- Modulo operator
- Modular arithmetics
- Divisibility rule
- 1 hard problem.

★ int range is $\{-2\times10^9, 2\times10^9\}$

★ long range is $\{-8\times10^{18}, 8\times10^{18}\}$

% Modulo / Remainder.

Divident = divisor * quotient + remainder

$10 \% 4 = 2$: $10 = 4 * \{\frac{10}{4}\}^{2} + 2$

$13 \% 5 = 3$: $13 = 5 * \{\frac{13}{5}\}^{2} + r$; $r = 3$.

$100 \% 7$: $100 = 7 * \{\frac{100}{7}\}^{14} + r$; $98 + r = 100$; $r = 2$

$-60 \% 9$: $-60 = 9 * \{-\frac{60}{9}\}^{-6} + r$; $-54 + r = -60$; $r = -6$

Java will provide this answer.

Concept

Remainder = divident − divisor * quotient

greatest multiple of divisor ≤ divident

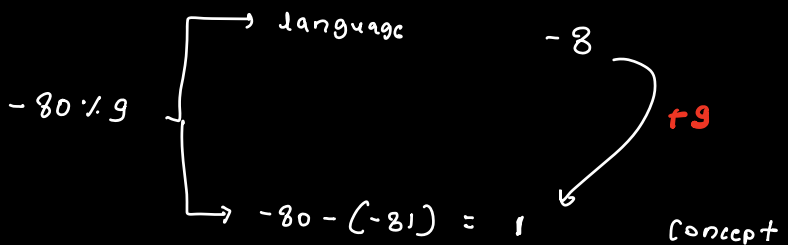
10 % 4

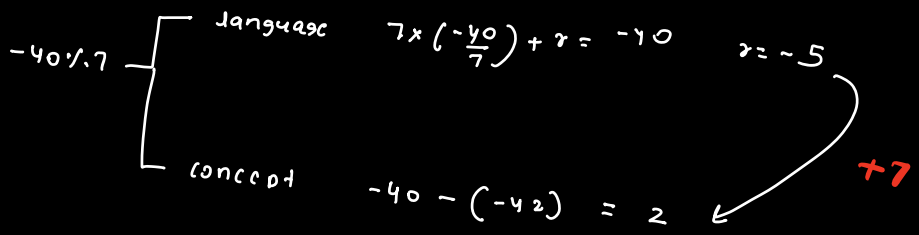
$10 \% 4 = 10 - 8 = 2$

$13 \% 5 = 13 - 10 = 3$

$100 \% 7 = 100 - 98 = 2$

$-60 \% 7 = -60 - (-63) = -60 + 63 = 3$





```
if (x < 0)
{
    ans = x % m + m
else
{
    ans = x % m
```

remainder is always +ve.

Modular arthimetics.

$a \% m \rightarrow [0, m-1]$

$(a+b) \% m = (a \% m + b \% m) \% m$

$[0, m-1] + [0, m-1] > m$

$a = 6 \quad b = 13 \quad m = 7$

$19 \% 7 = 5$

$6 \% 7 + 13 \% 7 = 6 + 6 = 12 \% 7 = 5$

$a = 21 \quad \% 10$

$32 \% 10$

$[0, m-1]$

$(a * b) \% m = (a \% m * b \% m) \% m$

$[0, m-1] \quad [0, m-1]$

$(a-b) \% m$

$(a/b) \% m$

} Advanced DSA

$a \% m \rightarrow [0, m-1]$

$10 \% 4 \rightarrow [0, 3]$

	%4
1	1
2	2
3	3
4	0
5	1
6	2
7	3
8	0
9	1
10	2
11	3
12	0
13	1
	2
	3
	0

Given a, n, m. Cal $(a^n) \% m$ → [0, m-1]

a = 2 N = 5 m = 10

$(2^5) \% 10 = 32 \% 10 = 2$

```
ans = 1
for ( i = 1 ; i ≤ N; i++)
{
    ans = ans * a
}
ans = ans % m
return ans
```

a > 0

a = 2 N = 10000 m = 50 ans [0, 49]

$(a^n) \% m$

```
ans = 1
for ( i = 1 ; i ≤ N; i++)
{
    ans = ans * a
}
ans = ans % m
return ans
```

ans = 2^{10000}

(a . a . a . a a a a . . .) % m

(a % m * a % m * a % m * a % m

```
ans = 1
for ( i = 1 ; i <= N ; i++)
{
    ans = (ans * a) % m
    ans = ( ans%.m * a%.m) %.m
ans = ans %. m
return ans
```

$2^5 \,\%.10$

ans = 1

ans = (1 * 2) %.10

ans = 2

ans = (2 * 2) %.10

ans = 4

ans = (4 * 2) %.10

ans = 8

ans = (8 * 2) %.10

ans = 6

ans = (6 * 2) %.10

ans = 2

$(2^5) \,\%.10$

$(32) \,\%.10 = 2$

ans = (ans%.m * a%.m) %.m → (0, m-1)

