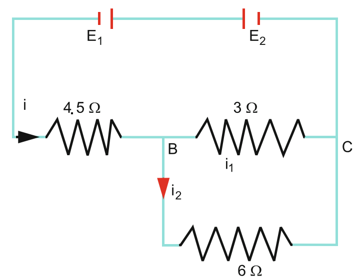
12.56 In the circuit shown in Fig. 12.30, the cells E_1 and E_2 have emfs 4 and 8 V and internal resistances 0.5 and 1 Ω, respectively. Calculate the current in each resistor and the potential difference across each cell.

[Indian Institute of Technology 1973]

Fig. 12.30



12.3 Solutions

12.3.1 Resistance, EMF, Current, Power

12.1 In the segment ACD, the two 3 Ω resistances give 6Ω, which with 6 Ω in parallel yields $\frac{6 \times 6}{6+6} = 3\,\Omega$. This together with 3 Ω, across DE in series, gives 6 Ω which together with 6 Ω across AE in parallel gives 3 Ω. By a similar reasoning resistance along AFB is 6 Ω, which with 3 Ω, across AB in parallel yields the effective resistance across AB:

$$R_{\mathrm{AB}} = \frac{6 \times 3}{6+3} = 2\,\Omega$$

12.2 $R = \frac{\rho l}{A} = \frac{\rho l^2}{Al} = \frac{\rho l^2}{v_0}$

where v_0 is the constant volume. Change in the resistance

$$\Delta R = 2\rho l \frac{\Delta l}{v_0}$$

$$\therefore \quad \frac{\Delta R}{R} \equiv 2\frac{\Delta l}{l} \equiv 2 \times \frac{0.1}{100} = \frac{0.2}{100} \quad \text{or} \quad 0.2\%$$

12.3 Let the resistances be R_1 and R_2

$$R_1 + R_2 = p \qquad \text{(series)} \tag{1}$$

$$\frac{R_1 R_2}{R_1 + R_2} = q \qquad \text{(parallel)} \tag{2}$$

Combining (1) and (2)

$$R_1 - R_2 = \pm\sqrt{n(n-4)}$$

Since R_1 and R_2 are real, $n \geq 4$.

12.4 Let a current i enter at A and leave at C. Currents in various branches are given by the junction theorem, Fig. 12.31. The potential difference

$$V_{\text{AB}} + V_{\text{BD}} = V_{\text{AD}}$$

$$\therefore \quad 3i_1 + 5i_2 = 6(i - i_1)$$

$$\text{or} \quad 9i_1 + 5i_2 = 6i \tag{1}$$

$$V_{\text{AC}} = V_{\text{AB}} + V_{\text{BC}} = V_{\text{AD}} + V_{\text{DC}}$$

$$\therefore \quad 3i_1 + 2(i_1 - i_2) = 6(i - i_1) + 4(i - i_1 + i_2)$$

$$\text{or} \quad 15i_1 - 6i_2 = 10i \tag{2}$$

Solving (1) and (2), $i_2 = 0$. Thus the middle branch BD is rendered ineffective.
Two resistances of 3 and 2 Ω in series in the upper branch are to be combined in parallel with two other resistances of 6 and 4 Ω in series in the lower branch to obtain the effective resistance between A and C. This is given by

$$R_{\text{eff}} = \frac{(3+2)(6+4)}{(3+2)+(6+4)} = 3.33\,\Omega$$

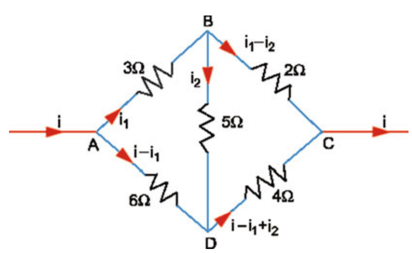


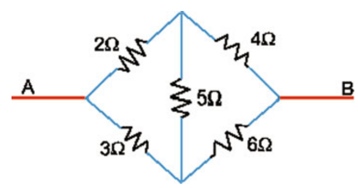
Fig. 12.31

This particular problem could have been easily solved by noticing that Wheatstone bridge balance requirement is fulfilled since $P/Q = R/S$; here $3/2 = 6/4$, in which case the current in the middle branch is zero (see prob. 12.33).

12.5 The network in Fig. 12.3 can be recast as shown in Fig. 12.32. Here again the balancing condition for Wheatstone bridge is satisfied: $\frac{P}{Q} = \frac{R}{S}, \frac{2}{4} = \frac{3}{6}$. Therefore the middle branch resistor of $5\,\Omega$ is rendered ineffective. The total resistance in the upper branch is $2 + 4 = 6\,\Omega$ and in the lower branch $3 + 6 = 9\,\Omega$. The equivalent resistance for 6 and $9\,\Omega$ in parallel will be

$$R_{\text{eq}} = \frac{6 \times 9}{6 + 9} = 3.6\,\Omega$$

Fig. 12.32



12.6 For convenience the given network can be recast as in Fig. 12.33. It is seen that this network is exactly identical with that in prob. (12.5) in which the Wheatstone bridge condition is satisfied. Therefore, the resistance in the middle branch CE is rendered ineffective. The total resistance in the upper branch is obviously equal to $2R$ which is also the case for the lower branch.

Therefore, the effective resistance between D and F will be

$$R_{\text{eff}} = \frac{2R \times 2R}{2R + 2R} = R$$

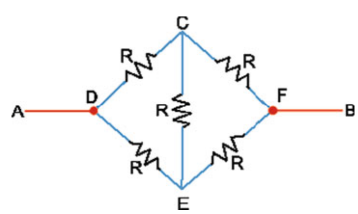


Fig. 12.33

12.7 Let R_{eq} be the equivalent resistance of the circuit. Let a current i enter at X and emerge at Y. The distribution of currents in various branches is shown in Fig. 12.34. The potential drop across X and Y

$$V_{xy} = i\ R_{eq} \tag{1}$$

Now $\quad V_{xy} = V_{XA} + V_{AY}$

$$= R(i - i_1) + 2R(i - i_1 - i_2) = 3Ri - 3Ri_1 - 2Ri_2 \tag{2}$$

From (1) and (2)

$$i\ R_{eq} = R(3i - 3i_1 - 2i_2) \tag{3}$$

i_1 and i_2 can be expressed in terms of i:

$V_{XB} = V_{XC} + V_{CB} = V_{XA} + V_{AB}$

$\therefore \quad 2Ri_1 = R(i - i_1) + Ri_2$

or $\quad 3i_1 - i_2 = i \qquad (4)$

Also, $\quad V_{AY} = V_{AB} + V_{BY} = V_{AD} + V_{DY}$

$\therefore \quad Ri_2 + R(i_1 + i_2) = 2R(i - i_1 - i_2)$

or $\quad 3i_1 + 4i_2 = 2i \qquad (5)$

Solving (4) and (5), $i_1 = 2i/5$ and $i_2 = i/5$.

