Lab 4: CIA and Multiple Regression

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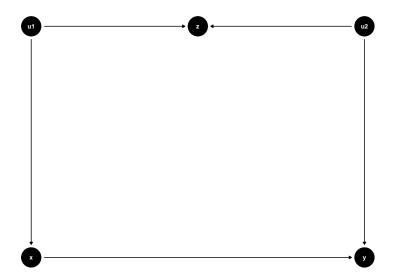
2024-02-23

Plan

- Refresher on DAGs, Bootstrap and FWL
- ► Treatment Effect Heterogeneity
- Omitted Variable Bias
- Specification Error

```
library(dagitty)
library(ggdag)
model <- dagitty("dag{x->y; u1->x; u1->z; u2->z; u2->y}")
latents(model) <- c("u1", "u2")
coordinates(model) <- list(
    x = c(x=1, u1=1, z=2, u2=3, y=3),
    y = c(x=1, u1=2, z=2, u2=2, y=1))</pre>
```

```
ggdag(model) + theme_dag()
```



Coefficients:
(Intercept)

-0.03282 1.01764

```
# simulate data (linear model)
n < -1e4
u1 \leftarrow rnorm(n)
u2 \leftarrow rnorm(n)
z \leftarrow u1 + u2 + rnorm(n)
x \leftarrow u1 + rnorm(n)
y \leftarrow x - 4*u2 + rnorm(n)
# unadjusted estimate is *not* confounded!
lm(y \sim x)
##
## Call:
## lm(formula = y \sim x)
##
```

х

```
# adjusting for Z induces bias!
lm(y \sim x + z)
##
## Call:
## lm(formula = y \sim x + z)
##
## Coefficients:
## (Intercept)
                         X
                                     Z
       -0.013 1.810 -1.600
##
```

Frisch-Waugh-Lovell theorem

- ▶ Linear model with K covariates. In matrix form: $y = X'\beta + \varepsilon$
- ► FWL gives a formula for the OLS estimate of the *k*th coefficient.

$$\hat{\beta}_k = (X_k' M_{[X_{-k}]} X_k)^{-1} X_k' M_{[X_{-k}]} y$$

Equivalent to the following:

- Regress the individual variable X_k on all the other covariates and take the residuals
- Regress the outcome variable y on all the covariates, except X_k , and take the residuals
- \triangleright Regress the residuals of y on the residuals for X
- Note that to get $\hat{\beta}_k$ it is enough to regress the non-residualized y on residualized X_k (why?), but the SE won't be right

FWL in R

```
set.seed(123)
N <- 1000
X <- rnorm(N, mean = 0, sd = 1)
# Generate binary treatment D, making D and X correlated
D <- rbinom(N, size = 1, prob = plogis(X))
Y <- 2*D + 0.5*X + rnorm(N, mean = 0, sd = 1)
model_ols <- lm(Y ~ D + X)
coef(model_ols)</pre>
```

```
## (Intercept) D X
## -0.02920341 2.05763264 0.43079560
```

FWL in R

```
resid_Y <- residuals(lm(Y ~ X))
resid_D <- residuals(lm(D ~ X))
model_fwl <- lm(resid_Y ~ resid_D - 1)
coef(model_fwl)</pre>
```

```
## resid_D
## 2.057633
```

Cluster bootstrap

- Most commonly used: Wild Cluster Bootstrap (Restricted)
- **Proof** Run the regression, estimate $\hat{\beta}$ and t
- ▶ To test the null hypothesis: run the regression under the null hypothesis (i.e. setting $\beta = 0$)
- ightharpoonup Resample clusters. For each cluster, multiply the residuals by +1 or -1 with equal chance
- ▶ Predict new *y*s with the new residuals and imposing $\beta = 0$
- lacktriangle Re-estimate the model for all parameters and find \hat{eta}_b
- ightharpoonup Compute *t*-statistic t_b from the new estimates
- Repeat many times
- Compute bootstrap p-values by counting the share of simulated t_b to the left/right of the observed one t

Cluster bootstrap

In R:

- Option cluster in sandwich::vcovBS()
- The package fwildclusterboot is a translation of Stata's boottest (same options)
- Function boottest works with objects of class lm, felm, fixest
- Another option is the package multiwaycov and the function cluster.boot which can be used for post-estimation SE calculation (e.g. in coeftest or stargazer)

