40 times per second about a horizontal axis. The magnetic field is vertical and has a magnitude of 0.5 T. Estimate the amplitude of the induced current.

13.73 A bar slides on rails separated by 20 cm, Fig. 13.17. If the current flowing through the resistor $R = 5\ \Omega$ is 0.4 A and the field $B = 1$ T, what is the speed of the bar?

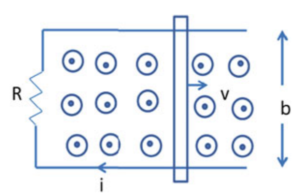


Fig. 13.17

13.74 A uniform magnetic field of induction B fills a cylindrical volume of radius R. A metal rod of length $2l$ is placed as in Fig. 13.18. If dB/dt is the rate of change of B show that the emf that is produced by the changing magnetic field that acts at the ends of the rod is given by

$$\xi = -\frac{dB}{dt} l\sqrt{R^2 - l^2}$$

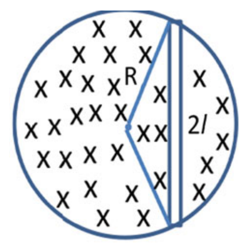


Fig. 13.18 Magnetic induction at the centre of a current-carrying wire made of three-fourths of a circle and a chord

13.75 A square wire of length l, mass m and resistance R slides without friction on parallel conducting resistance rails as in Fig. 13.19. The rails are interconnected at the bottom by resistance rails so that R, the wire and rails form a closed rectangular loop. The plane of the rails is inclined at an angle θ with the horizontal and a vertical uniform magnetic field B exists within the frame. Show that the wire acquires a steady velocity of magnitude

$$v = \frac{mgR\sin\theta}{B^2 l^2 \cos^2\theta}$$

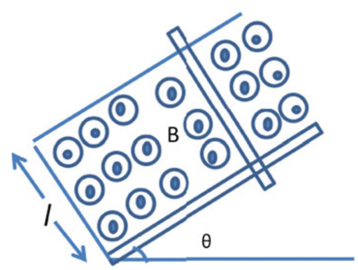


Fig. 13.19

13.76 A copper disc of 10 cm radius makes 1200 rotations per minute with its plane perpendicular to a magnetic field. If the induced emf between the centre and the edge of the disc is 6.28 mV, find the intensity of the field.

[Indian Administrative Services]

13.77 Show that Faraday's law $\xi = \mathrm{d}\varphi_B/\mathrm{d}t$ is dimensionally correct.

13.78 The magnetic field of an electromagnetic wave is given by the relation

$$B = 3 \times 10^{-12} \sin(4 \times 10^6 t)$$

where all quantities are in S.I. units. Find the magnitude of emf induced by the field in a 200-turn coil of 15 cm^2 area placed normal to the field.

13.79 Find the ratio of emf generated in a loop antenna by 100 MHz (typical television frequency) to that of 1 MHz (typical radio frequency) if both have equal field intensities.

13.80 Define magnetic flux and state Faraday's law, describing the relationship between the magnetic flux linked through a circuit and the current induced in the circuit. What is the force on a straight wire of length l carrying a current I in the presence of a magnetic field B?
Two long frictionless and resistanceless parallel rails, separated by a distance a, are connected by a resistanceless wire. A magnetic field B is oriented perpendicular to the plane containing the two rails. A frictionless conduction slider of resistance R and mass m is placed perpendicular to the rails and is given an initial velocity u along the rails. Obtain an expression for the force F on the slider while it moves at velocity v. Hence, find the maximum distance that the slider travels?

13.81 A flat, circular coil has 100 turns of wire, of radius 10 cm. A uniform magnetic field exists in a direction perpendicular to the plane of the coil. This field is increasing at a rate of 0.1 T/s. Calculate the emf induced in the coil.

13.82 A rectangular coil in the plane of the page has dimensions a and b. A long wire that carries a current I is placed directly on the coil, as shown in Fig. 13.20.

(i) Obtain an expression for the magnetic flux through the coil as a function of x for $0 \leq x \leq b$.
(ii) For what value of x is the net flux through the coil a maximum? For what value of x is the net flux a minimum?

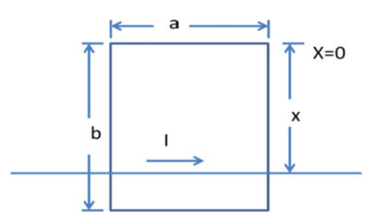


Fig. 13.20

(iii) Obtain an expression for the emf induced in the coil if the wire is placed at $x = b/4$ and the current varies with time according to $I = 2t$.

13.83 Show that in the betatron the magnetic flux Ø linking an electron orbit of radius R is given instantaneously by $\emptyset = 2\pi R^2 B$ where B is the instantaneous magnetic field.

[University of Newcastle upon Tyne 1965]

13.2.6 Hall Effect

13.84 What is the Hall effect and what is the significance of a positive Hall coefficient?
A potential difference is applied between the ends of a strip of copper and a current of 100 A flows along its length. The strip is 20 cm long in the x-direction of a rectangular system of coordinates, 2 cm wide in the y-direction and 1 mm thick in the z-direction. A uniform magnetic field of 10 Wb/m^2 is applied across the strip in the positive y- direction and the hall EMF is found to be 5 μV
Derive (a) the magnitude and direction of the Hall field when the current flows in the positive x-direction and (b) the concentration of free electrons.
[University of Manchester 1972]

13.85 The Hall coefficient and electrical conductivity of an n-type silicon are -7.3×10^{-5} m^3/C and 2×10^7 mho/m, respectively. Calculate the magnitude of the mobility of the electrons.

[University of Durham 1962]

13.3 Solutions

13.3.1 Motion of Charged Particles in Electric and Magnetic Fields

13.1 10^4 G = 1 T

$$f = \frac{Bq}{2\pi m} = \frac{1 \times 1.6 \times 10^{-19}}{2\pi \times 4.0026 \times 1.66 \times 10^{-27}} = 3.83 \times 10^6\,\text{Hz} = 3.83\,\text{MHz}$$

13.2 **(a)** $$K_\text{p} = \frac{1}{2}\frac{q^2r^2B^2}{m_\text{p}} = \frac{1}{2} \times \frac{(1.6 \times 10^{-19})^2(0.25)^2(1.5)^2}{1.66 \times 10^{-27}}$$

$$= 2.17 \times 10^{-13}\,\text{J} = \frac{2.17 \times 10^{-12}}{1.6 \times 10^{-13}}\,\text{MeV} = 13.56\,\text{MeV}$$