Using the values of i_1 and i_2 in (3) we find $R_{eq} = 7R/5$.

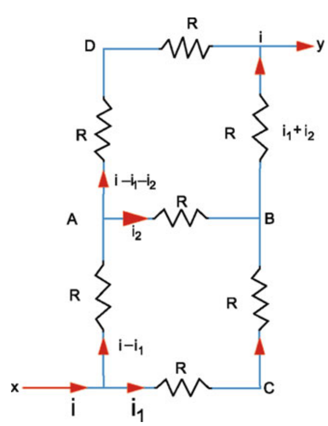


Fig. 12.34

12.8 **(i)** Let the current in AD, DE and R be i_1, i_2 and i_3, respectively:

$$
\begin{aligned}
& i_3 = 5 - 2 = 3\,\text{A} \qquad \text{(by junction theorem)}\\
& V_{\text{AD}} = V_{\text{AC}} + V_{\text{CD}}\\
& \therefore \quad i_1 \times 6 = 5 \times 4 + 5 \times 2\\
& \therefore \quad i_1 = 5\,\text{A}\\
& \therefore \quad i_2 = 2 + 5 = 7\,\text{A}\\
& V_{\text{CB}} = V_{\text{CD}} + V_{\text{DB}}\\
& 3R = 2 \times 5 + 7 \times 5\\
& \therefore \quad R = 15\,\Omega
\end{aligned}
$$

(ii) $V_{\text{AB}} = V_{\text{AC}} + V_{\text{CB}} = 4 \times 5 + 15 \times 3 = 65\,\text{V}$

(iii) Heat developed per second = power = $\sum i_n^2 R_n$

$$
\begin{aligned}
& = 5^2 \times 4 + 2^2 \times 5 + 5^2 \times 6 + 3^2 \times 15 + 7^2 \times 5\\
& = 650\,\text{J/s} = 650/4.18\,\text{Cal/s} = 155.5\,\text{Cal/s}
\end{aligned}
$$

12.9 The combination of 3 and 7 Ω resistance in series is 10 Ω. This in parallel with 10 Ω resistance yields 5 Ω. This in series with another 5 Ω resistance gives the combined resistance of 10 Ω. This being in parallel with 10 Ω resistance across A and B gives the effective resistance of 5 Ω across A and B.

12.10 Let the effective resistance between A and B be R. Then by adding one more section to infinite sections of resistors, the effective resistance will not change, Fig. 12.35.
The middle r is in parallel with R and the other two r's are in series. Then

$$R = r + \frac{Rr}{R + r} + r$$

Simplifying $R^2 - 2Rr - 2R^2 = 0$
whose solution is $R = r\left(1 + \sqrt{3}\right)$.
The second solution is ignored since R must be positive.

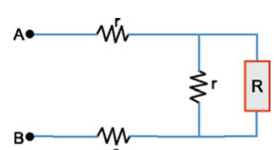


Fig. 12.35

12.11 $R_{\mathrm{I}} = \dfrac{\rho l}{A} = \dfrac{10 \times 10^{-6}\, l}{\pi (0.05)^2} = 1.274 \times 10^{-3} l$

$$R_{\mathrm{C}} = \frac{\rho' l}{A} = \frac{49 \times 10^{-6} l}{\pi (0.05)^2} = 6.24 \times 10^{-3} l$$

In parallel arrangement

$$R = \frac{R_{\mathrm{I}} R_{\mathrm{C}}}{R_{\mathrm{I}} + R_{\mathrm{C}}} = \frac{(12.739 \times 62.42) \times 10^{-6} l^2}{(12.739 + 62.42) \times 10^{-3} l}$$
$$= 10.58 \times 10^{-3} l = 2\,\Omega$$
$$\therefore \quad l = 1890\,\mathrm{cm} = 18.9\,\mathrm{m}$$

12.12 $R_2 = R_0(1 + \alpha t_2)$

$$R_1 = R_0(1 + \alpha t_1)$$
$$\therefore \quad \frac{R_2}{R_1} = \frac{1 + \alpha t_2}{1 + \alpha t_1}$$
$$\therefore \quad \alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)} = \frac{25.7 - 20.0}{20 \times (100 - 25)} = 3.8 \times 10^{-3}/^\circ\mathrm{C}$$

12.13 Resistance of each side = $10 \times 0.1 = 1\,\Omega$. Resistance of the diagonal $= \sqrt{2}\,\Omega$. The P.D., $V_{\mathrm{AB}} = V_{\mathrm{AD}}$ as $R_{\mathrm{AB}} = R_{\mathrm{AD}}$. Hence no current flows through the diagonal BD, Fig. 12.36. The effective resistance of the network is obtained by combining the resistance of AB and BC in series $(1 + 1)$ in parallel with that of AD and DC in series $(1 + 1)$:

$$R_{\mathrm{eff}} = \frac{2 \times 2}{2 + 2} = 1\,\Omega$$

Power dissipated $P = \dfrac{\xi^2}{R} = \dfrac{2^2}{1} = 4\,\mathrm{W}$.

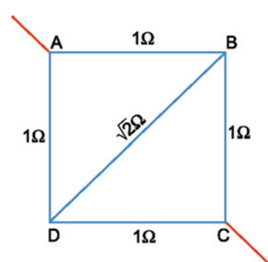


Fig. 12.36

12.14 $R = \dfrac{V^2}{P} = \dfrac{(100)^2}{60} = 166.7\,\Omega$

$R = R_0(1 + \alpha t)$

$t = \dfrac{R - R_0}{R_0\alpha} = \dfrac{166.7 - 20}{20 \times 0.0052} = 1410°\text{C}.$

12.15 Let a current of 6 A be sent through the corner a. Let a potential difference V_{ab} be established between a and b and current 6 A flow out from b. The currents in various branches are indicated in Fig. 12.37 from symmetry considerations. If R_{eq} is the equivalent resistance of this network across the body diagonal ab

$V_{ab} = 6\,R_{eq}$

But $\quad V_{ab} = V_{ac} + V_{cd} + V_{db}$

$= 2R + R + 2R = 5R$

$\therefore \quad 6\,R_{eq} = 5R$

or $\quad R_{eq} = \dfrac{5}{6}R = \dfrac{5}{6} \times 10 = 8.33\,\Omega$

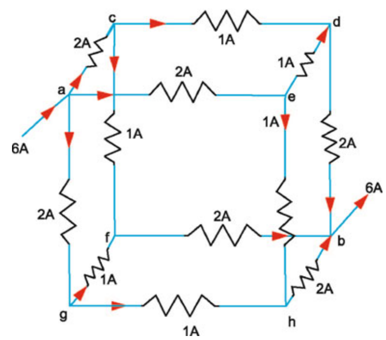


Fig. 12.37

12.16 $W = V^2/R$

$R_1 = \dfrac{V^2}{W_1} = \dfrac{(220)^2}{25} = 1936\,\Omega$

$R_2 = \dfrac{V^2}{W_2} = \dfrac{(220)^2}{100} = 484\,\Omega$

Joule heat in R_1, $H_1 = i^2 R_1 = 1936\, i^2$
Joule heat in R_2, $H_2 = i^2 R_2 = 484\, i^2$
Therefore, 25 W bulb glows brighter.