ATT (from Cyrus' slides)

► ATT (Average Treatment Effect on the Treated)

$$\rho_{\mathsf{ATT}} = \mathbb{E}[Y_{1i} - Y_{0i}|D_i = 1]
= \mathbb{E}_{X|D=1} \{ \mathbb{E}[Y_{1i} - Y_{0i}|X_i, D_i = 1] \}
= \mathbb{E}_{X|D=1} \{ \mathbb{E}[Y_{1i}|X_i, D_i = 1] - \mathbb{E}[Y_{0i}|X_i, D_i = 1] \}
= \mathbb{E}_{X|D=1} \{ \mathbb{E}[Y_{1i}|X_i, D_i = 1] - \mathbb{E}[Y_{0i}|X_i, D_i = 0] \}.$$

- $\blacktriangleright \text{ Let } \delta_{\mathsf{x}} = \mathbb{E}[Y_{1i}|X_i = \mathsf{x}, D_i = 1] \mathbb{E}[Y_{0i}|X_i = \mathsf{x}, D_i = 0].$
- ▶ For X_i discrete, unbiased "matching estimator'', $\hat{\rho}_{ATT}$:

$$\mathbb{E}[\hat{
ho}_{\mathsf{ATT}}] = \sum_{\mathsf{x}} \delta_{\mathsf{x}} \cdot \mathsf{Pr}[X_i = \mathsf{x} | D_i = 1]$$

 $\hat{\rho}_{\mathsf{ATT}} = \sum_{i} \hat{\delta}_{x} \cdot \mathsf{Pr}[X_{i} = x | D_{i} = 1]$

$$= \frac{\sum_{x} \delta_{x} \cdot \Pr[D_{i} = 1 | X_{i} = x] \cdot \Pr[X_{i} = x]}{\sum_{x} \Pr[D_{i} = 1 | X_{i} = x] \cdot \Pr[X_{i} = x]}$$

Multiple Regression (from Cyrus' slides)

► FWL computes the OLS estimator for the coefficient on *D_i*:

$$\hat{\delta}_R = rac{\sum_{i=1}^N Y_i ilde{D}_i}{\sum_{i=1}^N ilde{D}_i^2} imes rac{\mathsf{Cov}(Y_i, ilde{D}_i)}{\mathsf{Var}(ilde{D}_i)}$$

$$= \frac{\sum_{x} \mathsf{Cov}(Y_i, \tilde{D}_i | X_i = x) \mathsf{Pr}[X_i = x]}{\sum_{x} \mathsf{Var}(\tilde{D}_i | X_i = x) \mathsf{Pr}[X_i = x]}$$

$$\sum_{x} \operatorname{Var}(D_{i}|X_{i} = x) \operatorname{Pr}[X_{i} = x]$$

$$= \frac{\sum_{x} \operatorname{Cov}(Y_{0i} + \rho_{i}D_{i}, \tilde{D}_{i}|X_{i} = x) \operatorname{Pr}[X_{i} = x]}{\sum_{x} \operatorname{Var}(\tilde{D}_{i}|X_{i} = x) \operatorname{Pr}[X_{i} = x]}$$

$$= \frac{\sum_{x} \operatorname{Cov}(\rho_{i} D_{i}, \tilde{D}_{i} | X_{i} = x) \operatorname{Pr}[X_{i} = x]}{\sum_{x} \operatorname{Var}(\tilde{D}_{i} | X_{i} = x) \operatorname{Pr}[X_{i} = x]}$$

$$= \frac{\sum_{x} \mathbb{E}(\rho_{i} D_{i} \tilde{D}_{i} | X_{i} = x) \Pr[X_{i} = x]}{\sum_{x} \text{Var}(\tilde{D}_{i} | X_{i} = x) \Pr[X_{i} = x]}$$
$$- \sum_{x} \delta_{x} \text{Var}(D_{i} | X_{i} = x) \Pr[X_{i} = x]$$

$$= \frac{\sum_{x} \delta_{x} \text{Var}(D_{i}|X_{i} = x) \text{Pr}[X_{i} = x]}{\sum_{x} \text{Var}(D_{i}|X_{i} = x) \text{Pr}[X_{i} = x]}$$

$$= \frac{\sum_{x} \delta_{x} \text{Pr}[D_{i} = 1|X_{i} = x](1 - \text{Pr}[D_{i} = 1|X_{i} = x]) \text{Pr}[X_{i} = x]}{\sum_{x} \text{Pr}[D_{i} = 1|X_{i} = x](1 - \text{Pr}[D_{i} = 1|X_{i} = x]) \text{Pr}[X_{i} = x]}.$$

