3.40 A block is allowed to slide down a frictionless track freely under gravity. The track ends in a circular loop of radius R. Show that the minimum height from which the block must start is $2.5R$ so that it completes the circular track.

3.41 A nail is located at a certain distance vertically below the point of suspension of a simple pendulum. The pendulum bob is released from a position where the string makes an angle 60° with the vertical. Calculate the distance of the nail from the point of suspension such that the bob will just perform revolutions with the nail as centre. Assume the length of the pendulum to be 1 m.

[Indian Institute of Technology 1975]

3.42 A test tube of mass 10 g closed with a cork of mass 1 g contains some ether. When the test tube is heated, the cork flies out under the pressure of the ether gas. The test tube is suspended by a weightless rigid bar of length 5 cm. What is the minimum velocity with which the cork would fly out of the test tube so that the test tube describes a full vertical circle about the point of suspension? Neglect the mass of ether.

[Indian Institute of Technology 1969]

3.43 A car travels at a constant speed of 14.0 m/s round a level circular bend of radius 45 m. What is the minimum coefficient of static friction between the tyres and the road in order for the car to go round the bend without skidding?

[University of Manchester 2008]

3.3 Solutions

3.3.1 Motion in a Horizontal Plane

3.1 $x = a\cos t$ (1)

$y = a\sin t$ (2)

$z = t$ (3)

Squaring (1) and (2) and adding

$$x^2 + y^2 = a^2(\cos^2 t + \sin^2 t) = a^2$$

which is the equation of a circle.

Since $z = t$, the circular path drifts along the z-axis so that the path is a helix.

3.2 $a = \dfrac{dv}{dt} = k^2rt^2$

$$v = \int dv = k^2r\int t^2 dt = \frac{k^2r\,t^3}{3}$$

Power, $P = Fv = mav = mk^2r\,t^2\dfrac{k^2r\,t^3}{3} = \dfrac{mk^4r^2t^5}{3}$

3.3 $v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$

$= \sqrt{5^2 + (3 \times 4)^2} = 13\,\text{m/s}$

3.4 $a_\text{N} = \omega^2 r = a_\text{T} = 10$

$$\omega = \sqrt{\frac{10}{40}} = 0.5\,\text{rad/s}$$

$$\omega = \omega_0 + \alpha t = 0 + \frac{a_\text{T}\, t}{r}$$

$$t = \frac{\omega r}{a_\text{T}} = \frac{0.5 \times 40}{10} = 2\,\text{s}$$

3.5 $x = ct^3$

$$v = \frac{\mathrm{d}x}{\mathrm{d}t} = 3ct^2 = 3 \times 0.3 \times 10^{-2} t^2 = 0.4$$

$$t = \frac{20}{3}\,\text{s}$$

$$a_\text{N} = \frac{v^2}{r} = \frac{(0.4)^2}{0.04} = 4\,\text{m/s}^2$$

$$a_\text{T} = \frac{\mathrm{d}v}{\mathrm{d}t} = 6ct = 6 \times 0.3 \times 10^{-2} \times \frac{20}{3} = 0.12\,\text{m/s}^2$$

3.6 **(a)** $x = r\cos\theta$

$$y = r\sin\theta$$

$$\vec{r} = \hat{i}x + \hat{j}y$$

$$\theta = \omega t$$

where θ is the angle which the radius vector makes with the x-axis and ω is the angular speed.

$$\vec{r} = \hat{i}(r\ \cos\ \omega t) + \hat{j}(r\ \sin\ \omega t)$$

(b) $\dot{\vec{r}} = -\hat{i}(\omega r\ \sin\ \omega t) + \hat{j}(\omega r\ \cos\ \omega t)$

$$\vec{a} = \ddot{\vec{r}} = -\hat{i}(\omega^2 r\ \cos\ \omega t) - \hat{j}(\omega^2 r\ \sin\ \omega t)$$

$$= -\omega^2 r\,(\hat{i}\cos\ \omega t + \hat{j}\ \sin\ \omega t)$$

$$= -\omega^2(\hat{i}x + \hat{j}y)$$

$$\vec{a} = -\omega^2\vec{r}$$

where we have used the expression for the position vector $\vec{r}$. The last relation shows that by virtue of minus sign $\vec{a}$ is oppositely directed to $\vec{r}$, i.e. $\vec{a}$ is directed radially inwards.

