

Today's Content

- Prefix Sum introduction
- Problems based on prefix number.

Q. Given an array (N), and Q queries.

For each query, given L & R. we need to calculate sum of elements from [L, R]

arr[10] =

0	1	2	3	4	5	6	7	8	9
-3	6	2	4	5	2	8	-9	3	1

Q = 6.

L	R	sum
4	8	9
3	7	10
1	3	12
0	4	14
6	9	3
7	7	-9

→ For every query
iterate [L, R] & get sum.
= 6

Q, arr

while (Q--)

```
int L, R. read L, R.  
sum = 0  
for (i = L; i <= R; i++)  
{  
    sum = sum + arr[i]  
}  
print (sum)
```

TC: $O(N \times Q)$
SC: $O(1)$

Q. Given Indian Team score board, for last 10 overs.

41	42	43	44	45	46	47	48	49	50
288	312	330	349	360	383	394	406	436	439

Q. Runs scored in last 5 overs?

[46-50]

$$439 - 360 = 79$$

$$R[50] - R[45]$$

Q. Run scored in 50th over?

[50-50]

$$439 - 436 = 3$$

$$R[50] - R[49]$$

Q. Run scored in 49th over?

[49-49]

$$436 - 406 = 30$$

$$R[49] - R[48]$$

Q. Run scored in 42-47th over?

(42, 47]

$$R[47] - R[41]$$

$$\text{arr}[10] = \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ -3 & 6 & 2 & 4 & 5 & 2 & 8 & -9 & 3 & 1 \end{matrix}$$

$$\text{PF}[10] = \begin{matrix} -3 & 3 & 5 & 9 & 14 & 16 & 24 & 15 & 18 & 19 \end{matrix}$$

$$\text{sum}[0, 7] = \text{sum}[0-1] + \text{sum}[2-7]$$

$$\begin{aligned} \text{sum}[2, 7] &= \text{sum}[0-7] - \text{sum}[0-1] \\ &= 15 - 3 = 12 \end{aligned}$$

Array \rightarrow Given,

Cumulative sum = Prefix array

$$PF[i] = \text{sum of all elements } [0, i]$$

$$\text{sum}[4, 8] = PF[8] - PF[3]$$

$$\text{sum}[i, j] = PF[j] - PF[i-1]$$

$$\text{sum}[L, R] = PF[R] - PF[L-1]$$

$$\text{sum}[L, R] = \begin{cases} PF[R] & L=0 \\ PF[R] - PF[L-1] & \text{if } L > 0 \end{cases}$$

// Construct PF sum.

Given arr[n]

0	1	2	...	n-1

$$PF[0] = arr[0]$$

$$PF[1] = arr[0] + arr[1]$$

$$PF[2] = arr[0] + arr[1] + arr[2]$$

\downarrow
 $PF[1]$

$$PF[i] = arr[0] + arr[1] + arr[2] + \dots + arr[i-1] + arr[i]$$

\downarrow
 $PF[i-1]$

$$PF[i] = PF[i-1] + arr[i]$$

int PF[N]

$4N + 12$

PF[0] = arr[0]

for (i = 1; i < n; i++)

N

{ PF[i] = PF[i-1] + arr[i]

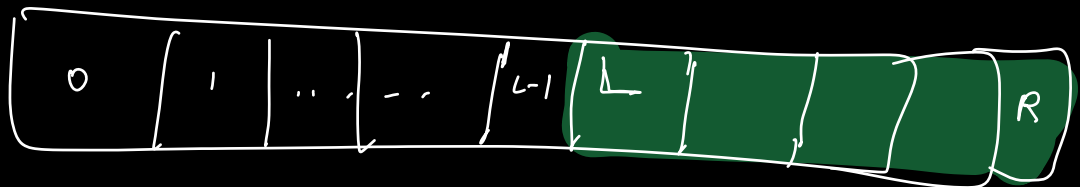
TC: $O(N+Q)$

int Q. read Q,

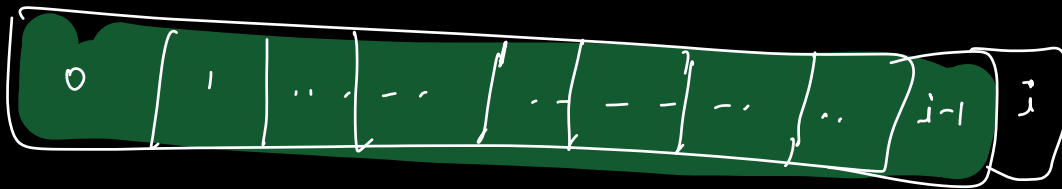
SC: $O(N)$

while (Q--)