ATT Matching vs Regression (from Cyrus' slides)

Compare:

$$\mathbb{E}[\hat{\rho}_{\mathsf{ATT}}] = \frac{\sum_{x} \delta_{x} \Pr[D_{i} = 1 | X_{i} = x] \Pr[X_{i} = x]}{\sum_{x} \Pr[D_{i} = 1 | X_{i} = x] \Pr[X_{i} = x]}.$$

versus

$$\hat{\delta}_R \stackrel{a}{\to} \frac{\sum_x \delta_x [\Pr[D_i = 1 | X_i = x] (1 - \Pr[D_i = 1 | X_i = x])] \Pr[X_i = x]}{\sum_x [\Pr[D_i = 1 | X_i = x] (1 - \Pr[D_i = 1 | X_i = x])] \Pr[X_i = x]}.$$

- ▶ Both are weighted averages of δ_x 's, but $\hat{\rho}_{ATT}$ aggregates via population weighting while $\hat{\delta}_R$ aggregates via conditional variance weighting wrt D_i .
- Population weighting is unbiased for population target.
- Conditional variance weighting is not.

Effective sample

► From Angrist and Krueger (1999), Angrist and Pischke (2009), Aronow and Samii (2016), the following result holds:

$$\hat{\beta} \stackrel{P}{\rightarrow} \frac{E[w_i \tau_i]}{E[w_i]}$$
, where $w_i = (D_i - E[D_i | X_i])^2$

where

$$E[w_i|X_i] = E[(D_i - E[D_i|X_i])^2|X_i)] = Var[D_i|X_i]$$

- Conditional variance weighting equivalent to run the regression on an effective sample different from the one we think we are working with
- ► To characterize the effective sample we can estimate the w_is

Effective sample

$$E[w_i|X_i] = E[(D_i - E[D_i|X_i])^2|X_i] = Var[D_i|X_i]$$

- ▶ If we assume linearity of the treatment assignment in X_i , the weight is equal to the square of the residual from regressing the treatment indicator on X_i
- ► Higher conditional variance of treatment ⇒ more variance not explained by the covariates ⇒ higher error term
- ► To estimate the regression weights:
 - Run the regression $D_i = X_i \gamma + e_i$
 - ► Take residual $\hat{e}_i = D_i X_i \hat{\gamma}$ and square it

Example #1

American Political Science Review (2018) 112, 4, 874-890

doi:10.1017/S0003055418000497

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Who Polices the Administrative State?

KENNETH LOWANDE University of Michigan

Scholarship on oversight of the bureaucracy typically conceives of legislatures as unitary actors. But most oversight is conducted by individual legislators who contact agencies directly. I acquire the correspondence logs of 16 bureaucratic agencies and re-evaluate the conventional proposition that ideological disagreement drives oversight. I identify the effect of this disagreement by exploiting the transition from George Bush to Barack Obama, which shifted the ideological orientation of agencies through turnover in agency personnel. Contrary to existing research, I find ideological conflict has a negligible effect on oversight, whereas committee roles and narrow district interests are primary drivers. The findings may indicate that absent incentives induced by public auditing, legislator behavior is driven by policy valence concerns rather than ideology. The results further suggest collective action in Congress may pose greater obstacles to bureaucratic oversight than previously thought.