12.17 By prob. (12.16), $R_1 = 1936\,\Omega$ and $R_2 = 484\,\Omega$:

$$H_1 = i_1 V = \frac{i V R_2}{R_1 + R_2}$$
$$H_2 = i_2 V = \frac{i V R_1}{R_1 + R_2}$$

Therefore, 100 W bulb glows brighter.

12.18 Effective resistance in parallel is given by (Fig. 12.38)

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{1}{2}$$
$$\therefore \quad R = 2\,\Omega$$

Total resistance, $R_{AB} = 2 + 1 + 2 = 5\,\Omega$
Current in the circuit

$$i = \frac{V_{AB}}{R_{AB}} = \frac{120}{5} = 24\,\text{A}$$
$$V_{AC} = i\,R_{AC} = 24 \times 2 = 48\,\text{V}.$$

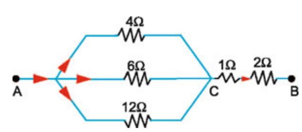


Fig. 12.38

12.19 $$i = \frac{\xi}{R + r}$$
$$P = i^2 R = \frac{\xi^2 R}{(R + r)^2}$$

Maximum power delivered to R is obtained by setting $\frac{\partial P}{\partial R} = 0$. This gives $R = r$:

$$\therefore \qquad P_{max} = \frac{\xi^2 r}{(r + r)^2} = \frac{\xi^2}{4r}$$

12.20 $H = P_1 t_1 = \frac{V^2}{R_1} t_1$

where H is joule heat and t is time

$$\therefore \quad t_1 = \frac{H}{V^2} R_1 \quad \text{(coil 1)}$$

$$t_2 = \frac{H}{V^2} R_2 \qquad \text{(coil 2)}$$

(a) $t = \frac{H}{V^2} R = \frac{H}{V^2}(R_1 + R_2) = t_1 + t_2 \quad$ (coils in series)

(b) $\frac{1}{t_1} = \frac{V^2}{H R_1} \qquad$ (coil 1)

$$\frac{1}{t_2} = \frac{V^2}{H R_2} \qquad \text{(coil 2)}$$

$$\therefore \qquad \frac{1}{t_1} + \frac{1}{t_2} = \frac{V^2}{H}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V^2}{HR} = \frac{1}{t}$$

$$\text{or} \qquad t = \frac{t_1 t_2}{t_1 + t_2} \qquad \text{(coils in parallel)}$$

12.21 **(a)** $i = \frac{\xi}{R_1 + R_2 + r} = \frac{24}{4 + 6 + 2} = 2\,\text{A}$

$$P_1 = i^2 R_1 = 2^2 \times 4 = 16\,\text{W}$$

$$P_2 = i^2 R_2 = 2^2 \times 6 = 24\,\text{W}$$

(b) Effective resistance of R_1 and R_2 in parallel is

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{4 \times 6}{4 + 6} = 2.4\,\Omega$$

$$i = \frac{\xi}{R + r} = \frac{24}{2.4 + 2} = 5.45\,\text{A}$$

$$i_1 = \frac{i R_2}{R_1 + R_2} = \frac{5.45 \times 6}{4 + 6} = 3.27\,\text{A}$$

$$i_2 = \frac{i R_1}{R_1 + R_2} = \frac{5.45 \times 4}{4 + 6} = 2.18\,\text{A}$$

$$P_1 = i^2{}_1 R_1 = (3.27)^2 \times 4 = 42.8\,\text{W}$$

$$P_2 = i^2{}_2 R_2 = (2.18)^2 \times 6 = 28.5\,\text{W}$$

12.22 Total resistance of the cable

$$R = 0.7 \times 30 = 21\,\Omega$$

$$\text{Voltage } V = 100\,\text{kV} = 10^5\,\text{V}$$

$$\text{Power } P = 10^4\,\text{kW} = 10^7\,\text{W}$$

$$\text{Current } I = \frac{P}{V} = \frac{10^7}{10^5} = 100\,\text{A}$$

$$\text{Power dissipated} = i^2 R = (100)^2 \times 21 = 2.1 \times 10^5\,\text{W}$$

$$\text{Fractional power loss} = \frac{2.1 \times 10^5}{10^7} = 0.021 \text{ or } 2.1\%$$

12.23 Let each of the three resistances be r. In the series arrangement the effective resistance, $R_1 = 3r$:

$$P_1 = \frac{\xi^2}{R_1} = \frac{\xi^2}{3r} = 10$$

$$\therefore \quad \frac{\xi^2}{r} = 30 \tag{1}$$

In the parallel arrangement the effective resistance $R_2 = r/3$:

$$P_2 = \frac{\xi^2}{R_2} = \frac{3\xi^2}{r} = 3 \times 30 = 90 \text{ W}$$

where we have used (1).

12.24 If the heater resistance is R_0,

$$R_0 = \frac{V^2}{P} = \frac{(100)^2}{1000} = 10\,\Omega$$

The combined resistance of R_0 and R in parallel is $\frac{10R}{R+10}$. As this is in series with $10\,\Omega$, the effective resistance of the circuit

$$R_e = 10 + \frac{10R}{R+10} = \frac{20R + 100}{R+10}$$

If P' is the power of the heater

$$R_e = \frac{20R + 100}{R+10} = \frac{V^2}{P'} = \frac{(100)^2}{62.5}$$

Solving for R, we find $R = 5\,\Omega$.

12.3.2 Cells

12.25 Let n cells of emf ξ and internal resistance r be wrongly connected. The effective emf of the battery is $(12–2n)\ \xi$. When the two cells and the battery aid each other, the net emf is $(12-2n)\xi+2\xi$ or $(14-2n)\xi$. The total internal resistance is $14r$. By problem, when the two cells and battery aid each other

$$(14 - 2n)\xi = 3 \times 14r \tag{1}$$

and when the two cells and battery oppose each other, the net emf is $(12-2n)\xi - 2\xi$ or $(10-2n)\xi$, the total internal resistance being $14r$. By problem

$$(10 - 2n)\xi = 2 \times 14r \tag{2}$$

Dividing (1) by (2)

$$\frac{14 - 2n}{10 - 2n} = \frac{3}{2}$$

whence $n = 1$.

