

MATH3075/3975

Financial Derivatives

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Tutorial sheet 2

Exercise 1 Consider the portfolios composed of shares and options:

- (a) one share of stock and put option with strike $K > 0$,
- (b) short position in one share of stock and call option with strike $K > 0$,
- (c) put with strike K_1 and short call with strike K_2 where $0 < K_1 < K_2$,
- (d) put with strike K_2 and call with strike K_1 where $0 < K_1 < K_2$.
- (e) call with strike K_1 and short call with strike K_2 where $0 < K_1 < K_2$.

All options are of European style and have the same maturity date T . Sketch the payoff profile at time T of each portfolio. Can you formulate a conjecture about a level of the initial value of each portfolio?

Exercise 2 Assume that you hold the following portfolio: one share, a short call with strike K_1 and a call with strike K_2 . Assume that $K_1 < K_2$ and denote by $\pi_0 = S_0 - C_0(K_1) + C_0(K_2)$ the price you paid for this portfolio's at time 0. Assume that the interest rate equals zero.

- (a) Sketch the graph of your profit and loss at time T as a function of S_T .
- (b) Find the maximum profit, the maximum loss and the break even point(s).
- (c) Determine any necessary condition(s) for this portfolio to have the property that both profits and losses may occur when S_T ranges from 0 to infinity.
- (d) What is the market view of an investor holding this portfolio?
- (e) Give answers to questions (a)-(d) when a short call with strike K_1 is replaced by two short calls with strike K_1 .

Exercise 3 Suppose that you hold the following portfolio: short call with strike K_1 and two short calls with strike K_2 with $K_1 < K_2$. Denote by $\pi_0 = C_0(K_1) + 2C_0(K_2) > 0$ the price you received at time 0 and assume that the simple interest rate between 0 and T equals $r > 0$.

- (a) How many shares with initial price S_0 would you need to buy/short at time 0 to augment your portfolio if you expect that the market value at time T of one share will be in the interval (K_1, K_2) ?
- (b) Compute the profit and loss at time T of your augmented portfolio as a function of the stock price S_T . Find the maximum profit, the maximum loss and the break even point(s) of your portfolio.
- (c) Assuming that $C_0(K_1)$ and $C_0(K_2)$ are given, derive a condition on the stock price S_0 for which you would make a profit with such a portfolio when $S_T \in (K_1, K_2)$.

Exercise 4 Let $0 < L < K$ be real numbers. Consider the contingent claim with the payoff X at maturity date $T > 0$ given as

$$X = \min(|S_T - K|, L), \quad \forall S_T \in [0, +\infty).$$

Notice that we interpret here S_T as a real variable, as opposed to the random variable $S_T(\omega)$.

- (a) Sketch the profile of the payoff X as a function of the stock price S_T at maturity date T and find the decomposition of the payoff X in terms of the payoffs of standard call and put options with different strikes and expiration date T , the payoff of long/short positions on the stock and a constant payoff of maturity T .
- (b) Using the law of one price, deduce from part (a) the decomposition of the price of the payoff X at time $t \in [0, T]$ in terms of the prices at time t of call and put options, the price at time t of the stock and the price at time t of the zero-coupon bond with maturity T .
- (c) Give answers to questions (a) and (b) for the payoff $Y = \max(|S_T - K|, L)$.

Exercise 5 (MATH3975) Let $g : \mathbb{R}_+ \rightarrow \mathbb{R}$ be an arbitrary continuous and piecewise linear function such that $g(0) = c \in \mathbb{R}$ and g is linear on each interval $[K_i, K_{i+1}]$ for $i = 0, 1, \dots, n-1$ where $K_0 = 0 < K_1 < K_2 < \dots < K_{n-1} < K_n = +\infty$ where $n \geq 2$ is any (fixed) natural number. Denote by α_i the derivative of g on the interval (K_i, K_{i+1}) for $i = 0, 1, \dots, n-1$.

- (a) Show that the payoff $X = g(S_T)$ can be represented by the terminal payoff of a static portfolio composed of long/short positions in a finite family of call and put options written on the stock S , with the expiration date T and strikes K_1, K_2, \dots, K_{n-1} . You may also include long/short positions in S (equivalently, call and put options with strike zero) and a constant payoff at time T . Notice that several alternative solutions may exist.
- (b) Using your solution from part (a), express the price at time $t \in [0, T]$ of the payoff $X = g(S_T)$ which settles at time T .
- (c) What is the price of $X = g(S_T)$ at time $t \in [0, U]$ if the payoff X is determined at time T but it settled at time U (that is, the actual transfer of the cash amount X between the counterparties occurs at time U).