ANOVA Sample Problem and Solution

One Way ANOVA

Problem Statement
A nutritionist conducted a study to investigate the effect of three different dietary supplements on weight loss. A total of 60 participants were randomly assigned to one of three treatment groups: Group A received Supplement X, Group B received Supplement Y, and Group C received Supplement Z. After a 12-week intervention, the participants' weight loss was measured in pounds. The nutritionist wants to determine if there are significant differences in weight loss among the three treatment groups.

The following table presents the weight loss data for each treatment group:

Treatment Group	Weight Loss (in pounds)
Group A	8, 9, 11, 10, 7, 12, 9
Group B	6, 5, 7, 8, 9, 6, 7
Group C	12, 14, 13, 10, 15, 12, 11

Hint:
Grand Total = 201

$$\sum x_{ij}^2 = 2{,}079$$

Part 1: Hypotheses and F-Test
State the null hypothesis (H0) and the alternative hypothesis (H1) to test whether there are significant differences in weight loss among the three treatment groups. Perform an F-test to determine if there is a significant difference at a 0.05 significance level. Calculate the F-statistic and compare it to the critical F-value.

Part 2: Tukey's Method for Paired Comparisons
If you find a significant difference in Part 1, apply Tukey's Method for paired comparisons to identify which treatment groups differ significantly from each other. Determine which pairs of treatment groups show significant differences in weight loss, and state your conclusions.

Hypotheses:

Null Hypothesis (H0): There is no significant difference in weight loss among the three treatment groups ($\mu1 = \mu2 = \mu3$, where $\mu1$ represents the mean weight loss for Group A, $\mu2$ for Group B, and $\mu3$ for Group C).

Alternative Hypothesis (H1): There is a significant difference in weight loss among the three treatment groups (at least one group mean is different).

One-Way ANOVA table with Mean Square (MS) and the F-statistic:

Source	df	SS	MS	F
Treatments Between Groups	2	108.14	MST= 54.428	21.167
Error	18	46.86	MSE=2.571	
Total	20	155.142		

Hint: Use the formula shown in the lecture to calculate the ANOVA table.

One-Way ANOVA table with Mean Square (MS) and the F-statistic:

Source	df	SS	MS	F
Treatments Between Groups	df1 2	108.14	MST= 54.428	21.167
Error	df2 18	46.86	MSE=2.571	
Total	20	155.142		

At a significance level of 0.05 and df1 = 2, df2 = 18, you can find the critical F-value from an F-table. For α=0.05, the critical F-value is approximately 3.552.

Degrees of freedom in denominator (df2)	p	Degrees of freedom in numerator (df1) 1	2	3	4	5	6	7	8	12	24	1000
10	0.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.28	2.18	2.06
	0.050	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	2.91	2.74	2.54
	0.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.62	3.37	3.09
	0.010	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.71	4.33	3.92
	0.001	21.04	14.90	12.55	11.28	10.48	9.93	9.52	9.20	8.45	7.64	6.78
12	0.100	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.15	2.04	1.91
	0.050	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.69	2.51	2.30
	0.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.28	3.02	2.73
	0.010	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.16	3.78	3.37
	0.001	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.00	6.25	5.44
14	0.100	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.05	1.94	1.80
	0.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.53	2.35	2.14
	0.025	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.05	2.79	2.50
	0.010	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	3.80	3.43	3.02
	0.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.13	5.41	4.62
16	0.100	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	1.99	1.87	1.72
	0.050	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.42	2.24	2.02
	0.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	2.89	2.63	2.32
	0.010	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.55	3.18	2.76
	0.001	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.20	5.55	4.85	4.08
18	0.100	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	1.93	1.81	1.66
	0.050	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.34	2.15	1.92
	0.025	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.77	2.50	2.20
	0.010	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.37	3.00	2.58
	0.001	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.13	4.45	3.69
20	0.100	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.89	1.77	1.61
	0.050	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.28	2.08	1.85
	0.025	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.68	2.41	2.09
	0.010	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.23	2.86	2.43
	0.001	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	4.82	4.15	3.40
30	0.100	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.77	1.64	1.46
	0.050	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.09	1.89	1.63

Compare the calculated F-statistic to the critical F-value:

Calculated F-Statistic (21.167) > Critical F-Value (3.552)
Since the calculated F-statistic is greater than the critical F-value, we reject the null hypothesis.

