Today Content:

- Number system Basics
- Binary to decimal or vice versa
- Add 2 binary numbers.
- Bitwise operators
  - Basic properties
  - Basic problems.

Number system. → Decimal number system → [0, 9] → [10]

$$734 = 7 \times 10^2 + 3 \times 10^1 + 4 \times 10^0$$

10^2 10^1 10^0

$$= 700 + 30 + 4$$

$$6594 = 6 \times 10^3 + 5 \times 10^2 + 9 \times 10^1 + 4 \times 10^0$$

Other number system

- binary
- ternary
- hexa
- octal

Octal → 0-7
→ 8

$(125)_8$

8^2 8^1 8^0

$1\times8^2 + 2\times8^1 + 5\times8^0$

$= 64 + 16 + 5 = (85)_{10}$

Binary number system → 0-1
→ [2]

$(10110)_2$

2^4 2^3 2^2 2^1 2^0

$= 1\times2^4 + 0\times2^3 + 1\times2^2 + 1\times2^1 + 0\times2^0$

$= 2^4 + 2^2 + 2^1$

$= 22$

$(1020)_3$ → invalid binary representation.

$(1010)_2$

2^3 2^2 2^1 2^0

$= 2^3 + 0 + 2^1 + 0$

$= 10$

Decimal to binary (Repeated division)

2	$(37)_{10}$
2	18
2	9
2	4
2	2
2	1
	0

1
0
0
1
0
1

$100101 = 2^5 + 2^2 + 2^0$

$= 32 + 4 + 1$

$= 37$

2	25
2	12
2	6
2	3
2	1
	0

1
0
0
1
1

$(11001)_2 = 2^4 + 2^3 + 2^0$

$= 16 + 8 + 1 = 25$

2	6
2	3
2	1
	0

0
1
1

$(110)_2$

binary → hexa

binary → decimal → hexa

Add 2 decimal number.

$$\begin{array}{r} {\scriptstyle 1}\quad {\scriptstyle 1} \\ 7\ 8\ 9 \\ +\quad 1\ 4\ 2 \\ \hline 9\ 3\ 1 \end{array}$$

$$\begin{array}{r} {\scriptstyle 17/10}\quad {\scriptstyle 8/10}\quad {\scriptstyle 17/10} \\ {\scriptstyle 11/10}\ {\scriptstyle 1}\quad 0\quad 1 \\ {\scriptstyle 1} \\ 7\ 8\ 3\ 9 \\ 3\ 9\ 4\ 8 \\ \hline {\scriptstyle 11\%10}\ {\scriptstyle 17/10}\ {\scriptstyle 8\%10}\ {\scriptstyle 17/10} \\ 1\quad 1\quad 7\quad 8\quad 7 \end{array}$$

Add 2 binary numbers.

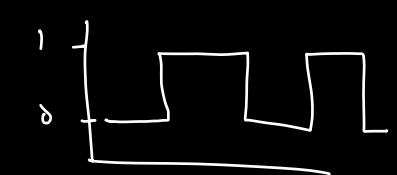
$$\begin{array}{r} {\scriptstyle 1/2}\quad {\scriptstyle 11/2}\quad {\scriptstyle 3/2}\quad {\scriptstyle 2/2}\quad {\scriptstyle 1/2} \\ 0\quad 1\quad 1\quad 0 \\ {\scriptstyle 0}\ (\ 1\ 0\ 1\ 1\ 0\ )_2 \\ (\ 0\ 0\ 1\ 1\ 1\ )_2 \\ \hline {\scriptstyle 1\%2}\quad {\scriptstyle 1/2}\quad {\scriptstyle 3/2}\quad {\scriptstyle 2/2}\quad {\scriptstyle 1/2} \\ (\ 1\quad 1\quad 1\quad 0\quad 1\ )_2 \end{array}$$

$(10)_{10}$

$(1010)_2$

$$\begin{array}{r} {\scriptstyle 2/2} \\ 1\quad (1)_2 \\ +\quad (1)_2 \\ \hline (1\ 0)_2 \end{array}$$

Why binary ?



