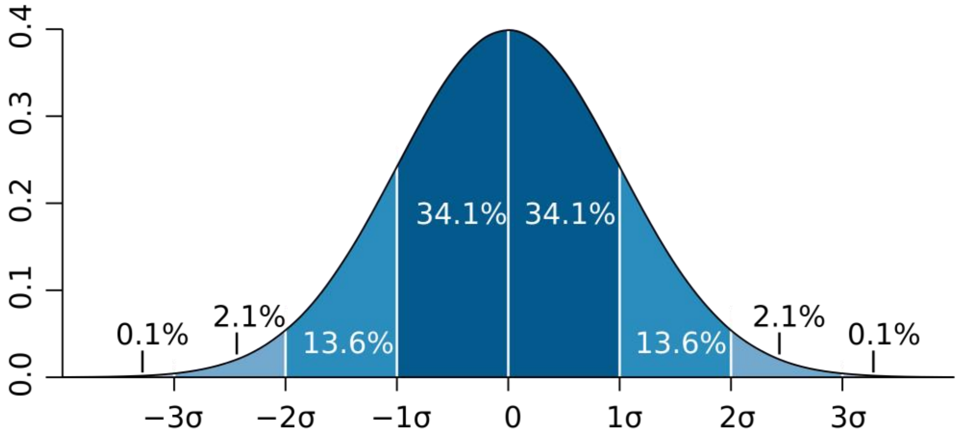
T-TESTS

Parametric testing

General Assumptions

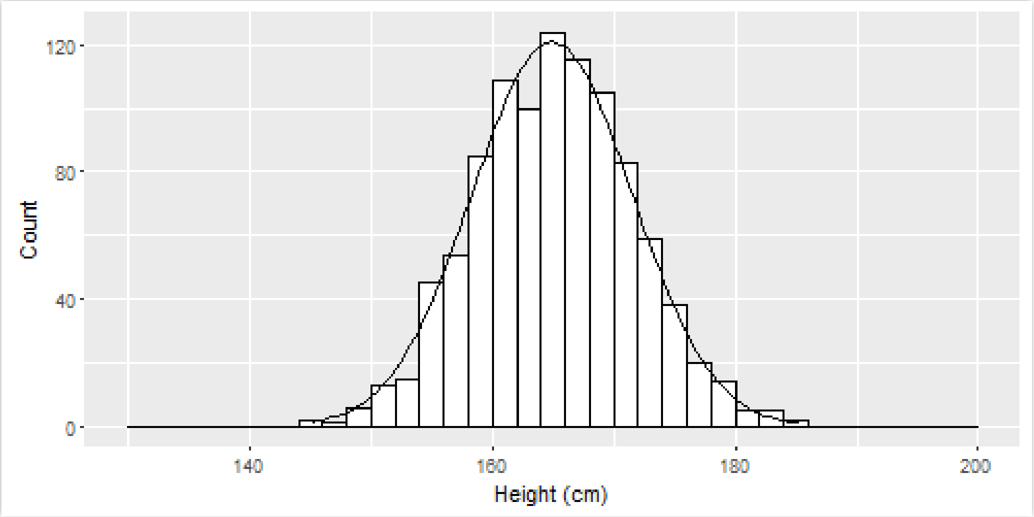
- Normal distribution
- There should be no significant outliers
- Homogenity of variance
- Independence (in most cases)



Testing the assumptions

- **Independence**
- **Normality of distribution**
- **Similar variance across groups**