12.26 The total number of cells is $N = m \times n$. The emf for each row of cells will be $n\xi$ and the combined internal resistance nr (Fig. 12.39). The effective emf for m rows would again be $n\xi$, but because the rows are in parallel, the effective internal resistance would become nr/m. The total resistance then becomes $R + nr/m$. The current through R will be

$$i = \frac{n\xi}{R + \frac{nr}{m}} = \frac{mnr}{Rm + nr} = \frac{N\,r}{Rm + nr} \tag{1}$$

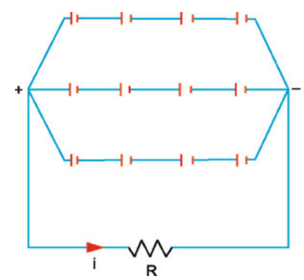
Writing $m = N/n$ in (1) and holding N as constant, maximum current i is found by setting $\frac{\partial i}{\partial n} = 0$. This gives

$$n^2 = \frac{RN}{r} = \frac{R\,m\,n}{r}$$

$$\text{or} \qquad R = \frac{n\,r}{m}$$

But the right-hand side is equal to the total internal resistance of the cells. Thus the current is maximum when the cells are arranged such that their total internal resistance is equal to the external resistance. In particular, for a single cell, $m = n = 1$, and the condition for maximum current is $R = r$, a result which is identical with that of prob. (12.19).

Fig. 12.39



12.27 Let ξ be the emf of each cell, i the current flowing in the circuit, r_1 and r_2 be the internal resistance of the first and the second cells, respectively. The potential drop across the first cell will be

$$V_1 = \xi - ir_1 = 0 \qquad \text{(by problem)} \tag{1}$$

$$i = \frac{2\xi}{r_1 + r_2 + R} \tag{2}$$

Combining (1) and (2)

$$\xi = ir_1 = \frac{2\xi r_1}{r_1 + r_2 + R}$$

$$\therefore \qquad R = r_1 - r_2$$

12.28 Total power $P = P_1 + P_2 + P_3 = \frac{10^3\,\text{W h}}{2\,\text{h}} = 500\,\text{W}$

$$P_1 : P_2 : P_3 = 5 : 3 : 2$$

$$\therefore \quad P_1 = 250\,\text{W}, \qquad P_2 = 150\,\text{W}, \qquad P_3 = 100\,\text{W}$$

$$R_1 = \frac{V^2}{P_1} = \frac{(200)^2}{250} = 160\,\Omega, \quad R_2 = \frac{V^2}{P_2} = \frac{(200)^2}{150} = 267\,\Omega,$$

$$R_3 = \frac{V^2}{P_3} = \frac{(200)^2}{100} = 400\,\Omega$$

In series total resistance $R' = R_1 + R_2 + R_3 = 160 + 267 + 400 = 827\,\Omega$. Required power for the series arrangement

$$P' = \frac{V'^2}{R'} = \frac{(248)^2}{827} = 74.4\ \text{W}$$

12.29 The current flows from positive to negative terminal inside the battery. The potential difference between the two terminals of the battery would be

$$V = \xi + ir = 2 + 5 \times 0.1 = 2.5\,\mathrm{V}$$

12.30 **(a)** The effective resistance in the circuit (Fig. 12.40) from 3 Ω in series with 5 and 2 Ω in parallel

$$R_0 = R + \frac{R_1 R_2}{R_1 + R_2} = 3 + \frac{2 \times 5}{2 + 5} = 4.43\,\Omega$$
$$i = \frac{\xi}{R_0} = \frac{6}{4.43} = 1.35\,\mathrm{A}$$

(b)
$$i_1 = \frac{i\,R_2}{R_1 + R_2} = \frac{1.35 \times 2}{5 + 2} = 0.386$$
$$P_1 = {i_1}^2\,R_1 = (0.386)^2 \times 5 = 0.74\,\mathrm{W}$$
$$i_2 = i - i_1 = 1.35 - 0.386 = 0.964\,\mathrm{A}$$
$$P_2 = {i_2}^2\,R_2 = (0.964)^2 \times 2 = 1.86\,\mathrm{W}$$
$$P = i^2\,R = (1.35)^2 \times 3 = 5.47\,\mathrm{W}$$

(c) Total power dissipated by the resistances
$= P_1 + P_2 + P = 0.74 + 1.86 + 5.47 = 8.07\,\mathrm{W} \simeq 8.1\,\mathrm{W}$

Power supplied by the battery $= \xi i = 6 \times 1.35 = 8.1\,\mathrm{W}$

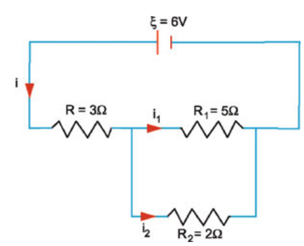


Fig. 12.40

12.3.3 Instruments

12.31 P.D across AB is $\dfrac{\xi\,R}{R + r} = \dfrac{l}{l_0}\,\xi_0$

$$\frac{\xi \times 20}{20 + r} = \frac{150}{l_0}\,\xi_0$$
$$\frac{\xi \times 10}{10 + r} = \frac{120}{l_0}\,\xi_0$$

Dividing the last two equations

$$\frac{2\,(10 + r)}{20 + r} = \frac{5}{4}$$

whence $r = 6.67\,\Omega$.

12.32 For the section ABC

$$i = \frac{\xi_1}{R + r} = \frac{2.2}{1 + 0.1} = 2\,\text{A}$$

In the section BCD also $i = 2\,\text{A}$.
The resistance of 26 cm wire $= \dfrac{26}{50} \times 1 = 0.52\,\Omega$.
Neglecting the internal resistance of the second cell

$$\xi_2 = 2 \times 0.52 = 1.04\,\text{V}$$

as no current flows through the galvanometer

12.33 **(a)** $V_{\text{AB}} = V_{\text{AD}}; \qquad V_{\text{BC}} = V_{\text{DC}}$

$$\therefore \quad i_1 P = i_2 R \tag{1}$$
$$i_1 Q = i_2 S \tag{2}$$

Dividing (1) by (2)

$$\frac{P}{Q} = \frac{R}{S} \tag{3}$$

(b) Assume that a non-zero current flows through the galvanometer of resistance G. Applying the junction theorem at A

$$i = i_1 + i_2 \tag{4}$$

Applying the loop theorem to the loop ABDA and noting that there is no emf in this loop

$$i_g G + i_1 P - i_2 R = 0 \tag{5}$$

Applying the loop theorem to the loop BCDB, which also does not have an emf

$$(i_1 - i_g)Q - i_g G - (i_2 - i_g)s = 0 \tag{6}$$

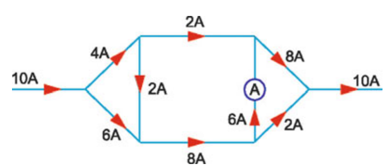
Combining (4), (5) and (6)

$$\frac{i_g}{i} = \frac{QR - PS}{G(Q + S) + (P + R)(G + Q + S)}$$

Note that $i_g = 0$ if $QR - PS = 0$, which is identical with the condition (3).