3.7 $a = \sqrt{a_N^2 + a_T^2}$ (Fig. 3.9)

$$\frac{a_N}{a_T} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$a_N = \frac{a_T}{\sqrt{3}} \tag{1}$$

$$a_T = \alpha R \tag{2}$$

$$\omega = \alpha t \tag{3}$$

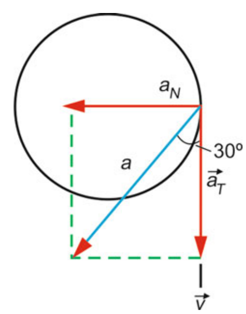
$$\therefore \quad a_N = \omega^2 R = \frac{a_T}{\sqrt{3}} = \frac{\alpha R}{\sqrt{3}}$$

$$\omega^2 = \frac{\alpha}{\sqrt{3}}$$

$$\alpha^2 t^2 = \frac{\alpha}{\sqrt{3}}$$

$$\alpha = \frac{1}{\sqrt{3} t^2} = \frac{1}{\sqrt{3}.1^2} = 0.577\,\text{rad/s}^2$$

Fig. 3.9



3.8 $a = \sqrt{a_N^2 + a_T^2}$

$$12\sqrt{10} = \sqrt{(\omega^2 R)^2 + \alpha^2 R^2} = \sqrt{(\alpha^2 t^2 R)^2 + \alpha^2 R^2}$$

$$= \alpha R \sqrt{\alpha^2 t^4 + 1} = 3R\sqrt{3^2 \times 1^2 + 1}$$

$$R = 4\,\text{cm}$$

3.9 **(i)** Equating the centripetal force to the frictional force

$$\frac{m v_{\max}^2}{R} = \mu m g$$

$$\therefore \quad v_{\max} = \sqrt{\mu g R}$$

(ii) $v_{\max} = \sqrt{0.85 \times 9.8 \times 150} = 35.35\,\text{m/s}$

(iii) $a_{\text{N}} = \dfrac{v^2}{R} = \dfrac{(35.35)^2}{150} = 8.33\,\text{m/s}$ towards the centre of the circle

(iv) $\tan\theta = \dfrac{v^2}{gR} = \dfrac{(35.35)^2}{9.8 \times 150} = 0.85$

$\therefore \quad \theta = 40.36°$

3.10 Equating the horizontal component of the tension to the centripetal force

$$T\,\sin\alpha = m\omega^2 R \tag{1}$$

Furthermore, the bob has no acceleration in the vertical direction.

$$T\,\cos\alpha = mg \tag{2}$$

$$\tan\alpha = \frac{R}{H} = \frac{\omega^2 R}{g}$$

$$\therefore \quad \omega = \sqrt{\frac{g}{H}}$$

3.11 Using the results of prob. (3.10), the difference in the level of the bob

$$\Delta H = H_1 - H_2 = g\left[\frac{1}{\omega_1^2} - \frac{1}{\omega_2^2}\right] \tag{1}$$

$$\omega_1 = 2\pi\, f_1 = \frac{140}{60}\pi \tag{2}$$

$$\omega_2 = 2\pi\, f_2 = \frac{160}{60}\pi \tag{3}$$

Using (2) and (3) in (1) and $g = 980\,\text{cm}$, $\Delta H = 31.95\,\text{cm}$.