➢ Data collection & study design
➢ Histogram



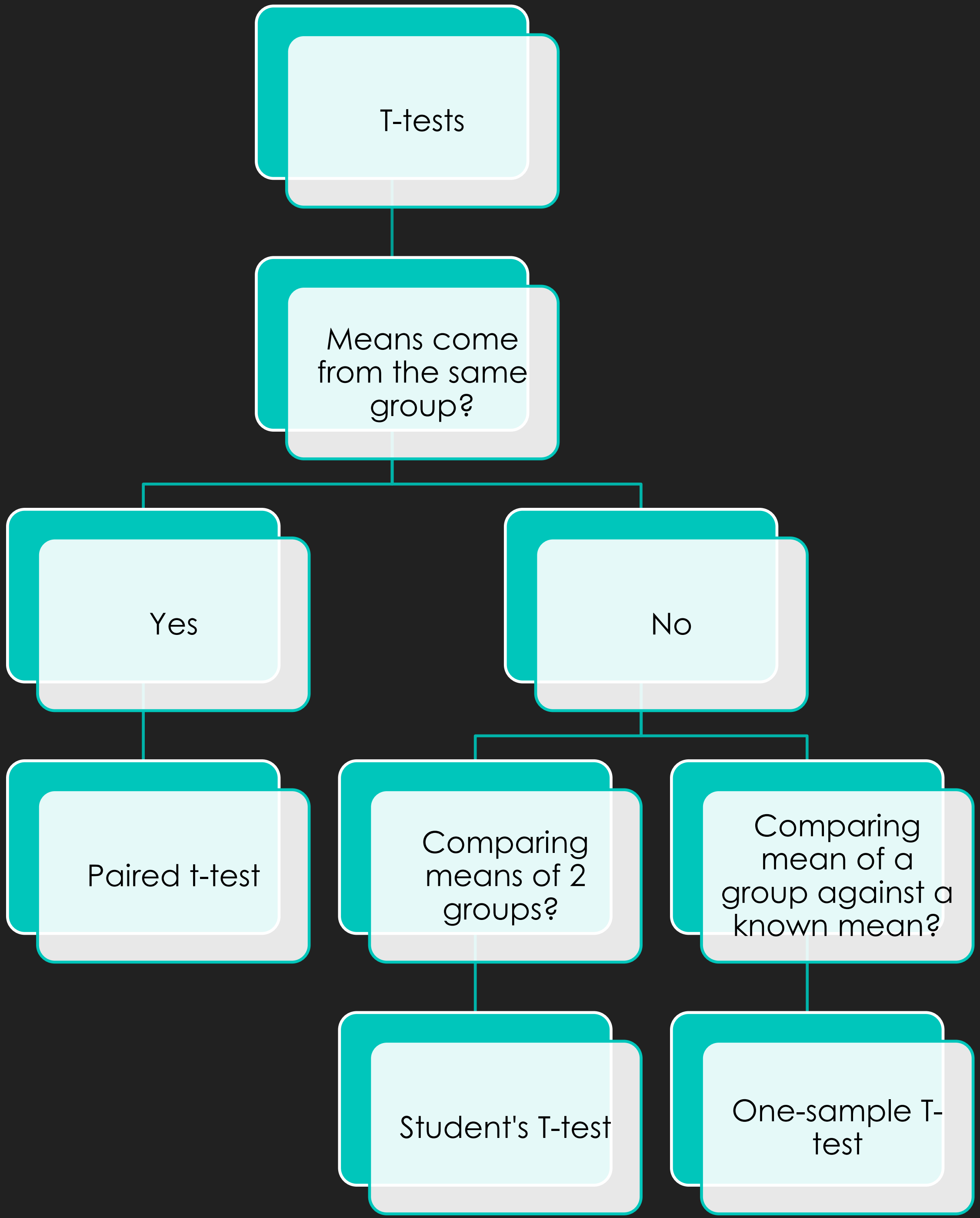
T-tests

- The *t* test is one type of inferential, parametric statistic
- Determine whether there is a significant difference between the means of two groups / conditions
- There are three main types

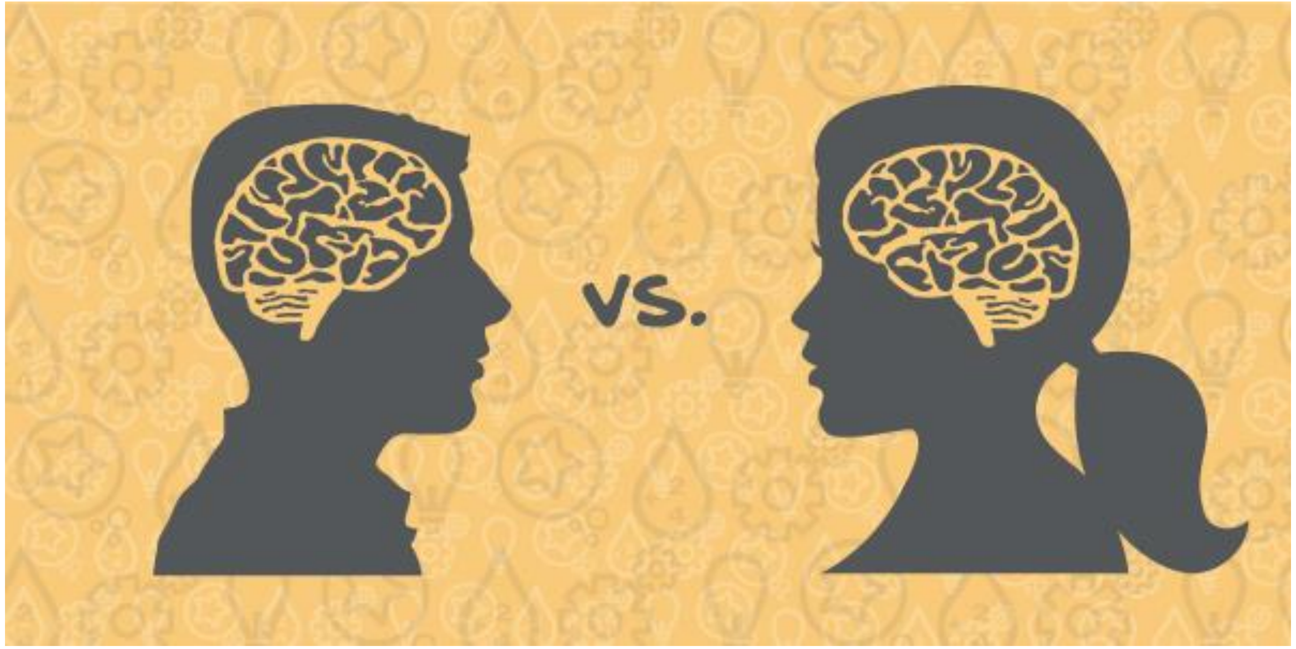
Student's T-test

How can we determine, to a reasonable degree of scientific certainty, if one variety of barley yields more than another?