12.34 Apply the junction theorem to obtain currents in various branches as indicated in Fig. 12.41. Current flowing through the ammeter is 6 A.

Fig. 12.41



12.35 Let the internal resistance of each battery be r, galvanometer resistance G and the external resistance R. Then in the series arrangement, total resistance in the circuit (Fig. 12.42)

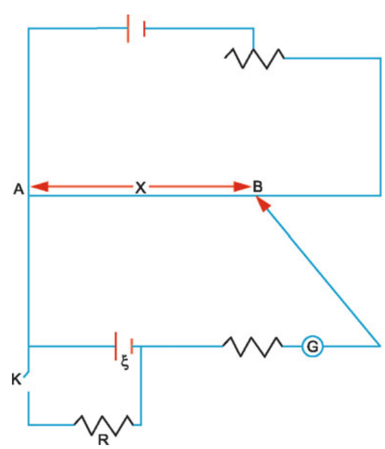


Fig. 12.42

$$R_{\text{eff}} = G + R + 2r$$
$$\text{Effective emf, } \xi_{\text{eff}} = 2 \times 1.5 = 3\,\text{V} \tag{12.17}$$
$$\xi_{\text{eff}} = i(G + R + 2r)$$
$$\therefore \quad 3 = 1 \times (G + R + 2r) \tag{1}$$

In the parallel arrangement, $\xi_{\text{eff}} = 1.5\,\text{V}$ and the combined internal resistance is $\dfrac{r \times r}{r + r} = 0.5\,r$.
Total resistance in the circuit

$$R_{\text{eff}} = G + R + 0.5\,r$$
$$1.5 = 0.6 \times (G + R + 0.5r)$$
$$\text{or} \quad 2.5 = G + R + 0.5r \tag{2}$$

Subtracting (2) from (1), $r = 0.333\,\Omega$.

12.36 When the key is closed P.D across R is

$$V = i\,R$$

and the emf of the cell is

$$\xi = i(R + r)$$

where r is the internal resistance of the cell.

$$\frac{\xi}{V} = \frac{R + r}{R} = 1 + \frac{r}{R}$$

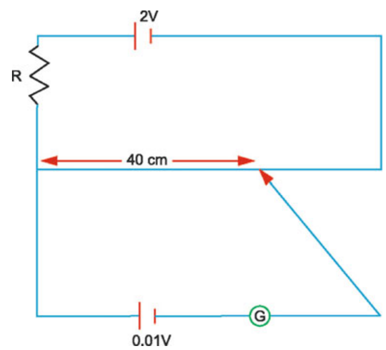
When the key is open, let the balancing length be X_1 cm from the end A against the emf ξ. When the key is closed, let the balancing length be X_2 against the P.D. of V volts:

$$\frac{\xi}{V} = \frac{X_1}{X_2} = 1 + \frac{r}{R}$$
$$\frac{52}{40} = 1 + \frac{r}{5}$$
$$\therefore \quad r = 1.5\,\Omega.$$

12.37 Resistance (Fig. 12.43) of 40 cm of potentiometer wire $= \dfrac{40}{100} \times 10 = 4\,\Omega$

P.D. across 40 cm wire due to 2 V cell is $V = \dfrac{2 \times 4}{10 + R} = \dfrac{8}{10 + R}$

Fig. 12.43



This is balanced by 0.01 V due to the second cell:

$$\therefore \quad \frac{8}{10+R} = 0.01$$
$$\therefore \quad R = 790\,\Omega$$

12.38 Resistance across ac (Fig. 12.20) is $\frac{1}{4} \times 12,000 = 3000\,\Omega$. The combined resistance of voltmeter (6000 Ω) in parallel with 3000 Ω resistance is $\frac{6000 \times 3000}{6000 + 3000} = 2000\,\Omega$.
Resistance across bc is $\frac{3}{4} \times 12,000 = 9000\,\Omega$. Effective resistance of the circuit $= 9000 + 2000 = 11,000\,\Omega$.
P.D. across ac is

$$V = \frac{220 \times 2000}{11,000} = 40\,\text{V}$$

Thus the voltmeter reads 40 V.

12.39 $$\frac{R}{3.5} = \frac{l}{100 - l} = \frac{30}{100 - 30} = \frac{3}{7}$$
$$\therefore \quad R = 1.5\,\Omega$$

12.40 **(a)** $$N = \frac{i}{i_1} = \frac{100\,\text{mA}}{1\,\text{mA}} = 100$$
$$S = \frac{G}{N-1} = \frac{80}{100-1} = 0.808\,\Omega$$

A shunt of 0.808 Ω should be provided for the moving coil meter.

(b) Initially $V = iG = 1 \times 10^{-3} \times 80 = 0.08\,\text{V}$

$$N = \frac{80}{0.08} = 1000$$
$$R = (N-1)G = (1000-1) \times 80 = 79,920\,\Omega$$

A resistance of 79,920 Ω must be connected in series with the moving coil galvanometer to give the required full scale deflection.

12.41 $\xi = i(R + r)$ (by Ohm's law)

$$9 = i(120 + 15)$$
$$\therefore \quad i = \frac{1}{15}\,\text{A}$$

The voltmeter would read

$$V = \xi - ir = 9 - \frac{1}{15} \times 15 = 8\,\text{V}$$

12.42 $S = \dfrac{G}{N-1}$

$$\frac{i_1}{i} = \frac{1}{N} = \frac{1}{100}$$

or $\quad N = 100$

$$\therefore \quad G = (N-1)S = (100-1) \times 1 = 99\,\Omega$$

12.3.4 Kirchhoff's Laws

12.43 By the junction theorem

$$i = i_1 + i_2 \tag{1}$$

Applying the loop theorem to the loop $BE_1\,AE_2B$, traversing clockwise

$$\xi_2 - i_2 r_2 - \xi_1 + i_1 r_1 = 0$$
$$\therefore\ 2i_2 - i_1 = 20 - 10 = 10 \tag{2}$$

Applying the loop theorem to the loop BE_1ARB

$$\xi_1 = i_1 r_1 + iR$$

$\therefore \quad 10 = i_1 + 30i$

or $\quad 31i_1 + 30i_2 = 10 \qquad (3)$

where we have used (1). Solving (2) and (3)