3.12 The centripetal force acting on the bob of the pendulum $= m\omega^2 r$, where r is the distance of the bob from the axis of rotation, Fig. 3.10. For equilibrium, the vertical component of the tension in the string of the pendulum must balance the weight of the bob

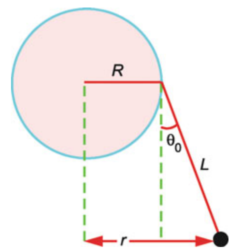
$$\therefore \quad T\,\cos\theta_0 = m\omega^2 r \tag{1}$$

Further, the horizontal component of the tension in the string must be equal to the centripetal force.

$$\therefore \quad T\,\sin\theta_0 = m\omega^2 r \tag{2}$$

Dividing (2) by (1)

Fig. 3.10



$$\tan\theta_0 = \frac{\omega^2 r}{g} = \frac{4\pi^2 n^2 r}{g} \tag{3}$$

where n = number of rotations per second. From the geometry of Fig. 3.10,

$$r = R + L\sin\theta_0$$
$$n = \frac{1}{2\pi}\sqrt{\frac{g\ \tan\theta_0}{R + L\ \sin\theta_0}}$$

3.13 The equilibrium condition requires that the centripetal force = the frictional force, $m\omega^2 r = \mu\, mg$

$$\therefore \quad f_{\max} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\mu g}{r}}$$

3.14 Let the spring length be stretched by x. Equating the centripetal force to the spring force

$$m\omega^2(L_0 + x) = kx$$
$$\therefore \quad x = \frac{m\omega^2 L_0}{k - m\omega^2}$$

Therefore, the new length L will be

$$L = L_0 + x = \frac{kL_0}{k - m\omega^2}$$

and the tension in the spring will be

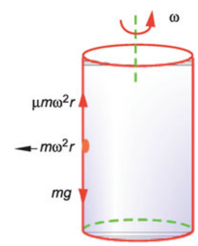
$$m\,\omega^2 L = \frac{m\,\omega^2 k L_0}{k - m\omega^2}$$

3.15 As the drum rotates with angular velocity ω, the normal reaction on the coin acting horizontally would be equal to $m\omega^2 r$, (Fig. 3.11). As the coin tends to slip down under gravity a frictional force would act vertically up.
If the coin is not to fall, the minimum frequency of rotation is given by the condition
Frictional force = weight of the coin

$$\mu m\omega^2 r = mg$$

$$\therefore \quad \omega = \frac{1}{2\pi}\sqrt{\frac{g}{\mu r}}$$

Fig. 3.11



3.16 The bead is to be in equilibrium by the application of three forces, the weight mg acting down, the centrifugal force $m\omega^2 R$ acting horizontally and the normal force acting radially along NO. Balancing the x- and z-components of forces (Fig. 3.12)

$$N \sin\theta = m\omega^2 R$$

$$N \cos\theta = mg$$

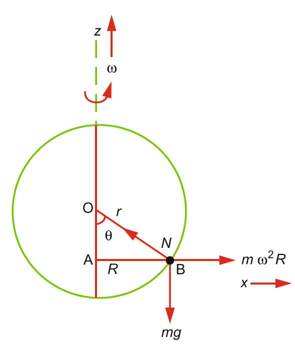


Fig. 3.12

Dividing the two equations

$$\tan\theta = \frac{\omega^2 R}{g} = \frac{\omega^2 r \sin\theta}{g}$$
$$\omega = \sqrt{\frac{g}{r\cos\theta}}$$

3.17 Since the wire is continuous, tension in the parts *AB* and *BC* will be identical. Equating the horizontal and vertical components of forces separately

$$\frac{mv^2}{r} = T\sin 30^\circ + T\sin 60^\circ \quad (1)$$
$$mg = T\cos 30^\circ + T\cos 60^\circ \quad (2)$$

As the right-hand sides of (1) and (2) are identical

$$\frac{mv^2}{r} = mg$$
$$\text{or} \quad v = \sqrt{gr}$$

3.18 Resolve the centripetal force along and normal to the funnel surface, Fig. 3.13. When the funnel rotates with maximum frequency, the cube tends to move up the funnel, and both the weight (mg) and the frictional force (μN) will act down the funnel surface, Fig. 3.13. Now

$$N = mg\cos\theta + m\omega^2 r\sin\theta$$

Taking the upward direction as positive, equation of motion is

$$m\omega^2 r\cos\theta - mg\sin\theta - \mu(mg\cos\theta + m\omega^2 r\sin\theta) = 0$$
$$\therefore \quad f_{\max} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{r}\frac{(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}}$$