{ int L, R. read L, R.
if (L == 0) print (PF[R])
else print (PF[R] - PF[L-1])



$$PF[R] - PF[L-1]$$



$$PF[i] = PF[i-1] + arr[i]$$

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interview

Q. Given an array, Return true if exists an equilibrium index.

EI = index of which

sum of left side = sum of right side

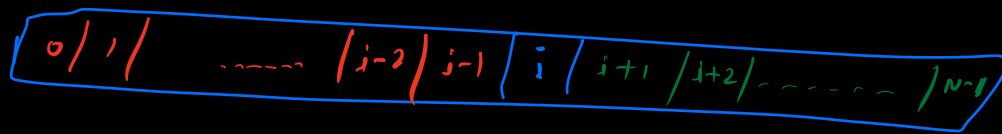
for index 0 $S_L = 0$
for index $N-1$ $S_R = 0$

a.

	x	x	x	x	
0	1	2	3	4	5
1	2	3	4	8	10

ans = True

Approach for every index, check if is a EI



For checking i

$$\text{sum}[0, i-1] \stackrel{L}{=} \stackrel{R}{=} \text{sum}[i+1, N-1]$$

$$\text{PF}[i-1] \qquad \text{PF}[N-1] - \text{PF}[i+1-1]$$

$$\text{PF}[i-1] \stackrel{L}{=} \stackrel{R}{=} \text{PF}[N-1] - \text{PF}[i]$$

// construct PF

for($i=0$; $i < N$; $i++$)

if($i == 0$) $S_L = 0$
 else $S_L = \text{PF}[i-1]$

if($i == N-1$) $S_R = 0$
 else $S_R = \text{PF}[N-1] - \text{PF}[i]$

if($S_L == S_R$) return True

return False

TC: $O(N)$

SC: $O(N)$

Q. Given an array, Calculate Q queries.

$L, R, O \Rightarrow$ sum of all odd-indexed elements

$L, R, E \Rightarrow$ sum of all even-indexed elements.

A =

	0	1	2	3	4	5	6	7
	2	3	1	-1	0	8	5	4

Q = 2

L R o/e

3 6 0

$$\text{arr}[3] + \text{arr}[5] = -1 + 8 = 7$$

1 5 5

$$\text{arr}[2] + \text{arr}[4] = 1 + 0 = 1$$

Let try to create 2 prefix sum.

$PS_e[i]$



sum of even indexed elements from $[0, i]$

$PS_o[i]$



sum of odd indexed elements from $[0, i]$

Arr =

	0	1	2	3	4
	2	4	3	1	5

PF_o 0 4 4 5 5

PF_e 2 2 5 5 10

$$PF_o[0] = 0$$

for ($j=1$; $j < N$; $j++$)

{
 if ($j \% 2 == 0$) $PF_o[j] = PF_o[j-1]$
 else $PF_o[j] = PF_o[j-1] + \text{arr}[j]$

$$PFE[0] = arr[0]$$

for (j=1 ; j<N ; j++)

$$\left\{ \begin{array}{l} \text{if (j \% 2 == 1)} \quad PFE[j] = PFE[j-1] \\ \text{else} \quad PFE[j] = PFE[j-1] + arr[j] \end{array} \right.$$

while (Q--)

$$\left\{ \begin{array}{l} L, R, O \\ \downarrow \\ PFO[R] - PFO[L-1] \\ \\ L, R, E \\ \downarrow \\ PFE[R] - PFE[L-1] \end{array} \right.$$

TC: $O(N+Q)$

SC: $O(N)$

Direct
Coderection
Google
JP Morgan.

Q, Given an array, Count the no of special index in the array.

Special index \geq An index is SF,
if after removing this index

$$\begin{array}{l} \text{sum of all} \\ \text{odd-indexed} \\ \text{elements} \end{array} = \begin{array}{l} \text{sum of all} \\ \text{even-indexed} \\ \text{elements.} \end{array}$$

$A =$

	0	1	2	3	4	5
	4	3	2	7	6	-2

$j=0$

	0	1	2	3	4
	3	2	7	6	-2

$S_o 8 = S_e (8)$ ✓

$j=1$

	0	1	2	3	4
	4	2	7	6	-2

$S_o 8 = S_e 9$ ✗

$j=2$

	0	1	2	3	4
	4	3	7	6	-2

$S_o 9 = S_e 9$ ✓

$j=3$

$j=4$

$j=5$

0	1	2	3	4	5
4	1	5	3	7	10

$$S_{odd} = 14$$

0	1	2	3	4
4	1	3	7	10

$$S_{odd} = 8$$

0	1	2	3	4	5
4	1	5	3	7	10

Sum of all 0 after deducting = Sum 0 in left + Sum E in Right before before

arr = []

PF₀ PF_E Sum₀ Sum_E

for (j = 0 ; j < n ; j++)

// j is SF or not