William Sealy Gosset
1908


T-tests
Means come from the same group?
Yes
No
Paired t-test
Comparing means of 2 groups?
Comparing mean of a group against a known mean?
Student's T-test
One-sample T-test

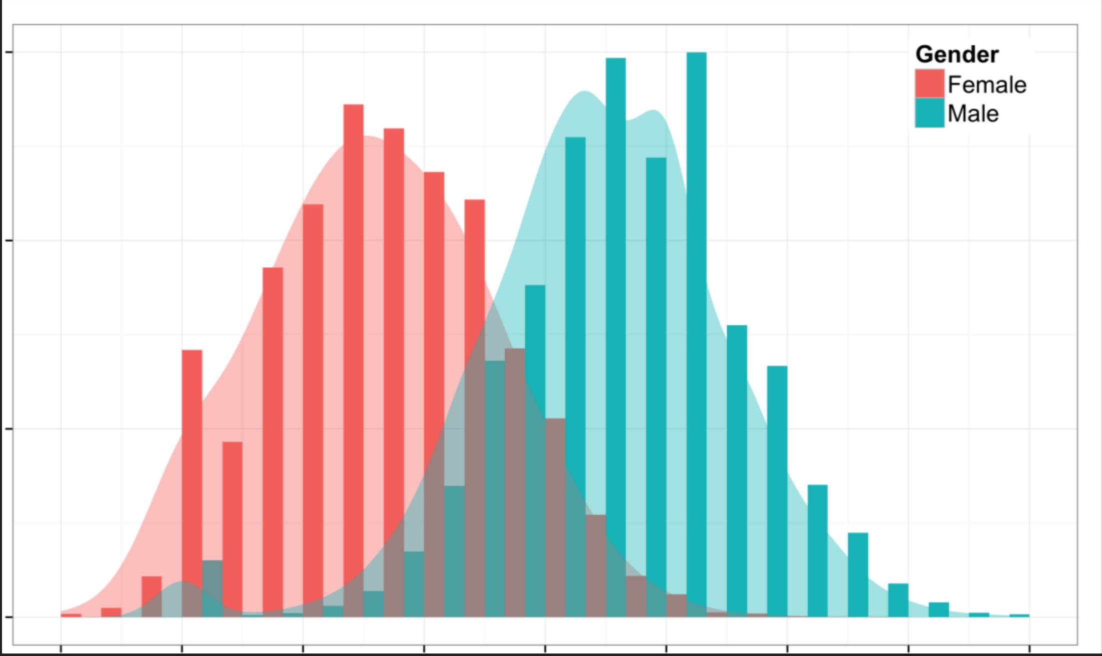
Student's T-test



- Determines whether there is a statistically significant difference between the means in two unrelated groups.

- **It is also known as independent samples t-test, two sample t-tests, between samples t-test and unpaired samples t-test.**