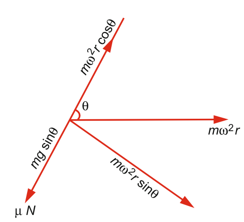


Fig. 3.13

3.19 At the minimum frequency of rotation, the cube tends to go down the surface and therefore the frictional force acts up the funnel. The equation of motion becomes

$$m\omega^2 r\cos\theta - mg\sin\theta + \mu(mg\cos\theta + m\omega^2 r\sin\theta) = 0$$

$$f_{\min} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{r}\frac{(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)}}$$

3.20 Tension is provided by the weight Mg

$$T = Mg \tag{1}$$

Three forces, weight (Mg), tension (T) and normal reaction ($m\omega^2 r$), are to be balanced:

$$T\sin\theta = m\omega^2 r \tag{2}$$

Further $r = L\sin\theta$ (3)

Combining (1), (2) and (3)

$$\omega^2 = \frac{Mg}{mL}$$

Frequency of rotation

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{Mg}{mL}}$$

3.21 The two forces acting at right angles are (i) weight (mg) and (ii) reaction (mv^2/r).

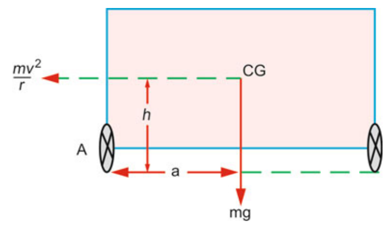
$$F = \sqrt{(mg)^2 + (mv^2/r)^2} = mg\sqrt{1 + (v^2/gr)^2}$$

Using $v = 7\,\text{m/s}$, $g = 9.8\,\text{m/s}^2$, $r = 100\,\text{m}$ and $m = 60\,\text{kg}$,

$$F = 60.075\,\text{kg wt.}$$

3.22 Figure 3.14 shows the rear of the carriage speeding with v, negotiating a circular curve of radius r. 'a' is half of the distance between the wheels and h is the height of the centre of gravity (CG) of the carriage above the ground. The centripetal force mv^2/r produces a counterclockwise torque about the left wheel at A. The weight of the carriage acting vertically down through the

Fig. 3.14



centre of gravity produces a clockwise torque. The condition for the maximum speed $v_{\max}$ is given by equating these two torques:

$$\frac{mv_{\max}^2}{r}h = mga$$

$$\text{or}\quad v_{\max} = \sqrt{\frac{gra}{h}} = \sqrt{\frac{9.8 \times 100 \times 0.75}{1.0}} = 27.11\,\text{m/s}$$

3.23 **(a)** When a vehicle takes a turn on a level road, the necessary centripetal force is provided by the friction between the tyres and the road. However, this results in a lot of wear and tear of tyres. Further, the frictional force may not be large enough to cause a sharp turn on a smooth road.

If the road is constructed so that it is tilted from the horizontal, the road is said to be banked. Figure 3.15 shows the profile of a banked road at an angle θ with the horizontal. The necessary centripetal force is provided by the horizontal component of the normal reaction N and the horizontal component of frictional force.

Three external forces act on the vehicle, and they are not balanced, the weight W, the normal reaction N, and the frictional force. Balancing the horizontal components

$$\frac{mv^2}{r} = \mu\, mg \cos^2\theta + N \sin\theta$$

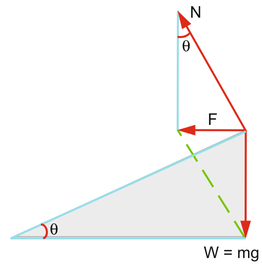
$$\text{or } N \sin\theta = \frac{mv^2}{r} - \mu\, mg \cos^2\theta \tag{1}$$

Balancing the vertical components

$$mg = N\cos\theta - \mu mg \cos\theta \sin\theta$$

$$\text{or } N\cos\theta = mg + \mu mg \cos\theta \sin\theta \tag{2}$$

Fig. 3.15



Dividing (1) by (2)

$$\tan\theta = \frac{v^2/r - \mu\, g \cos^2\theta}{g + \mu\, g\ \cos\theta \sin\theta}$$

$$v_{\max} = \sqrt{gr\,(\mu + \tan\theta)}$$

(b) For $\theta = 30°$, $\mu = 0.25$, $g = 9.8\,\text{m/s}^2$ and $r = 100\,\text{m}$, $v_{\max} = 28.47\,\text{m/s}$.