0, m-1 0, m-1 > m

Divisibility of 3 : Sum of digits divisible by 3

$789 \,\%.3 = 0$

$8215 \,\%.3 \neq 0$

$3458 \,\%.3$

$= (3\times10^3 + 4\times10^2 + 5\times10^1 + 8) \,\%.3$

$= ((3\times10^3)\%.3 + (4\times10^2)\%.3 + (5\times10^1)\%.3 + (8*1)\%.3) \,\%.3$

$= (((3\%.3) * (10^3\%.3))\%.3 + ((4\%.3) * (10^2\%.3))\%.3 + ((5\%.3) * (10^1\%.3))\%.3 + ((8\%.3) * (1\%.3))\%.3) \,\%.3$

$= (a \% m + b \% m) \% m = (a+b) \% m$

$= ((3 \% 3 + 10^{3} \% 3) + (4 \% 3 + 10^{2} \% 3) + (5 \% 3 + 10^{1} \% 3) + (8 \% 3 + 1 \% 3)) \% 3$

$= (3 \% 3 + 4 \% 3 + 5 \% 3 + 8 \% 3) \% 3$

$= (3 + 4 + 5 + 8) \% 3$

Divisibility of 4.

$34588 \% 4 = 0$ $\quad$ $88 \% 4 = 0$

$(3 \times 10^{4} + 4 \times 10^{3} + 5 \times 10^{2} + 8 \times 10^{1} + 8 \times 10^{0}) \% 4$

$((3 \times 10^{4}) \% 4 + (4 \times 10^{3}) \% 4 + (5 \times 10^{2}) \% 4 + (8 \times 10^{1}) \% 4 + 8 \% 4) \% 4$

$= ((8 \times 10^{1}) \% 4 + 8 \% 4) \% 4$

$= (8 \times 10 + 8) \% 4$

$= 88 \% 4$

Divisibility of 9. = sum of digits should be divisible by 9

$= ((3\times10^3)\%9 + (4\times10^2)\%9 + (5\times10^1)\%9 + (8*1)\%9)\%$

$(3+4+5+8)\%9$

Google
GS
micro

Q. Given 1 number in arr[] (contain single digit only), find num%m

arr[5] = 7, 8, 9, 6, 2 (0 1 2 3 4)

78962 %m

Constraints

$1 \le N \le 10^5$

$1 \le m \le 10^9$

N = 3 → $999 \approx 10^3$

N = 5 → $99999 \approx 10^5$

N = 11 → 10^{11}

⋮

$N = 10^5$ → 10^{100000}

$$(7\times10^4 + 8\times10^3 + 9\times10^2 + 6\times10^1 + 2\times10^0)\ \%\ m$$

$$([7\times10^4)\ \%\ m + (8\times10^3)\ \%\ m + (9\times10^2)\ \%\ m + (6\times10^1)\ \%\ m + 2\ \%\ m)\ \%\ m$$

$$= (\ (7\ \%\ m * (10^4)\ \%\ m)\ \%\ m + (8\ \%\ m * 10^3\ \%\ m)\ \%\ m + (9\ \%\ m * 10^2\ \%\ m)\ \%\ m + (\ \ )\ \%\ m * (\ \ )\ \%\ m + \ldots$$

$$(\text{arr}[0]\ \%\ m * 10^4\ \%\ m)\ \%\ m \quad \text{term1}$$

$$+ (\text{arr}[1]\ \%\ m * 10^3\ \%\ m)\ \%\ m \quad \text{term2}$$

$$+ (\qquad)\ \%\ m \quad \text{term3}$$

$$+ (\qquad)\ \%\ m \quad \text{termn}$$

```
fans = 0
tenpower = 1
for( i= n-1 ; i >=0 ; i--)
{
    tmp = (arr[i] %m * (tenpower) %m) %m
    fans = (fans + tmp) %m
    tenpower = (tenpower * 10) %m
}
return fans;
```

$$(a + b \% m) \% m = (a+b) \% m$$

$$(a \% m + xm \% m + b \% m) \% m = (a+b) \% m$$

$a = xm + a \% m$

$(1000) \% 7$

$(10 \times 10 \times 10) \% 7$

$(10 \% 7 * 10 \% 7 * 10 \% 7) \% 7$

$60 \% 7$

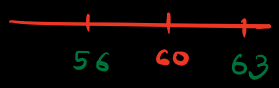
int division

$60 = 7 * \left(\frac{60}{7}\right) + r$ (8)

$60 = 56 + r$

$r = 4$

$r = 60 - (56) = 4$



$-30 \% 4$ $\quad -7 \quad -7.$

$-30 = 4\left(\frac{-30}{4}\right) + r$

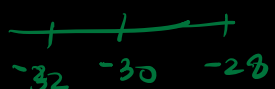
$-30 = -28 + r$

$r = 28 - 30 = -2$

$-30 - (-32)$

$= -30 + 32 = 2$

+4



4 → 1 → 0 → -1 → 0 1 1 1 1 1 1

int

fans int

tenpow.

tenpow = (tenpow x 10) % m

tenpow = [0, m-1]

↓

10^9

$= [0, 10^9]$