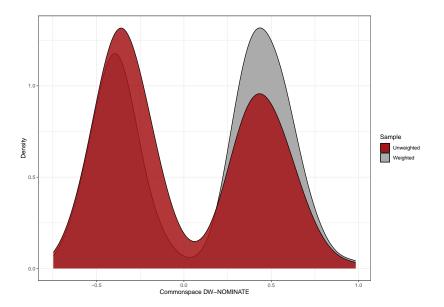
Oversight and Ideology (Table 1 from the paper)

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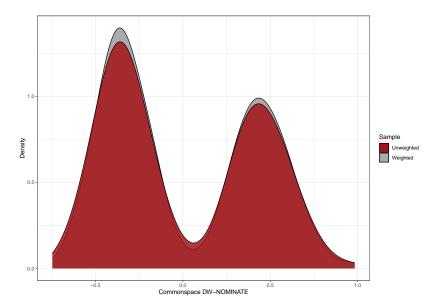
	Regression Results						
## ======: ## ##	Dependent variable:						
## ##	Casework (1)	Policy (2)	Both (3)				
## ## Committee ## ##	0.039***		0.063***				
## ## Chair ## ##		0.111** (0.047)					
## Ranking Member ## ##		0.143*** (0.049)					
## Distance ## ##	(0.021)	-0.001 (0.020)	(0.021)				
## ## Observations ## R2	16,455 0.500	16,455 0.430	17,552 0.503				
## ======= ## Note:		**p<0.05;					

Ideology of Effective Legislator Sample

Ideological Distance



Committee Membership



Example 2 (code taken from Giacomo Lemoli)

Turning Personal Experience into Political Attitudes: The Effect of Local Weather on Americans' Perceptions about Global Warming

Patrick J. Egan New York University Megan Mullin Temple University

How do people translate their personal experiences into political attitudes? It has been difficult to explore this question using observational data, because individuals are typically exposed to experiences in a selective fashion, and self-reports of exposure may be biased and unreliable. In this study, we identify one experience to which Americans are exposed nearly at random—their local weather—and show that weather patterns have a significant effect on people's beliefs about the evidence for global warming.

Application: weather and global warming beliefs

```
# Import the data
library(haven)
library(dplyr)
d <- read_dta("gwdataset.dta")</pre>
# Import state IDs
zips <- read_dta("zipcodetostate.dta")</pre>
zips <- zips %>% select(c(statenum, statefromzipfile)) %>% unique()
zips <- zips %>% filter(!(statenum == 8 & statefromzipfile == "NY"))
# Import population data
pops <- read.csv("population_ests_2013.csv")</pre>
# Format.
pops$state <- tolower(pops$NAME)</pre>
d$getwarmord <- as.double(d$getwarmord)</pre>
```

Weather and global warming beliefs

```
# Estimate primary model of interest:
d$doi <- factor(d$doi)
d$statenum <- factor(d$statenum)</pre>
d$wbnid_num <- factor(d$wbnid_num)</pre>
Y <- "getwarmord"
D <- "ddt week"
X \leftarrow names(d)[c(15,17,42:72)]
reg_formula <- paste0(Y, "~", D, "+", paste0(X, collapse = "+"))</pre>
reg_out <- lm(as.formula(reg_formula), d)</pre>
# Or
out <- lm(getwarmord~ddt_week+educ_hsless+educ_coll+educ_postgrad+
          educ_dk+party_rep+party_leanrep+party_leandem+
          party_dem+male+raceeth_black+raceeth_hisp+
          raceeth_notwbh+raceeth_dkref+age_1824+age_2534+
          age_3544+age_5564+age_65plus+age_dk+ideo_vcons+
          ideo_conservative+ideo_liberal+ideo_vlib+ideo_dk+
          attend_1+attend_2+attend_3+attend_5+attend_6+
          attend 9+as.factor(doi)+as.factor(statenum)+
          as.factor(wbnid_num),d)
```

Base Model

summary(reg_out)\$coefficients[1:10,]

```
## Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.945740062 0.771478843 2.5220913 0.01169077
## ddt_week 0.004857915 0.002475887 1.9620908 0.04979656
## wbnid_num3103 0.843451519 0.922666490 0.9141456 0.36067588
## wbnid_num3154 1.575071541 0.973391215 1.6181280 0.10568587
## wbnid_num3159 1.903629413 1.021302199 1.8639237 0.06237963
## wbnid_num3804 1.406498119 0.794035963 1.7713280 0.07655528
## wbnid_num3810 1.330878449 0.806312016 1.6505750 0.09887602
## wbnid_num3811 1.082204367 0.798796489 1.3547936 0.17553267
## wbnid_num3813 0.986084952 0.829563706 1.1886790 0.23461152
```

