MATH3075/3975
Financial Derivatives

School of Mathematics and Statistics
University of Sydney

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Tutorial sheet 5

Background: Section 2.2 – Single-Period Market Models.

Exercise 1 Consider the market model $\mathcal{M} = (B, S)$ introduced in Exercise 3 (Week 4). We thus have $k = 3$, $r = \frac{1}{9}$, $S_0 = 5$ and the stock prices at time 1 are given by the following table

	ω_1	ω_2	ω_3
S_1	$\frac{60}{9}$	$\frac{40}{9}$	$\frac{30}{9}$

Are there any values for K such that the call option $(S_1 - K)^+$ represents an attainable contingent claim?

Exercise 2 Consider the stochastic volatility model $\mathcal{M} = (B, S)$ introduced in Example 2.2.3 from the course notes. Hence $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, the volatility v is the random variable on Ω given by

$$v(\omega) = \begin{cases} h & \text{if } \omega = \omega_1, \omega_4, \\ l & \text{if } \omega = \omega_2, \omega_3, \end{cases}$$

where $0 < l < h < 1$ and the stock price S_1 satisfies: $S_0 > 0$ and

$$S_1(\omega) = \begin{cases} (1 + v(\omega))S_0 & \text{if } \omega = \omega_1, \omega_2, \\ (1 - v(\omega))S_0 & \text{if } \omega = \omega_3, \omega_4, \end{cases}$$

We assume, in addition, that $0 \le r < h$.

(a) Characterise the class of all attainable contingent claims in $\mathcal{M}$ and check whether the model $\mathcal{M}$ is complete.

(b) Describe the class $\mathbb{M}$ of all risk-neutral probability measures for $\mathcal{M}$.

(c) (MATH3975) Describe the set of all arbitrage prices for the call option $(S_1 - K)^+$ where the strike K satisfies $S_0(1 + l) < K < S_0(1 + h)$.

(d) (MATH3975) Assume that $r = 0$. Check directly whether the call option with strike K such that $S_0(1 + l) < K < S_0(1 + h)$ is attainable and find the range of values of its arbitrage price.