3.24 At latitude λ the distance r of a point from the axis of rotation will be $r = R\cos\lambda$
where R is the radius of the earth.
The angular velocity, however, is the same as for earth's rotation

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{86,400} = 7.27 \times 10^{-5}\,\text{rad/s}$$

The linear velocity

$$v = \omega r = \omega R\cos\lambda = 7.27 \times 10^{-5} \times 6.4 \times 10^6 \times \cos 60°$$
$$= 232.64\,\text{m/s}$$

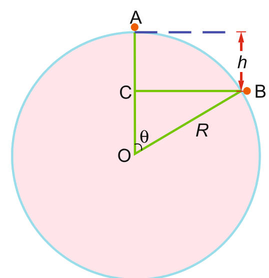
3.25 The speed of the plane must be equal to the linear velocity of a point on the surface of the earth. Suppose the plane is flying close to the earth's surface, $\omega = 7.27 \times 10^{-5}$ rad/s (*see* prob. 3.24)

$$v = \omega R = 7.27 \times 10^{-5} \times 6.4 \times 10^6 = 465.28\,\text{m/s}$$
$$= 1675\,\text{km/h}.$$

3.3.2 Motion in a Vertical Plane

3.26 Let a particle of mass m be placed at A, the highest point on the sphere of radius R with the centre of O. Let it slide down from rest along the arc of the great circle and leave the surface at B, at depth h below A, Fig. 3.16. Let the radius OB make an angle θ with the vertical line OA. The centripetal

Fig. 3.16



force experienced by the particle at B is mv^2/R, where v is the velocity of the particle at this point. Now the weight mg of the particle acts vertically down so that its component along the radius BO is $mg\cos\theta$. So long as $mg\cos\theta >$ mv^2/R the particle will stick to the surface. The condition that the particle will leave the surface is

$$mg\cos\theta = \frac{mv^2}{R} \tag{1}$$

$$\text{or} \quad \cos\theta = \frac{v^2}{gR} \tag{2}$$

Now, in descending from A to B, the potential energy is converted into kinetic energy

$$mgh = \frac{1}{2}mv^2 \tag{3}$$

$$\text{or} \quad \frac{v^2}{g} = 2h \tag{4}$$

using (4) in (2)

$$\cos\theta = \frac{2h}{R} \tag{5}$$

Drop a perpendicular BC on AO.

$$\text{Now} \ \cos\theta = \frac{\text{OC}}{\text{OB}} = \frac{R-h}{R} \tag{6}$$

Combining (5) and (6), $\quad h = \dfrac{R}{3}$

Thus the particle will leave the sphere at a point whose vertical distance below the highest point is $\dfrac{R}{3}$.

3.27 At the highest point A the tension T_A acts vertically up, the centrifugal force also acts vertically up but the weight acts vertically down. We can then write

$$T_A = \frac{mv_A^2}{r} - mg \tag{1}$$

where m is the mass of the sphere, v_A is its speed at the point A and r is the radius of the vertical circle.
At the lowest point B both the centrifugal force and the weight act vertically down and both add up to give the tension T_B. If v_B is the speed at B, then we can write

$$T_B = \frac{mv_B^2}{r} + mg \tag{2}$$

By problem

$$T_B = 3T_A \tag{3}$$

Combining (1), (2) and (3), we get

$$v_B^2 = 3v_A^2 - 4gr \tag{4}$$

Conservation of mechanical energy requires that loss in potential energy = gain in kinetic energy. Therefore, in descending from A to B,