Tukey's Method for Paired Comparisons

Use Tukey's method to determine which of the three population means differ from the others.

	Group A	**Group B**	**Group C**
	$T_1 = 66$	$T_2 = 48$	$T_3 = 87$
Means	66/7 = 9.43	48/7 = 6.86	87/7 = 12.43

Tukey's Method for Paired Comparisons

Use Tukey's method to determine which of the three population means differ from the others.

$$\omega = q(2,18) * \frac{\sqrt{MSE}}{\sqrt{n}}$$

MSE = 2.571

Treatment Sample =7

*The critical values for q corresponding to alpha = .05 (top) and alpha = .01 (bottom)

df for Error Term	k= Number of Treatments								
	2	3	4	5	6	7	3	9	10
5	3.64 5.70	4.60 6.98	5.22 7.80	5.67 8.42	6.03 8.91	6.33 9.32	6.58 9.67	6.80 9.97	6.99 10.24
6	3.46 5.24	4.34 6.33	4.90 7.03	5.30 7.56	5.63 7.97	5.90 8.32	6.12 8.61	6.32 8.87	6.49 9.10
7	3.34 4.95	4.16 5.92	4.68 6.54	5.06 7.01	5.36 7.37	5.61 7.68	5.82 7.94	6.00 8.17	6.16 8.37
8	3.26 4.75	4.04 5.64	4.53 6.20	4.89 6.62	5.17 6.96	5.40 7.24	5.60 7.47	5.77 7.68	5.92 7.86
9	3.20 4.60	3.95 5.43	4.41 5.96	4.76 6.35	5.02 6.66	5.24 6.91	5.43 7.13	5.59 7.33	5.74 7.49
10	3.15 4.48	3.88 5.27	4.33 5.77	4.65 6.14	4.91 6.43	5.12 6.67	5.30 6.87	5.46 7.05	5.60 7.21
11	3.11 4.39	3.82 5.15	4.26 5.62	4.57 5.97	4.82 6.25	5.03 6.48	5.20 6.67	5.35 6.84	5.49 6.99
12	3.08 4.32	3.77 5.05	4.20 5.50	4.51 5.84	4.75 6.10	4.95 6.32	5.12 6.51	5.27 6.67	5.39 6.81
13	3.06 4.26	3.73 4.96	4.15 5.40	4.45 5.73	4.69 5.98	4.88 6.19	5.05 6.37	5.19 6.53	5.32 6.67
14	3.03 4.21	3.70 4.89	4.11 5.32	4.41 5.63	4.64 5.88	4.83 6.08	4.99 6.26	5.13 6.41	5.25 6.54
15	3.01 4.17	3.67 4.84	4.08 5.25	4.37 5.56	4.59 5.80	4.78 5.99	4.94 6.16	5.08 6.31	5.20 6.44
16	3.00 4.13	3.65 4.79	4.05 5.19	4.33 5.49	4.56 5.72	4.74 5.92	4.90 6.08	5.03 6.22	5.15 6.35
17	2.98 4.10	3.63 4.74	4.02 5.14	4.30 5.43	4.52 5.66	4.70 5.85	4.86 6.01	4.99 6.15	5.11 6.27
18	2.97 4.07	3.61 4.70	4.00 5.09	4.28 5.38	4.49 5.60	4.67 5.79	4.82 5.94	4.96 6.08	5.07 6.20
19	2.96	3.59	3.98	4.25	4.47	4.65	4.79	4.92	5.04

Tukey's Method for Paired Comparisons

1.Interpreting Tukey's Pairwise Comparisons:

1. Tukey's Method determines which specific pairs of groups are significantly different from each other based on mean differences and confidence intervals.
2. It provides a binary decision for each pair: either there is a significant difference or there isn't, based on the calculated critical values and confidence intervals.
3. Tukey's Method helps identify which pairwise comparisons have sufficient evidence to conclude that the groups are different in terms of the measured variable (e.g., weight loss).

2.Considering the Effect Size (ω):

1. The effect size (ω) is a measure of the proportion of total variance in the dependent variable (e.g., weight loss) that can be attributed to the differences between the treatment groups.
2. It provides a continuous measure of the magnitude of the treatment effect, indicating the strength of the relationship between the groups and the outcome.
3. A larger effect size (ω) suggests that the treatment groups explain a greater proportion of the total variance.