Student's T-test

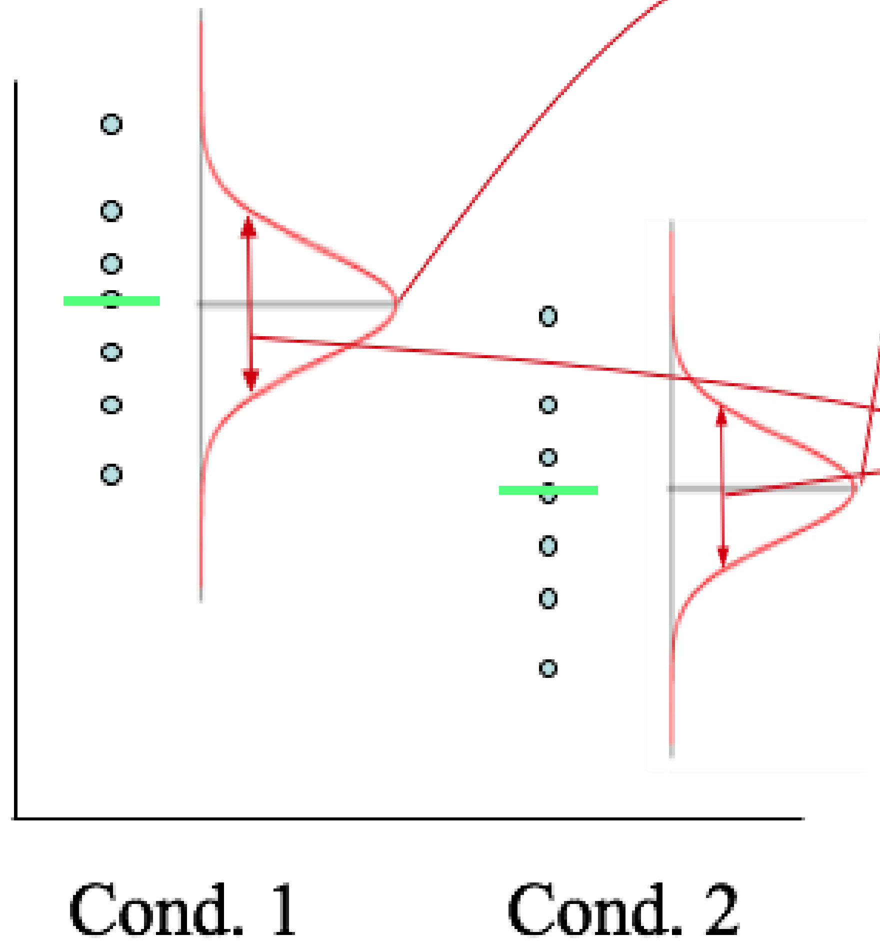


- Independent groups
- Independent measurements
- One independent, categorical variable that has two levels/groups
- One continuous dependent variable

FORMULA

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_{\bar{x}_1 - \bar{x}_2}}$$

The difference between the mean divided by the pooled standard error of the mean.



$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_{\bar{x}_1 - \bar{x}_2}}$$

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^{\,2}}{n_1} + \frac{s_2^{\,2}}{n_2}}$$

$$\text{t-value} = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

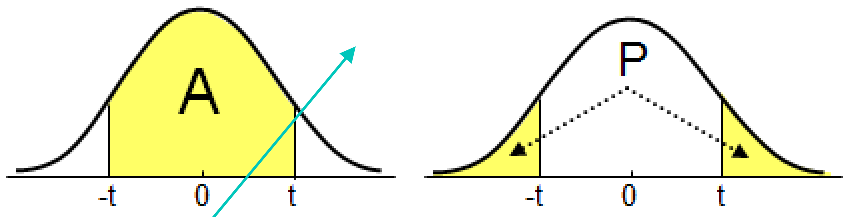
$$\text{t-value} = \frac{|15.31 - 15.71|}{\sqrt{\frac{0.4080^2}{18} + \frac{0.4349^2}{18}}}$$

$$= 2.8486$$

Field 1	Field 2
15.2	15.9
15.3	15.9
16.0	15.2
15.8	16.6
15.6	15.2
14.9	15.8
15.0	16.2
15.4	15.6
15.6	15.6
15.7	15.8
15.5	16.2
15.2	15.6
15.5	15.8
15.2	15.5
15.5	15.5
15.1	15.5
15.3	14.9
15.0	15.9
15.31	**15.71**

T test

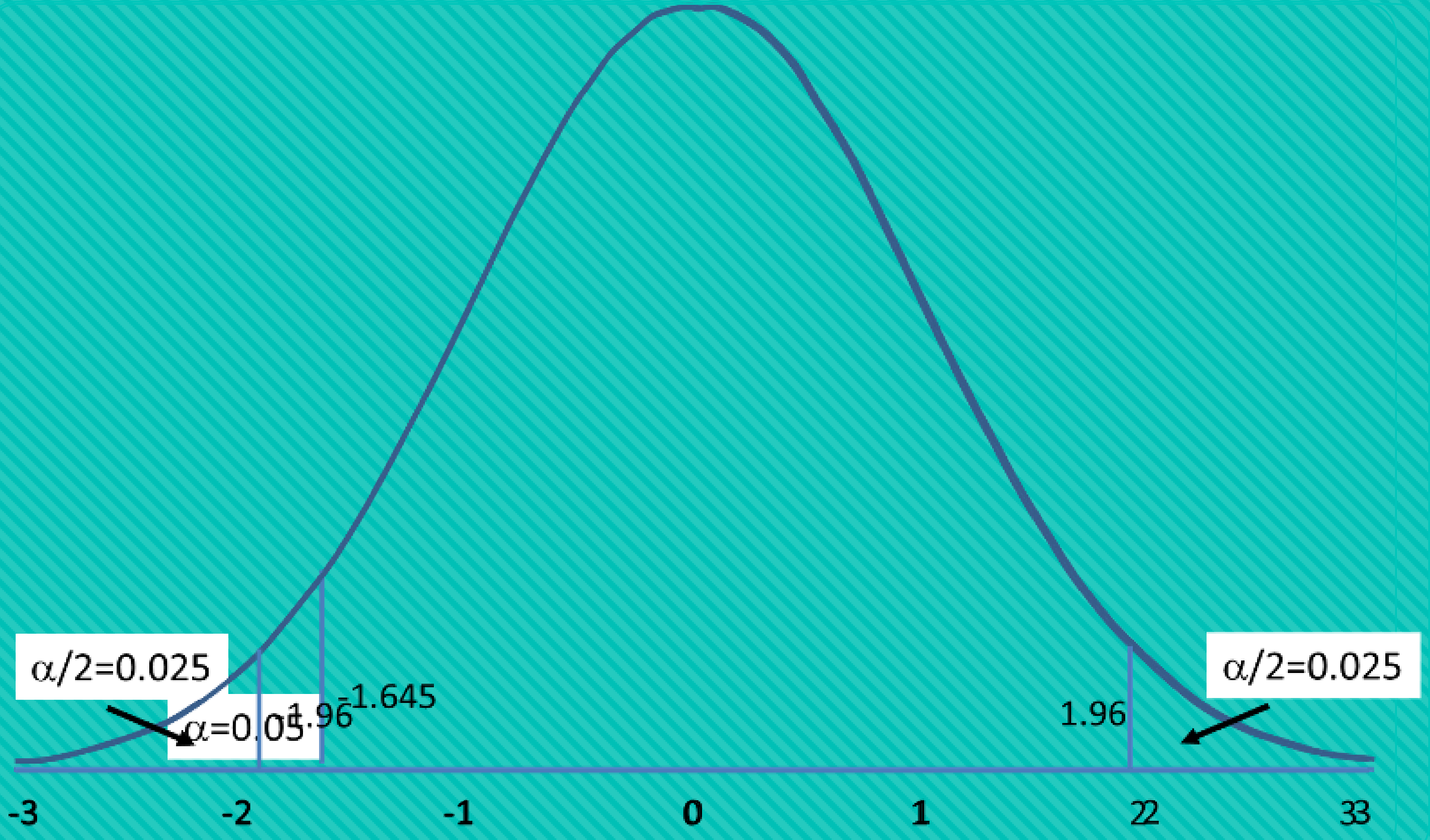
- Significance level: 0.05
- Degrees of freedom: (n1 + n2) - 2
- Degrees of freedom: (18 + 18) - 2 = 34
- Critical Value: 2.032
  - T-Value: 2.8486
    - We reject H_0