$$mg2r = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 \tag{5}$$

$$\text{or} \quad v_B^2 = v_A^2 + 4gr \tag{6}$$

From (4) and (6) we get

$$v_A = \sqrt{4gr} = \sqrt{4 \times 980 \times 30} = 343\,\text{cm/s}$$

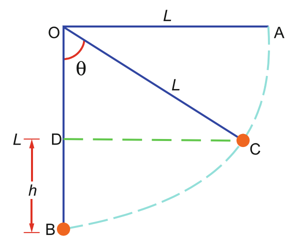
3.28 Measure potential energy from the equilibrium position B, Fig. 3.17. At A the total mechanical energy $E = mgL$ as the pendulum is at rest. As it passes through C let its speed be v. The potential energy will be mgh, where h = BD and CD is perpendicular on the vertical OB. Now

$$h = L - L\cos\theta = L(1 - \cos\theta) \tag{1}$$

Energy conservation gives

$$mgL = mgL\,(1 - \cos\theta) + \frac{1}{2}mv^2 \tag{2}$$

Fig. 3.17



The tension in the string at C will be

$$T = \frac{mv^2}{L} + mg\,\cos\theta \tag{3}$$

By problem, $T = mg$ (4)

Combining (2), (3) and (4) we get $\cos\theta = \frac{1}{3}$ or $\theta = \cos^{-1}\left(\frac{1}{3}\right)$.

3.29 At the top of the sphere, v is in the horizontal direction and the frictional force acts upwards. The condition that the motorcyclist may not fall is
Friction force = Weight

$$\mu\frac{mv^2}{r} = mg$$

$$v = \sqrt{\frac{gr}{\mu}} = \sqrt{\frac{9.8 \times 10}{0.8}} = 11\ \mathrm{m/s}$$

3.30 At the lowest point A, Fig. 3.18, the tension in the string is

$$T_{\mathrm{A}} = \frac{mv_{\mathrm{A}}^2}{L} + mg \tag{1}$$

where v_{A} is the velocity at point A.

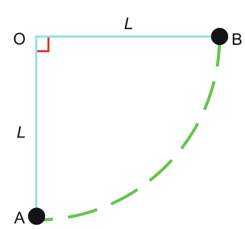


Fig. 3.18

Measure potential energy with respect to A, the equilibrium position. At the point B, at height L the mechanical energy is entirely potential energy as the bob is at rest. As the vertical height is L, the potential energy will be mgL.
When the bob is released, at the point A, the energy is entirely kinetic, potential energy being zero, and is equal to $\frac{1}{2}mv_A^2$.
Conservation of mechanical energy requires that

$$\frac{1}{2}mv_A^2 = mgL$$

$$\text{or} \quad v_A^2 = 2gL \tag{2}$$

Using (2) in (1)

$$T_A = 2mg + mg = 3mg$$

Thus the minimum strength of the string that it may not break upon passing through the lowest point is three times the weight of the bob.

3.31 Let the ball be deflected through a small angle θ from the equilibrium position A, Fig. 3.19.

$$\theta = \frac{s}{L} \tag{1}$$

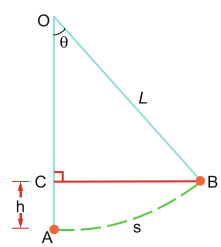


Fig. 3.19

where s is the corresponding arc. Drop a perpendicular BC on AO, so that the height through which the bob is raised is AC $= h$.
Now, $h = \mathrm{AC} = \mathrm{OA} - \mathrm{OC} = L - L\cos\theta = L(1-\cos\theta)$

$$= L\left[1 - 1 + \frac{\theta^2}{2!} + \cdots\right]$$

$$\therefore \quad h = \frac{L\theta^2}{2} = \frac{s^2}{2L} \tag{2}$$

where we have used (1).