3.Comparing Tukey's Results with Effect Size:

1. When Tukey's Method indicates a significant difference between two groups, it means that, on average, those groups are distinct in terms of the measured variable.
2. The effect size (ω) complements this by quantifying the extent of the difference. A larger effect size suggests a more substantial difference between the groups.
3. If Tukey's Method indicates significance and the effect size (ω) is large, it confirms both the existence and the practical importance of the difference.

Calculate the mean difference for each pair:

•For Groups A and B (Mean Difference_AB):

- (Group A Mean) ≈ 9.43
- (Group B Mean) ≈ 6.86

$MeanDifference\ AB = 9.43 - 6.86 \approx 2.57$

•For Groups A and C (Mean Difference_AC):

- (Group A Mean) ≈ 9.43
- (Group C Mean) ≈ 12.57

$MeanDifference\ AC = 9.43 - 12.57 \approx -3.14$

•For Groups B and C (Mean Difference_BC):

- (Group B Mean) ≈ 6.86
- (Group C Mean) ≈ 12.57

$MeanDifference\ BC = 6.86 - 12.57 \approx -5.71$

For example:

If Tukey's Method finds a significant difference between Group A and Group C and the effect size (ω) is large (indicating a substantial proportion of variance explained), it suggests that not only is there a statistically significant difference between these groups, but it is also practically meaningful.

Tukey's Method for Paired Comparisons

Comparison between Group A and Group B:
Mean Difference (Group A - Group B) ≈ 2.57 pounds
Effect Size (ω) ≈ 0.712
Explanation: Although the mean difference between Group A and Group B is approximately 2.57 pounds, the effect size (ω) of 0.712 suggests that a substantial proportion of the total variance in weight loss can be attributed to the treatment groups. However, based on Tukey's Method analysis, this particular mean difference is not statistically significant, indicating that despite the large effect size, there is no strong evidence of a significant difference in weight loss between these two groups.

Comparison between Group A and Group C:
Mean Difference (Group A - Group C) ≈ -3.14 pounds
Effect Size (ω) ≈ 0.712
Explanation: The mean difference between Group A and Group C is approximately -3.14 pounds. Importantly, the effect size (ω) of 0.712 suggests that a substantial proportion of the total variance in weight loss is attributable to the treatment groups. Moreover, based on Tukey's Method analysis, this mean difference is statistically significant, indicating that Group C has a significantly higher weight loss compared to Group A. The combination of a large effect size and statistical significance underscores the meaningful difference in weight loss outcomes between these two groups.

Comparison between Group B and Group C:
Mean Difference (Group B - Group C) ≈ -5.71 pounds
Effect Size (ω) ≈ 0.712

Explanation: The mean difference between Group B and Group C is approximately -5.71 pounds. Similar to the previous comparisons, the effect size (ω) of 0.712 suggests that a substantial proportion of the total variance in weight loss is explained by the treatment groups. Furthermore, based on Tukey's Method analysis, this mean difference is statistically significant, indicating that Group C has a significantly higher weight loss compared to Group B. The large effect size reinforces the meaningful difference in weight loss outcomes between these two groups.

Calculate the mean difference for each pair:

•For Groups A and B (Mean Difference_AB):
- (Group A Mean) ≈ 9.43
- (Group B Mean) ≈ 6.86

$MeanDifference\ AB = 9.43 - 6.86 \approx 2.57$

•For Groups A and C (Mean Difference_AC):
- (Group A Mean) ≈ 9.43
- (Group C Mean) ≈ 12.57

$MeanDifference\ AC = 9.43 - 12.57 \approx -3.14$

•For Groups B and C (Mean Difference_BC):
- (Group B Mean) ≈ 6.86
- (Group C Mean) ≈ 12.57

$MeanDifference\ BC = 6.86 - 12.57 \approx -5.71$

Problem: Two-Way ANOVA Analysis

A pharmaceutical company is conducting a study to evaluate the effectiveness of a new drug treatment for a medical condition. The study considers two independent variables: dosage level (Low, Medium, High) and patient age group (Young, Middle-aged, Senior). The company is interested in assessing whether the drug's effectiveness varies based on these variables. The data collected represents the number of patients who experienced symptom relief after treatment.

	Age Group			
Dosage	**Young**	**Middle-Aged**	**Senior**	T_i
Low	35	28	21	84
Medium	52	47	36	135
High	65	59	45	169
B_j	152	134	102	388

$$\sum x_{ij}^2 = 18{,}390$$

Questions:

1. Perform the two-way ANOVA and provide the F-statistics, p-values, and any significant main effects or interactions.
2. Interpret the results and discuss the effects of dosage level, patient age group, and their interaction on the drug's effectiveness.