DF	A P	0.80 0.20	0.90 0.10	0.95 0.05	0.98 0.02	0.99 0.01	0.995 0.005	0.998 0.002	0.999 0.001
1		3.078	6.314	12.706	31.820	63.657	127.321	318.309	636.619
2		1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
31		1.309	1.695	2.040	2.453	2.744	3.022	3.375	3.633
32		1.309	1.694	2.037	2.449	2.738	3.015	3.365	3.622
33		1.308	1.692	2.035	2.445	2.733	3.008	3.356	3.611
34		1.307	1.691	2.032	2.441	2.728	3.002	3.348	3.601
35		1.306	1.690	2.030	2.438	2.724	2.996	3.340	3.591
36		1.306	1.688	2.028	2.434	2.719	2.991	3.333	3.582
37		1.305	1.687	2.026	2.431	2.715	2.985	3.326	3.574

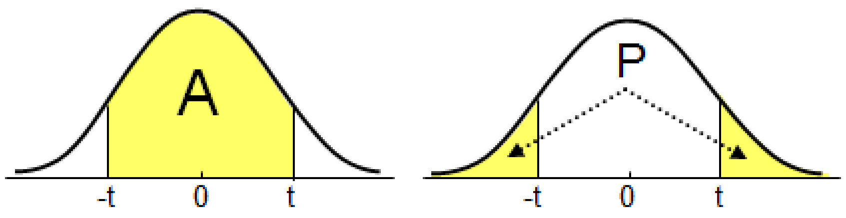
Two tails P-value: .05

P= ≤ 0.05



One-tailed and two-tailed tests

T test



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Tail probability P-value

P-value: .005

Paired T-test

- Estimate whether the means of two related measurements are significantly different from one another
- Used when two continuous variables are related
  - Same participant at different times
  - Different sites on the same person
  - Cases and their matched controls.
- Also known as within-subjects, repeated-measures and dependent-samples.

Paired T-test

- The outcome variable has a continuous scale
- The differences between the pairs of measurements are normally distributed
- The interest is on the difference in the outcome measurements between each pair

Paired T-test

The procedure for a paired sample t-test can be summed up in four steps.

- D = Differences between two paired samples
- d_i = The i^{th} observation in D
- n = The sample size
- $\bar{d}$ = The sample mean of the differences
- $\hat{\sigma}$ = The sample standard deviation of the differences
- T = The critical value of a t-distribution with $(n - 1)$ degrees of freedom
- t = The t-statistic (t-test statistic) for a paired sample t-test
- p = The p-value (probability value) for the t-statistic.

Paired T-test

The four steps are listed below:

- 1. Calculate the sample mean.
  - $\bar{d} = \dfrac{d_1 + d_2 + \cdots + d_n}{n}$
- 2. Calculate the sample standard deviation.
  - $\hat{\sigma} = \sqrt{\dfrac{(d_1 - \bar{d})^2 + (d_2 - \bar{d})^2 + \cdots + (d_n - \bar{d})^2}{n - 1}}$
- 3. Calculate the test statistic.
  - $t = \dfrac{\bar{d} - 0}{\hat{\sigma}/\sqrt{n}}$

Paired T-test

- 3. Calculate the test statistic.
  - $t = \dfrac{\bar{d} - 0}{\hat{\sigma}/\sqrt{n}}$
- 4. Calculate the probability of observing the test statistic under the null hypothesis. This value is obtained by comparing *t* to a *t*-distribution with $(n - 1)$ degrees of freedom. This can be done by looking up the value in a table, such as those found in many statistical textbooks, or with statistical software for more accurate results.
  - $p = 2 \cdot Pr(T > |t|)$ (two-tailed)
  - $p = Pr(T > t)$ (upper-tailed)
  - $p = Pr(T < t)$ (lower-tailed)

determine whether the results provide sufficient evidence to reject the null hypothesis in favor of the alternative hypothesis.