From energy conservation, $mgh = \frac{1}{2}mv^2$

$\therefore \quad v = \sqrt{2gh} = s\sqrt{\frac{g}{L}}$

3.32 In coming down from angular displacement of 60° to 45°, loss of potential energy is given by

$$\begin{aligned} mg(h_1 - h_2) &= mgL(1 - \cos 60°) - mgL(1 - \cos 45°) \\ &= 0.207\, mgL \end{aligned}$$

Gain in kinetic energy $= \frac{1}{2}mv^2$

$\therefore \quad \frac{1}{2}mv^2 = 0.207\, mgL = 0.207\, mg \quad (\because \quad L = 1\text{ m})$

or $\quad v = 2.014\,\text{m/s}$

The tension in the string would be

$$\begin{aligned} T &= \frac{mv^2}{L} + mg\cos 45° \\ &= mg\left[\frac{4.056}{gL} + 0.707\right]\text{N} = 1.12\, mg\text{ N} \end{aligned}$$

3.33 When the bob is displaced through angle θ, the potential energy is $mgL(1 - \cos\theta)$. At the lowest position the energy is entirely kinetic

$$\frac{1}{2}mv^2 = mg\,L\,(1 - \cos\theta) \tag{1}$$

The tension in the string will be

$$T = mg + \frac{mv^2}{L} = mg + 2mg\,(1 - \cos\theta) \tag{2}$$

where we have used (1)
By problem

$$T = 2mg \tag{3}$$

From (2) and (3) we find $\cos\theta = \frac{1}{2}$ or $\theta = 60°$

3.3.3 Loop-the-Loop

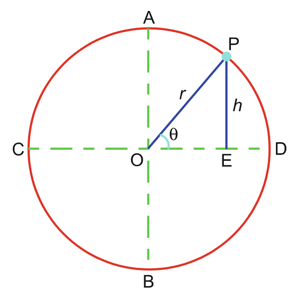
3.34 If the bob of the pendulum has velocity u at B, the bottom of the vertical circle of radius r such that

$$\sqrt{2gr} < u < \sqrt{5gr} \tag{1}$$

then the bob would leave some point P on the are DA (Fig. 3.20). Here

$$\sqrt{2gr} = \sqrt{2 \times 9.8 \times 1} = 4.427\,\text{m/s and}\,\sqrt{5gr} = \sqrt{5 \times 9.8 \times 1} = 7\,\text{m/s}$$

Fig. 3.20



Therefore (1) is satisfied for $u = 6\,\text{m/s}$.
Drop a perpendicular PE on the horizontal CD. Let PE $= h$ and PO make an angle θ with OD. When the bob leaves the point P, the normal reaction must vanish.

$$\frac{mv^2}{r} - mg\sin\theta = 0 \tag{2}$$

Loss in kinetic energy = gain in potential energy

$$\frac{1}{2}m\,u^2 - \frac{1}{2}mv^2 = mg(h + r) \tag{3}$$

$$\sin\theta = \frac{h}{r} \tag{4}$$

Eliminating v^2 between (2) and (3) and using (3), with $u = 6\,\text{m/s}$ and $r = 1.0\,\text{m}$,

$$h = \frac{1}{3}\left[\frac{u^2}{g} - 2r\right] = 0.558\,\text{m}$$

3.35 Let the height of the incline be h. Then the velocity of the block at the bottom of the vertical circle will be $v = \sqrt{2gh}$. Minimum height is given by the condition that $v = \sqrt{5gr}$ which is barely needed for the completion of the loop.

$$\sqrt{2gh} = \sqrt{5gr}$$
$$\text{or } h = \frac{5}{2}r = \frac{5}{2} \times 12 = 30\,\text{cm}$$

3.36 The analysis is similar to that of prob. (2.34). The velocity of the particle at the bottom of the circular groove will be given by

$$v = \sqrt{(2g)(2r)} = \sqrt{4gr} \tag{1}$$

which satisfies the condition

$$\sqrt{2gr} < u < \sqrt{5gr}$$

The particle leaves the circular groove at a height h above the centre of the circle, Fig. 3.20.

$$h = \frac{1}{3}\left[\frac{u^2}{g} - 2r\right] \tag{2}$$

But $u^2 = 4gr$ (1)

$$\therefore \quad h = \frac{2}{3}r$$

Thus, the particle leaves the circular groove at a height of $h + r = \frac{5}{3}r$ above the lowest point.