Estimate the weights

```
# Regress treatment indicator on the vector of covariates
D_formula <- pasteO(D, "~", pasteO(X, collapse = "+"))
outD <- lm(as.formula(D_formula), d)
# Extract the residuals and take their square
eD2 <- residuals(outD)^2</pre>
```

Effective sample statistics

```
# Take some relevant variables
compare_samples<- d[, c("wave", "ddt_week", "ddt_twoweeks",
    "ddt_threeweeks", "party_rep", "attend_1", "ideo_conservative",
    "age_1824", "educ_hsless")]

# Compute statistics with and without weights
compare_samples <- t(apply(compare_samples,2,function(x)
    c(mean(x),sd(x),weighted.mean(x,eD2),
        sqrt(weighted.mean((x-weighted.mean(x,eD2))^2,eD2)))))
colnames(compare_samples) <- c("Nominal Mean", "Nominal SD",
    "Effective Mean", "Effective SD")</pre>
```

Effective Sample Statistics

compare_samples

##		Nominal Mean	${\tt Nominal \ SD}$	Effective Mean	${\tt Effective \; SD}$
##	wave	3.09693726	1.4252527	3.20788200	1.5609143
##	ddt_week	3.83548593	5.9047249	5.11579140	10.8980228
##	ddt_twoweeks	3.85505617	5.4572382	5.00137435	9.2262827
##	ddt_threeweeks	3.96719696	4.7689594	5.10859485	8.4348180
##	party_rep	0.29527208	0.4561989	0.28978321	0.4536617
##	attend_1	0.11433244	0.3182383	0.12343459	0.3289354
##	${\tt ideo_conservative}$	0.31132917	0.4630715	0.29325249	0.4552532
##	age_1824	0.07195956	0.2584402	0.06881146	0.2531333
##	educ_hsless	0.34151056	0.4742516	0.31219962	0.4633908

Effective sample maps

```
# Construct the "effective sample weights" for each state
wts_by_state <- tapply(eD2, d$statenum, sum)</pre>
wts_by_state <- wts_by_state/sum(wts_by_state)*100
wts_by_state <- data.frame(eff = wts_by_state,</pre>
                            statenum = as.numeric(names(wts_by_state)))
# Merge to the state name variable
data_for_map <- merge(wts_by_state, zips, by="statenum")</pre>
# Construct the "nominal sample weights" for each state
wts_by_state <- tapply(rep(1,6726),d$statenum,sum)</pre>
wts_by_state <- wts_by_state/sum(wts_by_state)*100
wts_by_state <- data.frame(nom = wts_by_state,
                            statenum = as.numeric(names(wts by state)))
# Add to the other data
data_for_map <- merge(data_for_map, wts_by_state, by="statenum")</pre>
```

Effective sample maps

```
# Get correct state names
require(maps,quietly=TRUE)
data(state.fips)
# Add them to the dataset
data_for_map <- left_join(data_for_map, state.fips,
                          by = c("statefromzipfile" = "abb"))
# More data prep
data for map$state <- sapply(as.character(data for map$polyname).
                             function(x)strsplit(x,":")[[1]][1])
data_for_map <- data_for_map %>% group_by(statefromzipfile) %>%
 summarise_all(first) %>% ungroup() %>% select(-polyname)
# Diff between nominal and effective weights
data for map$diff <- data for map$eff - data for map$nom
# Merge with population data
data for map <- left join(data for map, pops, by="state")
# Actual "weight" of each state in the US
data_for_map$pop_pct <- data_for_map$POPESTIMATE2013/sum(
 data for map$POPESTIMATE2013)*100
# Different representativity of the two samples
data for map <- mutate(data for map.
                       pop_diff_eff = eff - pop_pct,
                       pop_diff_nom = nom - pop_pct)
data for map <- mutate(data for map.
                       pop_diff = pop_diff_eff - pop_diff_nom)
require(ggplot2,quietly=TRUE)
state map <- map data("state")
```