A study was conducted to determine the effectiveness of a new hydraulics pressure level for new type of car engine. The table below shows

The before and after pressure level of 10 cars in the experiment.

(a) Is this experiment effective for adjusting pressure level ?(Use a 5% significance level).

(b) Construct a 95% confidence interval and determine the margin of error.

S.	Before	After
1	185	169
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x_d = Difference between two pair of samples = After - Before

Hypotheses

$$H_0: \mu_D \geq 0$$
$$H_a: \mu_D < 0$$

- 1. Calculate the sample mean.

- $\bar{d} = \frac{d_1 + d_2 + \cdots + d_n}{n}$

Mean, $\overline{x_d} = -13.1$

Using the data provided:
x̄d = (-16 - 5 - 13 - 1 - 31 + 3 - 28 - 22 + 5 - 23) / 10
= -131 / 10 = -13.1

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x_d = Difference between two pair of samples = After - Before

- 2. Calculate the sample standard deviation.

- $$\hat{\sigma} = \sqrt{\frac{(d_1 - \bar{d})^2 + (d_2 - \bar{d})^2 + \cdots + (d_n - \bar{d})^2}{n - 1}}$$

Standard deviation of the differences: 13.03

$$S_d = 13.03$$

significance level $\alpha = 0.05$
Degree of freedom, df = n-1 = 10-1 = 9
T – value with 9 degree of freedom with 0.05 is 2.262

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T – value with 9 degree of freedom with 0.05 is 2.262

Two-tails test

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

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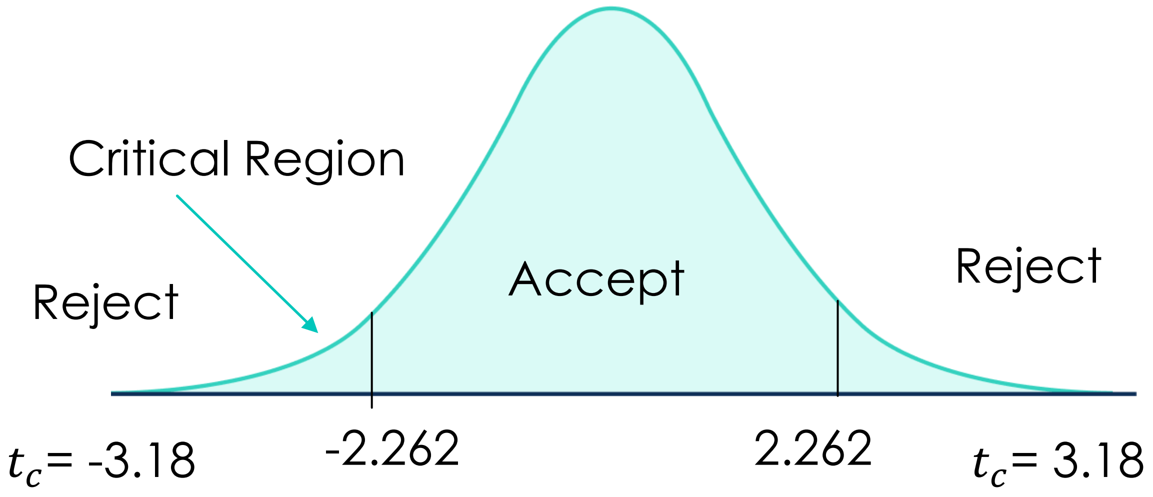
Here we are using left tail t-test we can use ± 2.262 as critical value.

T-value, $t_c = \dfrac{\overline{x_d}-\mu_d}{\frac{s_d}{\sqrt{n}}} = \dfrac{-13.1-0}{\frac{13.03}{\sqrt{10}}} = -3.18$, here μ_d=0 from null hypothesis