3.37 Let the velocity at B be v.

Kinetic energy gained = potential energy lost

$$\frac{1}{2}mv^2 = mg(5R - R)$$
$$\therefore \quad m\frac{v^2}{R} = 8\,mg$$

which is the centrifugal force acting on the track horizontally. The weight acts vertically down. Hence the resultant force

$$F = \sqrt{(8\,mg)^2 + (mg)^2} = \sqrt{65}\,mg$$

3.38 Gain in kinetic energy = loss of potential energy

$$\frac{1}{2}mv^2 = mg(h - 2R)$$
$$v^2 = 2g\,(h - 2R) \tag{1}$$

The force exerted on the track at the top

$$F = \frac{mv^2}{R} - mg \tag{2}$$

By problem

$$F = mg \tag{3}$$
$$\therefore \quad \frac{mv^2}{R} - mg = mg$$
$$\text{or} \quad v^2 = 2gR \tag{4}$$

Using (4) in (1) we find $h = 3R$.

3.39 Let the particle velocity at the lowest position be $u = 0.8944\sqrt{5gR}$ and v at point P.
Loss in kinetic energy = gain in potential energy

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mg(R + R\sin\theta)$$
$$\text{or} \quad v^2 = \left(0.8944\sqrt{5gR}\right)^2 - 2gR(1 + \sin\theta) \tag{1}$$

The particle would leave at P (Fig. 3.7) when

$$\frac{mv^2}{R} = mg\sin\theta$$
$$\text{or} \quad v^2 = gR\sin\theta \tag{2}$$

Using (2) in (1) and solving
$\sin\theta = 2/3$ or $\theta = 41.8°$

3.40 Let the minimum height be h. The velocity of the block at the beginning of the circular track will be

$$v = \sqrt{2gh} \tag{1}$$

For completing the circular track

$$v = \sqrt{5gR} \tag{2}$$

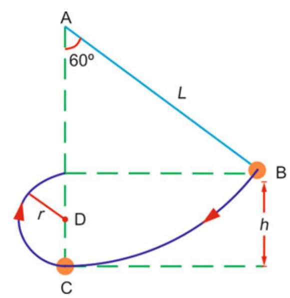
From (1) and (2)

$$h = 2.5R$$

3.41 Let the nail be located at D at distance x vertically below A, the point of suspension, Fig. 3.21. Initially the bob of the pendulum is positioned at B at height h above the equilibrium position C.

$$h = L(1 - \cos\theta) = L(1 - \cos 60^\circ) = \frac{1}{2}L \tag{1}$$

Fig. 3.21



where $L = 1$ m is the length of the pendulum. When the pendulum is released its velocity at C will be

$$\upsilon = \sqrt{2gh} = \sqrt{gL} \tag{2}$$

The velocity needed at C to make complete revolution in the vertical circle centred at the nail and radius r is

$$\upsilon = \sqrt{5gr} \tag{3}$$

From (2) and (3)

$$r = \frac{1}{5}L \tag{4}$$

Therefore $x = \mathrm{AD} = \mathrm{AC} - \mathrm{DC} = L - \dfrac{L}{5} = 0.8\,L$

$$= 0.8 \times 1\,\mathrm{m} = 80\,\mathrm{cm}$$

3.42 If M and m are the mass of the test tube and cork, respectively, and their velocity V and υ respectively, momentum conservation gives

$$MV = mv \tag{1}$$

$$\text{or} \quad v = \frac{M}{m}V = \frac{10}{1}V = 10\,V \tag{2}$$

Condition for describing a full vertical circle is that the minimum velocity of the test tube should be

$$V = \sqrt{5gr} = \sqrt{5 \times 980 \times 5} = 156.5\,\text{cm/s}$$

Therefore the minimum velocity of the cork which flies out ought to be

$$v = 10\ V = 1565\,\text{cm/s} = 15.65\,\text{m/s}$$

3.43 Equating centripetal force to frictional force

$$\frac{mv^2}{r} = \mu\, mg$$

$$\mu = \frac{v^2}{gr} = \frac{(14)^2}{9.8 \times 45} = \frac{4}{9}$$