==0 (0)

>0 (1)

int x = 25

↓

In your system stored in binary

x = 11001

print(x) ← binary to decimal

Bitwise operators

= { &, |, ^, ~, <<, >> }

And, OR, XOR, inverse, left shift, right shift

Truth Table

a	b	a & b (If both are 1, then 1)	a | b (If one of them is one, 1)	a ^ b (Same Same puppy shame)	~a	~b
0	0	0	0	0	1	1
0	1	0	1	1	1	0
1	0	0	1	1	0	1
1	1	1	1	0	0	0

(12) & (13)

```
  1100
& 1101
------
  1100
```

```
 12  1100
----------
~12  0011
```

$(0011)_2$

12 & 13 = 12

~12 = 3

if(12 && 13) True

if(!12) False

Q. a = 29 , b = 19.

print(a & b) = 17
print(a | b) = 31
print(a ^ b) = 14

a : 1 1 1 0 1
b : 1 0 0 1 1

a & b : 1 0 0 0 1 = 17
a | b : 1 1 1 1 1 = 31
a ^ b : 0 1 1 1 0 = 14

a = 1 1 0 1
b = 1 0 1 0

a & b = 1 0 0 0
a | b = 1 1 1 1
a ^ b = 0 1 1 1

Properties

1. a & 1 → even: 0, odd: 1
2. a & 0 = 0
3. a & a = a

a = (10)$_{10}$ → (1 0 1 0)$_2$

a & 1

0 0 0 1
0 0 0 0

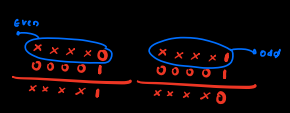
a = 14 print(a & 1) = 0

1 1 1 0
0 0 0 1
0 0 0 0

4. $a \wedge 0 = a$

5. $a \wedge a = 0$

6. $a \wedge 1$ — Even → $a+1$; odd → $a-1$



$a = 13$ $\quad$ print$(a \& 1) = 1$

$$\begin{array}{r} 1101 \\ 0001 \\ \hline 0001 \end{array}$$

$$\begin{array}{r} x\,x\,x\,x\,x\,x\,x \\ 0\,0\,0\,0\,0\,0\,1 \\ \hline 0\,0\,0\,0\,0\,0\,0 \end{array}$$

odd — unit place is 1 $\Rightarrow a \& 1 \Rightarrow 1$

unit place is 0 $\rightarrow a \& 1 \Rightarrow 0$

Even

$a = 13$

$$\begin{array}{r} 1101 \\ 0001 \\ \hline 1100 \end{array} = 12$$

$a = 14$

$$\begin{array}{r} 1110 \\ 0001 \\ \hline 1111 \end{array} = 15$$

ToDo

7. $a \mid 1 = \neq a$

8. $a \mid 0 = a$

9. $a \mid a = a$

Commutative Property

$a \& b = b \& a$

$a \mid b = b \mid a$

$a \wedge b = b \wedge a$

Associative property.

$a \& b \& c = a \& (b \& c)$

$= (a \& b) \& c = (a \& c) \& b$

Similary for $\mid$ and $\wedge$

Q. 5 numbers

a ^ b ^ a ^ d ^ b

= a ^ a ^ b ^ d ^ b

= 0 ^ b ^ d ^ b

= b ^ d ^ b

= b ^ b ^ d

= 0 ^ d

= d

Q 7 numbers.

e ^ f ^ a ^ f ^ e ^ g ^ a = ?

= g

V.V.V.V.V. Important

Q. Given N array elements, every element repeats twice except 1 (once).

find that unique number.

arr[5] = { 6, 9, 10, 9, 8, 6, 8 }

= 10

Take XOR of entire array.

```
ans = 0
for(i=0; i<N; i++)
{  ans = ans ^ arr[i]
return ans.
```

TC: O(N)
SC: O(1)

6, 8, 10, 8, 6 = 10

0 1 1 0
1 0 0 0
1 1 1 0
1 0 1 0
0 1 0 0

1 0 0 0
1 1 0 0
0 1 1 0
$(1\,0\,1\,0)_2 = 10$