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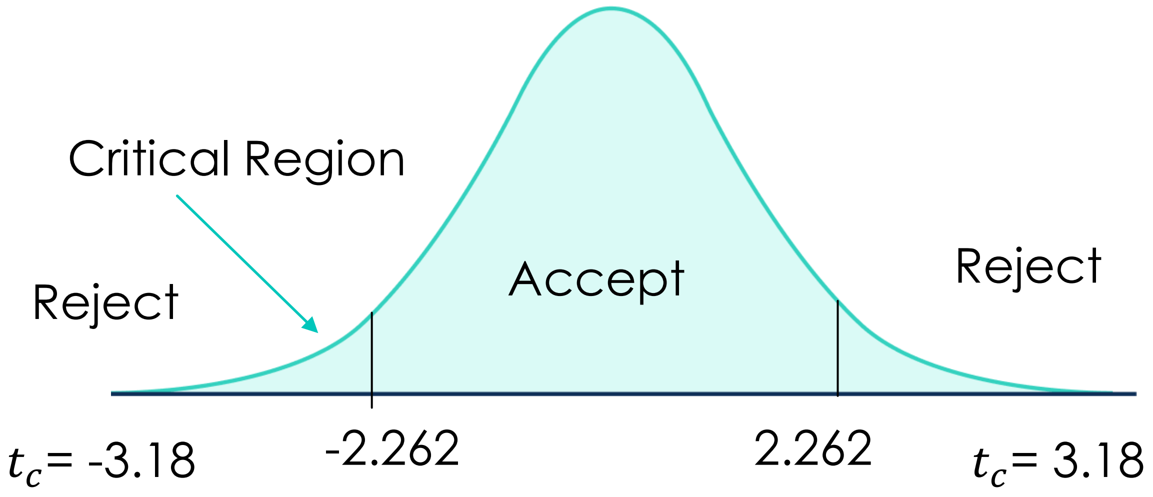
- The **absolute value** of a number is the non-negative value of that number without regard to its sign. So, for a t-statistic of −3.18, its absolute value is |−3.18|=3.18.
- The **critical value** of −2.262 also has an absolute value of |−2.262|=2.262.

Since 3.18 is than 2.262, this means that the test statistic is in the rejection region, and the null hypothesis can be rejected at the 5% significance level. Therefore, the result is statistically significant, and it indicates that the change in hydraulic pressure level is effective.



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Formula for confidence interval, t = 2.262
CI = $\overline{x_d} \pm t\frac{s_d}{\sqrt{n}} = -13.1 \pm 2.262\frac{13..03}{\sqrt{10}} = \pm 9.32$

CI (-22.19,-3.78)
margin of error = 9.32

Spring 2023 – Question Solution

A computer science student is conducting an experiment to compare the performance of two algorithms for solving a particular machine learning problem. The student randomly selects a sample of 7 machine learning problems and records the time it takes to solve each problem using both algorithms. The data is as follows:

Algorithm 1:	10.2	9.5	8.8	10.7	11.1	9.3	10.5
Algorithm 2:	11.5	11.7	10.8	11.9	12.3	11.1	10.9

The student wants to analyze the data and draw conclusions using statistical tests (Use 5% significance level) to determine whether there is a statistically significant difference between the means in two unrelated groups of algorithms.

a) State the null and alternative hypotheses.

b) Calculate the test statistic and determine the critical value.

c) Compare the test statistic with the critical value and draw a conclusion based on the results.

Null Hypothesis (H0): There is no statistically significant difference between the means of Algorithm 1 and Algorithm 2 for solving the machine learning problem. Alternative Hypothesis (Ha): There is a statistically significant difference between the means of Algorithm 1 and Algorithm 2 for solving the machine learning problem.

Mathematically: H0: $\mu_1 - \mu_2 = 0$ Ha: $\mu_1 - \mu_2 \neq 0$

Where:

- μ_1 is the population mean of Algorithm 1.
- μ_2 is the population mean of Algorithm 2.

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b) Calculate the test statistic and determine the critical value:

Algorithm 1 data: n1 = 7 x̄1 = 10.01 (sample mean)
s1 = 0.83 (sample standard deviation)

Algorithm 2 data: n2 = 7 (sample size) x̄2 = 11.46 (sample mean) s2 = 0.55 (sample standard deviation)

Now, calculate the test statistic (t-statistic) using the two-sample t-test formula:
t-statistic ≈ 3.82

Degrees of freedom: (7 + 7) - 2 = 12

The calculated t-statistic is 3.82, and the critical t-value at a 5% significance level with 12 degrees of freedom is approximately 2.179.

Two tailed test

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

Spring 2023 – Question Solution

A computer science student is conducting an experiment to compare the performance of two algorithms for solving a particular machine learning problem. The student randomly selects a sample of 7 machine learning problems and records the time it takes to solve each problem using both algorithms. The data is as follows:

The student wants to analyze the data and draw conclusions using statistical tests (Use 5% significance level) to determine whether there is a statistically significant difference between the means in two unrelated groups of algorithms.

a) State the null and alternative hypotheses.
b) Calculate the test statistic and determine the critical value.
c) Compare the test statistic with the critical value and draw a conclusion based on the results.

Algorithm 1:	10.2	9.5	8.8	10.7	11.1	9.3	10.5
Algorithm 2:	11.5	11.7	10.8	11.9	12.3	11.1	10.9

c) Compare the test statistic with the critical value and draw a conclusion based on the results:

The calculated t-statistic is 3.82, and the critical t-value at a 5% significance level with 12 degrees of freedom is approximately 2.179.

Since the absolute value of the calculated t-statistic (3.22) is greater than the critical t-value (2.179), we reject the null hypothesis (H0). This means that, at the 5% significance level, there is enough evidence to conclude that there is a statistically significant difference between the means of Algorithm 1 and Algorithm 2 for solving the machine learning problem.