$i_1 = -3.04\,\text{A}, \; i_2 = 3.48\,\text{A}$

Power dissipated through E_1 is $i^2{}_1 r_1 = (3.04)^2 \times 1 = 9.24\,\text{W}$
Power dissipated through E_2 is $i^2{}_2 r_2 = (3.48)^2 \times 2 = 24.2\,\text{W}$
Power dissipated through R is $i^2 R = (0.44)^2 \times 30 = 5.8\,\text{W}$

$(\because \; i = i_1 + i_2 = -3.04 + 3.48 = 0.44\,\text{A})$

12.44 Referring to Fig. 12.44

$i_1 + i_2 = i \qquad$ (junction theorem) $\qquad (1)$

Traversing the loop BE_2AE_1B counterclockwise the loop theorem gives

$\xi_2 - r_2 i_2 - (\xi_1 - r_1 i_1) = 0$

$\therefore \quad 1 - 2i_1 - (2 - 1i_1) = 0$

or $\quad i_1 - 2i_2 = 1 \qquad (2)$

For the loop BE_1ARB

$\xi_1 - i_1 r_1 - iR = 0$

$\therefore \quad 2 - 1i_1 - 10i = 0$

or $\quad 11i_1 + 10i_2 = 2 \qquad (3)$

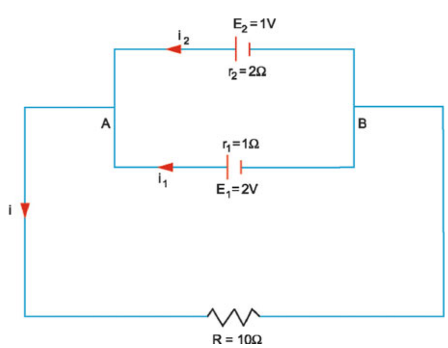


Fig. 12.44

where we have used (1). Application of Kirchhoff's laws to the loop BE_2ARB does not yield anything extra. From (1), (2) and (3) we find $i_1 = 0.44$ A, $i_2 = -0.28$ A and $i = 0.16$ A. The negative sign of i_2 shows that its direction is opposite to that has been assumed.

12.45 $i_1 + i_2 + i_3 = i = 5\,\text{A}$ (1)

As $\rho =$ const and $R = \pi\ \text{d}^2/4$ (Fig. 12.45)

$$R_1 : R_2 : R_3 = \frac{l_1}{d_1^2} : \frac{l_2}{d_2^2} : \frac{l_3}{d_3^2}$$

$$\therefore \quad R_1 : R_2 : R_3 = \frac{2}{3^2} : \frac{3}{4^2} : \frac{4}{5^2} = \frac{2}{9} : \frac{3}{16} : \frac{4}{25} \tag{2}$$

Since P.D across all the resistors is identical,

$$i_1 R_1 = i_2 R_2 = i_3 R_3 \tag{3}$$

$$\therefore \quad i_2 = i_1 \frac{R_1}{R_2} = \frac{2}{9} \times \frac{16}{3} i_1 = \frac{32}{27} i_1 \tag{4}$$

$$i_3 = i_1 \frac{R_1}{R_3} = \frac{2}{9} \times \frac{25}{4} = \frac{25}{18}\ i_1 \tag{5}$$

$$\therefore \quad i_1 + \frac{32}{27} i_1 + \frac{25}{18} i_1 = i = 5 \tag{6}$$

$$\therefore \quad i_1 = 1.4\,\text{A},\ i_2 = 1.66\,\text{A},\ i_3 = 1.94\,\text{A}$$

where we have used (1), (4) and (5).

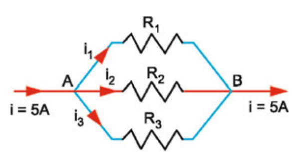


Fig. 12.45

12.46 The P.D across 8 and 2 Ω resistors are equal, Fig. 12.46:

$$2i_2 = 8i_3 \tag{1}$$

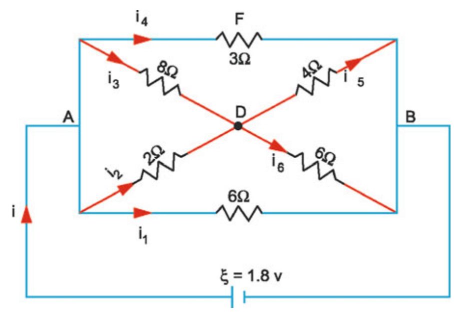
As P.D across 3 and 6 Ω resistors are equal

$$3i_4 = 6i_1 \tag{2}$$

As P.D across 4 and 6 Ω resistors are equal

$$6i_6 = 4i_5 \tag{3}$$

Fig. 12.46



Applying junction theorem

$$i = i_1 + i_2 + i_3 + i_4 \tag{4}$$
$$i_2 + i_3 = i_5 + i_6 \tag{5}$$
$$V_{AB} = V_{AD} + V_{DB}$$
$$\therefore \quad 3i_4 = 8i_3 + 4i_5 \tag{6}$$

Applying the loop theorem to CAFBC

$$\xi - ir - 3i_4 = 0$$
$$\frac{2}{3}i + 3i_4 = 1.8 \tag{7}$$

Solving (1), (2), (3), (4), (5), (6) and (7), $i_4 = 0.4$ A and $i = 0.9$ A. Applying the loop theorem to the entire circuit

$$\xi - ir - iR = 0 \tag{8}$$

where R is the equivalent resistance of the circuit

$$R = \frac{\xi}{i} = \frac{1.8}{0.9} - \frac{2}{3} = \frac{4}{3}$$

Power dissipated in the entire circuit is

$$P = i^2(R + r) = (0.9)^2\left(\frac{4}{3} + \frac{2}{3}\right) = 1.62\,\text{W}$$

12.47 Let A and B be the midpoints of the coils. As no current flows through the galvanometer, P.D across AB is zero. Applying the loop theorem to the main circuit, Fig. 12.47

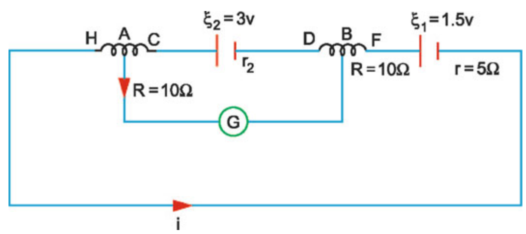


Fig. 12.47

$$\xi_1 + \xi_2 - i(R + R + r_1 + r_2) = 0$$

$$\therefore \quad i = \frac{\xi_1 + \xi_2}{2R + r_1 + r_2} = \frac{1.5 + 3}{2 \times 10 + 5 + r_2} = \frac{4.5}{r_2 + 25} \tag{1}$$