More setup

```
# Plot the weights in each sample
plot eff <- ggplot(data for map, aes(map id = state)) +
  geom map(aes(fill=eff), map = state map) +
 expand_limits(x= state_map$long, y = state_map$lat) +
 scale fill continuous("% Weight", limits=c(0,17), low="white", high="black") +
 labs(title = "Effective Sample") +
 theme(legend.position=c(.2,.1),legend.direction = "horizontal",
        axis.line = element_blank(), axis.text = element_blank(),
        axis.ticks = element blank(), axis.title = element blank(),
        panel.background = element_blank(),
        plot.background = element_blank(),
        panel.border = element blank().
        panel.grid = element_blank())
plot nom <- ggplot(data for map, aes(map id = state)) +
  geom_map(aes(fill=nom), map = state_map) +
 expand_limits(x=state_map$long, y=state_map$lat) +
 scale fill continuous("% Weight", limits=c(0,17), low="white", high="black") +
 labs(title="Nominal Sample") +
 theme(legend.position=c(.2,.1),legend.direction = "horizontal",
        axis.line = element blank(), axis.text = element blank(),
        axis.ticks = element_blank(), axis.title = element_blank(),
        panel.background = element_blank(),
        plot.background = element blank().
        panel.border = element blank(), panel.grid = element blank())
```

Maps

```
require(gridExtra,quietly=TRUE)
grid.arrange(plot_nom,plot_eff,ncol=2)
```

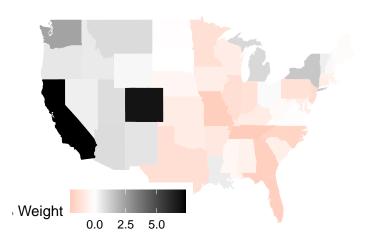


Setup comparison plot

Difference in weights

plot_diff

Effective Weight minus Nominal Weight



Sensitivity analysis

- Suppose the true model includes covariates;
- \triangleright If omit covariates, then the coefficient on D_i is

$$\frac{\mathsf{Cov}(Y_i, D_i)}{\mathsf{Var}(D_i)} = \rho + \underbrace{\gamma' \delta}_{\text{"omitted variable bias"}}$$

- ► Following Cinelli & Hazlett (2020) OVB = "confounder impact $\gamma \times$ imbalance δ "
- According to Cinelli & Hazlett

$$|\hat{\mathsf{bias}}| = \hat{\mathsf{se}}(\hat{r}_{\mathsf{res}}) \sqrt{\frac{R_{Y \sim Z|D,X}^2 R_{D \sim Z|X}^2}{1 - R_{D \sim Z|X}^2}} (\mathsf{df}).$$

Example

American Political Science Review

Vol. 103, No. 2 May 2009

doi:10.1017/S0003055409090212

From Violence to Voting: War and Political Participation in Uganda CHRISTOPHER BLATTMAN Yale University

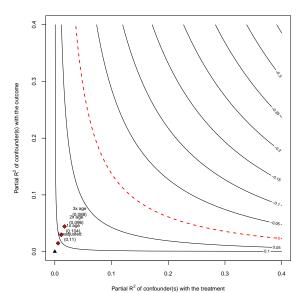
That is the political legacy of violent conflict? I present evidence for a link from past violence to increased political engagement among excombatants. The evidence comes from northern Uganda, where rebel recruitment generated quasiexperimental variation in who was conscripted by abduction. Survey data suggest that abduction leads to substantial increases in voting and community leadership, largely due to elevated levels of violence witnessed. Meanwhile, abduction and violence do not appear to affect nonpolitical participation. These patterns are not easily explained by conventional theories of participation, including mobilization by elites, differential costs, and altruistic preferences. Qualitative interviews suggest that violence may lead to personal growth and political activation, a possibility supported by psychological research on the positive effects of traumatic events. Although the generalizability of these results requires more evidence to judge, the findings challenge our understanding of political behavior and point to important new avenues of research.

Impact of Abduction on Social and Political Participation

Impact of Abduction on Social and Political Participation

```
##
   Regression Results
##
                     Dependent variable:
##
##
                         Voted in 2005
## Abducted
                           0.113***
                            (0.040)
##
##
   Observations
                              533
## R.2
                             0 176
## Note:
                 *p<0.1; **p<0.05; ***p<0.01
```

Sensitivity Analysis



Sensitivity Analysis

```
plot(sensitivity_1,sensitivity.of = "t-value")
```

