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In [1]:

```python
from scipy.stats import lognorm, norm, expon
import numpy as np
```

Q1

Let X denote the amount of time for which a book on 2-hour reserve at a college library is checked out by a randomly selected student and suppose that X has density function: $f(x)$ = {0.5x 0 ≤ x ≤ 2, 0 $otherwise$} Calculate the following probabilities:

a. $P(X \leq 1)$

$$P(X <= B) = CDF(X) = F(X) = \int_0^B \frac{x}{2}\,dx = \frac{x^2}{4}$$
$$P(X <= 1) = F(1) = \frac{1}{4} = .25$$

b. $P(0.5 \leq X \leq 1.5)$

$$F(1.5) - F(.5) = \frac{1.5^2}{4} - \frac{.5^2}{4} = \frac{2.25 - .25}{4} = .5$$

c. $P(X > 1.5)$

$$1 - F(1.5) = 1 - \frac{1.5^2}{4} = \frac{4 - 2.25}{4} = \frac{1.75}{4} = .4375$$

Q2

Extensive experience states that the time to failure of fans in certain types of diesel engines follow an exponential distribution with mean time to failure 10,000 hours.

a. What is the probability that the fan will last at least 15,000 hours

In [2]:
```
mean = 10000
exp_dist = expon(scale = mean)
print(1-exp_dist.cdf(15000))
```

```
0.2231301601484298
```

$$X = time\ to\ failure$$
$$distribution = exponential$$
$$\lambda = \frac{1}{mean} = \frac{1}{10000}$$
$$P(X > 15,000) = 1 - F(15000) = .2231$$

b. What is the standard deviation of time to failure?

$$in\ exponential\ distribution\ we\ have:$$
$$mean = \frac{1}{\lambda} = \sigma$$
$$std = \sigma = 10000$$

In [3]:
```
exp_dist.std()
```

Out[3]:
```
10000.0
```

c. What is the probability that a fan will last longer than the mean by more than 2 standard deviations?

$$P(X > \mu + 2 * \sigma) = P(X > 30000) = 1 - F(30,000) = .04979$$

In [4]:
```
print(1 - exp_dist.cdf(mean+mean+mean))
```

```
0.04978706836786395
```

Q3

A person can be in an upright position either sitting or standing. For children between the ages of eight and fifteen, time per day in an upright position follows a normal probability model with mean 6.2 hours and standard deviation 1.4 hours.

a. What is the probability that a randomly selected child's time per day in an upright position is more than 7 hours?

$$X = time\ per\ day\ in\ an\ upright\ position$$
$$Distribution = Normal$$
$$\mu = 6.2$$
$$\sigma = 1.4$$
$$P(X > 7) = 1 - F(7) = .28385$$

In [5]:
```
mean = 6.2
std = 1.4
normal_dist = norm(mean, std)
print(1 - normal_dist.cdf(7))
```

```
0.2838545830986763
```

b. What is the probability that a randomly selected child's time per day in an upright position is less than 3 hours?

$$P(X < 3) = F(3) = .01114$$

In [6]:
```
print(normal_dist.cdf(3))
```

```
0.01113548947961638
```

c. Fill in the blank: 5% of children have less than ____ hours per day in an upright position.

$$P(X < A) = F(A) = .05$$
$$A = 3.89$$

In [7]:
```
norm.ppf(.05,mean,std)
```

Out[7]:
```
3.8972049222679384
```

d. Children in the highest 1% for time spent upright were in this position the longest. Therefore, 1% of children have more than ___ hours per day in an upright position.

$$P(X > A) = 1 - F(A) = .01$$
$$F(A) = .99$$
$$A = 9.4569$$

In [8]:
```
norm.ppf(.99,mean,std)
```

Out[8]:
```
9.456887023657178
```

Q4

The life of an Infinity brand car battery is normally distributed with mean 810 days and standard deviation 42 days.

a. What is the probability that a randomly selected Infinity car battery has a lifetime between 825 and 900 days?

$$X = Battery\ lifetime$$
$$Distribution = Normal$$
$$\mu = 810$$
$$\sigma = 42$$
$$P(825 < X < 900) = F(900) - F(825) = .3444$$

In [9]:

```
mu = 810
std = 42
normal_dist = norm(mu, std)
print(normal_dist.cdf(900) - normal_dist.cdf(825))
```

```
0.3444301453470071
```

b. What is the probability that a randomly selected Infinity car battery has a lifetime more than 850 days?

$$P(X > 850) = 1 - F(850) = .1705$$

In [10]:

```
print(1 - normal_dist.cdf(850))
```

```
0.17045190796406517
```

c. What is the 37th percentile of brand Infinity lifetimes?

$$P(X < A) = F(A) = .37$$
$$A = 796.06$$

In [11]:

```
percentile = .37
percentile_value = norm.ppf(percentile, loc=mu, scale=std)
print(f"The 37th percentile of brand Infinity lifetimes is approximate
```

```
The 37th percentile of brand Infinity lifetimes is approximately 796.06 d
ays.
```

d. One Infinity battery had a lifetime of 700 days. Is this unusual? EXPLAIN.

in this question we have

$$\frac{|700-\mu|}{\sigma} = \frac{810-700}{42} = 2.6$$

Which means X is more than 2.6 times of standard deviation far from the mean. which makes it very unlikely and unusual the probablity of such an event is:

$$P(X < 700) = F(700) = .0044$$

Which is less than half percent

In [12]:
```
normal_dist.cdf(700)
```

Out[12]: 0.004408781892886983

Q5

Suppose that X has a lognormal distribution with parameters μ = 6 and σ = 0.3. Determine the following:

a. $P(X < 300)$

In [13]:
```
mu = 6  # Mean of the underlying normal distribution
sigma = .3  # Standard deviation of the underlying normal distributior
scale = np.exp(mu)  # Scale parameter (corresponds to the median)
x = 300
lognorm_dist = lognorm(scale=scale, s=sigma)
print(lognorm_dist.cdf(300))
```

0.1617253133244424

b. The value for x such that $P(X \le x)$ = 0.95

In [14]:
```
lognorm_dist.ppf(.95)
```

Out[14]: 660.8032153804501

c. $P(100 < X < 500)$

In [15]:
```
lognorm_dist.cdf(500)-lognorm_dist.cdf(100)
```

Out[15]: 0.7628051258906872