The P.D of the point B with respect to the negative terminal of the first cell is

$$V_B = \xi_1 - \left(\frac{10}{2} + r_1\right) i = 1.5 - 10\,i \tag{2}$$

The P.D of the point A with respect to the negative terminal of the first cell is

$$V_A = \xi_1 + \xi_2 - \left(\frac{10}{2} + r_2 + 10 + r_1\right) i$$
$$= 1.5 + 3 - (5 + r_2 + 10 + 5)i = 4.5 - (20 + r_2)$$

But $V_B = V_A$ ($\because$ no current flows through the galvanometer)

$$\therefore \quad 1.5 - 10i = 4.5 - (20 + r_2)i$$

$$\therefore \quad i = \frac{3}{10 + r_2} \tag{3}$$

From (1) and (2), $r_2 = 20\,\Omega$

12.48 **(a)** **(i)** $R_{eq} = \dfrac{R_1 R_2}{R_1 + R_2} + \dfrac{R_3 R_4}{R_3 + R_4} = \dfrac{8 \times 2}{8 + 2} + \dfrac{3.2 \times 3.2}{3.2 + 3.2} = 3.2\,\text{k}\Omega$

(ii) $V_{p_1} = 320 \times \dfrac{1.6}{3.2} = 160\,\text{V}$

(iii) $V_{p_2} = 320 \times \dfrac{1.6}{3.2} = 160\,\text{V}$

(iv) $I_T = \frac{\xi}{R_{eq}} = \frac{320}{3.2 \times 10^3} = 0.1\,\text{A}$

$$I_1 = I_T \times \frac{R_2}{R_1 + R_2} = 0.1 \times \frac{2}{8+2} = 0.02\,\text{A}$$

$$I_3 = I_T \times \frac{R_4}{R_3 + R_4} = 0.1 \times \frac{3.2}{3.2+3.2} = 0.05\,\text{A}$$

(v) $W = I_T^2 R_{eq} = (0.1)^2 \times 3.2 \times 10^3 = 32\,\text{J}$

$$W_3 = I_3^2 R_3 = (0.05)^2 \times 8 \times 10^3 = 20\,\text{J}$$

(b) $P = \frac{\xi^2}{r} = \frac{(24)^2}{0.01} = 5.76 \times 10^4\,\text{J}$

12.49 **(i)** $i = \frac{\xi}{R+r} = \frac{24}{140+0.02} = 0.1714\,\text{A}$

(ii) $V = \xi - ir = 24 - 0.1714 \times 0.02 = 23.9966\,\text{V}$

(iii) $P_R = i^2\text{R} = (0.1714)^2 \times 140 = 4.113\,\text{W}$

$$P_r = i^2 r = (0.1714)^2 \times 0.02 = 5.87 \times 10^{-4}\,\text{W}$$

(iv) $V = \xi = 24\,\text{V}$. Full voltage is available in the absence of load resistance.

12.50 **(i)** $R_{eq} = \frac{R_1 R_2}{R_1 + R_2} + R_5 + \frac{R_3 R_4}{R_3 + R_4}$

$$= \frac{80 \times 80}{80+80} + 20 + \frac{40 \times 40}{40+40} = 80\,\text{k}\Omega$$

(ii) $I_T = \frac{V}{R_{eq}} = \frac{300}{80 \times 10^3} = 3.75 \times 10^{-3}\,\text{A}$

$$I_1 = I_T \times \frac{R_2}{R_2 + R_1} = 3.75 \times 10^{-3} \times \frac{80}{80+80} = 1.875 \times 10^{-3}\,\text{A}$$

$$I_3 = I_T \times \frac{R_4}{R_4 + R_1} = 3.75 \times 10^{-3} \times \frac{40}{40+40} = 1.875 \times 10^{-3}\,\text{A}$$

(iii) $V_1 = \frac{V}{R_{eq}} \times \frac{R_1 R_2}{(R_1 + R_2)} = \frac{300}{80} \times \frac{80 \times 80}{(80+80)} = 150\,\text{V}$

$$V_2 = \frac{V\,R_5}{R_{eq}} = 300 \times \frac{20}{80} = 75\,\text{V}$$

$$V_3 = \frac{V}{R_{eq}} \times \frac{R_3 R_4}{(R_3 + R_4)} = \frac{300}{80} \times \frac{40 \times 40}{(40+40)} = 75\,\text{V}$$

(iv) $P_5 = I^2{}_T R_5 = (3.75 \times 10^{-3})^2 \times 20 \times 10^3 = 0.281\,\text{W}$

$$P = I^2{}_T R_{eq} = (3.75 \times 10^{-3})^2 \times 80 \times 10^3 = 1.125\,\text{W}$$

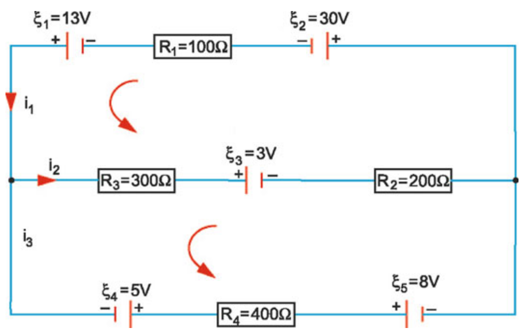


Fig. 12.48

12.51 $i_1 = i_2 + i_3$ (junction theorem) (1)

Traversing the top loop counterclockwise (Fig. 12.48)

$$\xi_1 - i_2 R_3 - \xi_3 - i_2 R_2 - \xi_2 - i_1 R_1 = 0 \quad \text{(loop theorem)}$$

$$\therefore \quad 13 - 300\, i_2 - 3 - 200\, i_2 - 30 - 100\, i_1 = 0$$

$$\text{or} \quad 100\, i_1 + 500 i_2 + 20 = 0 \qquad (2)$$

Traversing the bottom loop counterclockwise

$$\xi_4 - i_3 R_4 - \xi_5 + i_2 R_2 - \xi_3 + i_2 R_3 = 0$$

$$\therefore \quad 5 - 400\, i_3 - 8 + 200\, i_2 - 3 + 300\, i_2 = 0$$

$$\text{or} \quad 500\, i_2 - 400\, i_3 - 6 = 0 \qquad (3)$$

Required equations for the unknown currents are (1), (2) and (3).

12.52 **(i)** Kirchhoff's rule 1 (junction theorem) At any junction of an electric network (branched circuit) the algebraic sum of the currents flowing towards that junction is zero (Fig. 12.49).
Kirchhoff's rule 2 (loop theorem)
Sum of the changes in the potential encountered in traversing a loop (closed circuit) in a particular direction (clockwise or counterclockwise) is zero.
If a resistor is traversed in the direction of the current, the change in the potential is $-iR$, while in the opposite direction it is $+iR$.
If a seat of emf is traversed in the direction of emf, the change in potential is $+\xi$, while in the opposite direction it is $-\xi$.