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
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11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	$\boxed{2.179}$	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
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23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
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30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
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z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

Spring 2023 – Question Solution

A professor wants to determine if there is a significant change of students' scores on the midterm and final exams in a particular course. The professor randomly selects a sample of 7 students and records their scores on both exams. The data is as follows:
The professor wants to analyze the data and draw conclusions based on the results of your statistical test (Use 5% significance level) to determine if test score significantly changes after each exam.
a) State the null and alternative hypotheses.
b) Calculate the test statistic and determine the critical value.
c) Compare the test statistic with the critical value and draw a conclusion based on the results.

Midterm Exam Scores:	80	85	72	90	78	92	88
Final Exam Scores:	85	88	70	92	80	95	90

a) State the null and alternative hypotheses:
Null Hypothesis (H0): There is no significant change in students' scores from the midterm to the final exam; the mean score difference is zero. Alternative Hypothesis (Ha): There is a significant change in students' scores from the midterm to the final exam; the mean score difference is not zero.
Mathematically: $H_0: \mu_d = 0$ $H_a: \mu_d \neq 0$
Where:
- μ_d is the population mean difference between final exam scores and midterm exam scores.

Spring 2023 – Question Solution

A professor wants to determine if there is a significant change of students' scores on the midterm and final exams in a particular course. The professor randomly selects a sample of 7 students and records their scores on both exams. The data is as follows:
The professor wants to analyze the data and draw conclusions based on the results of your statistical test (Use 5% significance level) to determine if test score significantly changes after each exam.

a) State the null and alternative hypotheses.
b) Calculate the test statistic and determine the critical value.
c) Compare the test statistic with the critical value and draw a conclusion based on the results.

Midterm Exam Scores:	80	85	72	90	78	92	88
Final Exam Scores:	85	88	70	92	80	95	90

b) Calculate the test statistic and determine the critical value:
Calculate the differences: Differences = Final Exam Scores - Midterm Exam Scores

Differences = (85 - 80, 88 - 85, 70 - 72, 92 - 90, 80 - 78, 95 - 92, 90 - 88) = (5, 3, -2, 2, 2, 3, 2)

Calculate the sample mean and standard deviation of the differences:
$\bar{x}d = (5 + 3 - 2 + 2 + 2 + 3 + 2) / 7 \approx 2.14$ (sample mean of differences)
$sd = \sqrt{[\Sigma(xi - \bar{x}d)^2 / (n - 1)]} \approx 2.12$ (sample standard deviation of differences)

Now, calculate the test statistic (t-statistic) using the paired sample t-test formula:
t-statistic = $(\bar{x}d - \mu d) / (sd / \sqrt{n})$
t-statistic = $(2.14 - 0) / (2.12 / \sqrt{7})$
t-statistic ≈ 2.68

Spring 2023 – Question Solution

A professor wants to determine if there is a significant change of students' scores on the midterm and final exams in a particular course. The professor randomly selects a sample of 7 students and records their scores on both exams. The data is as follows:
The professor wants to analyze the data and draw conclusions based on the results of your statistical test (Use 5% significance level) to determine if test score significantly changes after each exam.
a) State the null and alternative hypotheses.
b) Calculate the test statistic and determine the critical value.
c) Compare the test statistic with the critical value and draw a conclusion based on the results.

Midterm Exam Scores:	80	85	72	90	78	92	88
Final Exam Scores:	85	88	70	92	80	95	90

t-statistic ≈ 2.38
Next, determine the degrees of freedom (df) for the test. In a paired sample t-test, df is equal to the sample size minus 1:
df = n - 1 = 7 - 1 = 6
Now, at a 5% significance level and with df = 6, you can find the critical t-value using a t-table or a t-distribution calculator. For a two-tailed test, the critical t-value is approximately ±2.447.

Two tailed test

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
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4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
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7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

When comparing the calculated t-statistic with the critical value:

•If the absolute value of the calculated t-statistic is **less** than the critical value, then the result is not considered statistically significant, and we fail to reject the null hypothesis. This would mean that any observed difference could likely be due to chance rather than a systematic effect.

•If the absolute value of the calculated t-statistic is **greater** than the critical value, then the result is considered statistically significant, and we reject the null hypothesis. This would suggest that the observed difference is unlikely to be due to chance, and there is a statistically significant difference in the scores.

Spring 2023 – Question Solution

A professor wants to determine if there is a significant change of students' scores on the midterm and final exams in a particular course. The professor randomly selects a sample of 7 students and records their scores on both exams. The data is as follows:
The professor wants to analyze the data and draw conclusions based on the results of your statistical test (Use 5% significance level) to determine if test score significantly changes after each exam.
a) State the null and alternative hypotheses.
b) Calculate the test statistic and determine the critical value.
c) Compare the test statistic with the critical value and draw a conclusion based on the results.

Midterm Exam Scores:	80	85	72	90	78	92	88
Final Exam Scores:	85	88	70	92	80	95	90

c) Compare the test statistic with the critical value and draw a conclusion based on the results:
The calculated t-statistic is 2.68, and the critical t-value at a 5% significance level with 6 degrees of freedom is approximately ±2.447.
Since the calculated t-statistic (2.68) is greater than the critical value (2.447), we reject the null hypothesis. This indicates that there is a statistically significant difference in the scores from the midterm to the final exam, meaning that the scores did significantly change.

Two tailed test

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										