Two-Way ANOVA Table

Source	df	SS	MS	F
Treatments (Dosage)	2	1220.222	610.111	161.500
Blocks (Age Group)	2	427.556	213.778	56.588
Error	4	15.111	3.778	
Total	11	1662.889		

Hint: Use the formulas given in the slides to calculate the table

For either treatment or block means, we can test:

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \ldots \quad \text{versus}$$

$$H_1 : \text{at least one mean is different}$$

Two-Way ANOVA Table

Source	df	SS	MS	F
Treatments (Dosage)	2	1220.222	610.111	161.500
Blocks (Age Group)	2	427.556	213.778	56.588
Error	4	15.111	3.778	
Total	11	1662.889		

To test the H_0 that treatment (or block) means are equal

$$\text{Test Statistic}: F = \frac{\text{MST}}{\text{MSE}} \left(\text{or } F = \frac{\text{MSB}}{\text{MSE}}\right)$$

Reject H_0 if $F > F_\alpha$ with k-1 (or $b-1$) and $(b-1)(k-1)$ df.

THE SEEDLING PROBLEM-ANOVA

Source	df	SS	MS	F
Treatments (Dosage)	2	1220.222	610.111	161.500
Blocks (Age Group)	2	427.556	213.778	56.588
Error	4	15.111	3.778	
Total	11	1662.889		

df_2	a	1	2
1	.100	39.86	49.50
	.050	161.4	199.5
	.025	647.8	799.5
	.010	4052	4999.5
	.005	16211	20000
2	.100	8.53	9.00
	.050	18.51	19.00
	.025	38.51	39.00
	.010	98.50	99.00
	.005	198.5	199.0
3	.100	5.54	5.46
	.050	10.13	9.55
	.025	17.44	16.04
	.010	34.12	30.82
	.005	55.55	49.80
4	.100	4.54	4.32
	.050	7.71	6.94
	.025	12.22	10.65
	.010	21.20	18.00
	.005	31.33	26.28
5	.100	4.06	3.78
	.050	6.61	5.79
	.025	10.01	8.43
	.010	16.26	13.27
	.005	22.78	18.31
6	.100	3.78	3.46
	.050	5.99	5.14
	.025	8.81	7.26
	.010	13.75	10.92
	.005	18.63	14.54

To test for a difference due to dosage:

Null Hypothesis (H0) for Dosage:

•H0: There is no significant difference in Symptom Relief Count among the different dosage levels (Low, Medium, High).

Alternative Hypothesis (Ha) for Dosage:

•Ha: There is a significant difference in Symptom Relief Count among at least two of the dosage levels (Low, Medium, High).

THE SEEDLING PROBLEM-ANOVA

Source	df	SS	MS	F
Treatments (Dosage)	2	1220.222	610.111	161.500
Blocks (Age Group)	2	427.556	213.778	56.588
Error	4	15.111	3.778	
Total	11	1662.889		

To test for a difference due to dosage:

F_Dosage (F-statistic for Dosage) ≈ 161.5

For critical F-value for α = 0.05 is 6.94
Here F(Dosage) 161.5 > Critical F-Value 6.94

We reject H0 and conclude that there is symptom Relief Count among the different dosage levels (Low, Medium, High).

df_2	a	1	2
1	.100	39.86	49.50
	.050	161.4	199.5
	.025	647.8	799.5
	.010	4052	4999.5
	.005	16211	20000
2	.100	8.53	9.00
	.050	18.51	19.00
	.025	38.51	39.00
	.010	98.50	99.00
	.005	198.5	199.0
3	.100	5.54	5.46
	.050	10.13	9.55
	.025	17.44	16.04
	.010	34.12	30.82
	.005	55.55	49.80
4	.100	4.54	4.32
	.050	7.71	6.94
	.025	12.22	10.65
	.010	21.20	18.00
	.005	31.33	26.28
5	.100	4.06	3.78
	.050	6.61	5.79
	.025	10.01	8.43
	.010	16.26	13.27
	.005	22.78	18.31
6	.100	3.78	3.46
	.050	5.99	5.14
	.025	8.81	7.26
	.010	13.75	10.92
	.005	18.63	14.54