(ii) $i_1 = i_2 + i_3$ (junction theorem) (1)

Traversing the top loop counterclockwise

$$- \xi_1 - i_2 R_2 + \xi_3 + \xi_2 - i_1 R_1 = 0 \text{ (loop theorem)}$$
$$\therefore \quad -11 - 200\, i_2 + 33 + 22 - 100\, i_1 = 0$$
$$\text{or} \quad 100\, i_1 + 200 i_2 - 44 = 0 \qquad (2)$$

Traversing the bottom loop counterclockwise

$$- \xi_4 - i_3 R_3 + \xi_5 - \xi_3 + i_2 R_2 = 0$$
$$\therefore \quad -44 - 300\, i_3 + 55 - 33 + 200\, i_2 = 0$$
$$\text{or} \quad 200\, i_2 - 300\, i_3 - 22 = 0 \qquad (3)$$

Equations (1), (2) and (3) are the required equations in the three unknown currents i_1, i_2 and i_3.

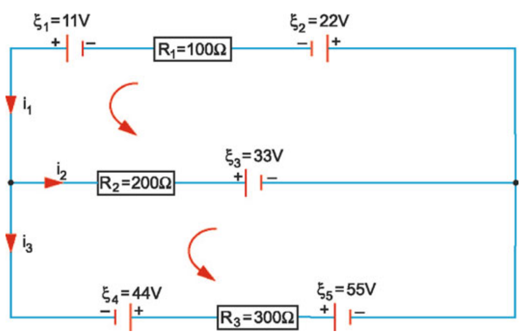


Fig. 12.49

12.53 **(i)** The equivalent resistance of the circuit is

$$R_{\text{eq}} = R_1 + R_2 + \frac{R_3 R_4}{R_3 + R_4}$$
$$= 5 + 10 + \frac{20 \times 60}{20 + 60} = 30\,\text{k}\Omega$$
$$V_1 = V \times \frac{R_1}{R_{\text{eq}}} = 300 \times \frac{5}{30} = 50\,\text{V}$$
$$V_2 = V \times \frac{R_2}{R_{\text{eq}}} = 300 \times \frac{10}{30} = 100\,\text{V}$$

(ii) $V_\text{p} = V \times \dfrac{R_3 R_4}{R_\text{eq}(R_3 + R_4)} = 300 \times \dfrac{20 \times 60}{30(20 + 60)} = 150\,\text{V}$

(iii) $I_\text{T} = \dfrac{V}{R_\text{eq}} = \dfrac{300}{30 \times 10^3} = 0.01\,\text{A}$

$$I_2 = I_\text{T}\frac{R_4}{R_4 + R_3} = 0.01 \times \frac{60}{60 + 20} = 0.0075\,\text{A}$$

$$I_4 = I_\text{T}\frac{R_3}{R_4 + R_3} = 0.01 \times \frac{20}{60 + 20} = 0.0025\,\text{A}$$

12.54 $i_1 - i_2 - i_3 = 0$ (junction theorem) (1)

Traversing clockwise the top loop (Fig. 12.50)

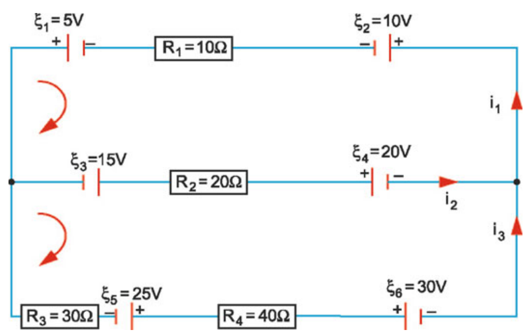


Fig. 12.50

$\xi_1 + i_1 R_1 - \xi_2 - \xi_4 + i_2 R_2 + \xi_3 = 0$ (loop theorem)

$\therefore \quad 5 + 10i_1 - 10 - 20 + 20i_2 + 15 = 0$

or $\quad i_1 + 2i_2 - 1 = 0$ (2)

Traversing clockwise the bottom loop

$-\xi_3 - i_2 R_2 + \xi_4 - \xi_6 + i_3 R_4 + \xi_5 + i_3 R_3 = 0$ (loop theorem)

$\therefore \quad -15 - 20i_2 + 20 - 30 + 40i_3 + 25 + 30i_3 = 0$

or $\quad 7i_3 - 2i_2 = 0$ (3)

The required equations are (1), (2) and (3) in three unknown currents.

12.55 $i_1 - i_2 - i_3 = 0$ (junction theorem) (1)

Traversing the top loop clockwise (Fig. 12.51)

$\xi_4 - i_1 R_4 - \xi_5 - i_2 R_3 - \xi_2 - i_2 R_2 = 0$

$\therefore \quad 50 - 200i_1 - 30 - 80i_2 - 10 - 120i_2 = 0$

or $\quad 20i_1 + 20i_2 - 1 = 0$ (2)

Traversing the outer loop clockwise

$\xi_4 - i_1 R_4 - \xi_5 - i_3 R_5 - \xi_3 - i_3 R_1 + \xi_1 - i_3 R_6 = 0$

$50 - 200i_1 - 30 - 100i_3 - 20 - 60i_3 + 40 - 40i_3 = 0$

or $\quad 5i_1 + 5i_2 - 1 = 0$ (3)

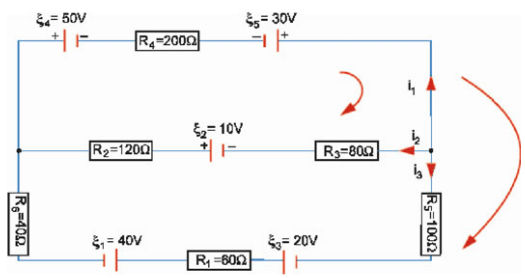


Fig. 12.51

The required equations are (1), (2) and (3) in three unknown currents i_1, i_2 and i_3. Note that by considering the bottom loop no new information is provided as it is already contained in the other two loops. In general it is sufficient to consider any two loops out of three.

12.56 Traversing the loop ABCDA clockwise

$-iR - i_1 R_1 + \xi_2 - ir_2 - \xi_1 - ir_1 = 0$

$i_1 = 2i/3, \; i_2 = i/3$

$\therefore \quad -4.5i - 3 \times \dfrac{2i}{3} + 8 - i - 4 - 0.5i = 0$

$\therefore \quad i = 0.5\,\text{A}, \; i_1 = 0.33\,\text{A}, \; i_2 = 0.165\,\text{A}$

P.D over E_1

$V_1 = \xi_1 + ir_1 = 4 + 0.5 \times 0.5 = 4.25\,\text{V}$

P.D over E_2

$V_2 = \xi_2 - ir_2 = 8 - 0.5 \times 1 = 7.5\,\text{V}$