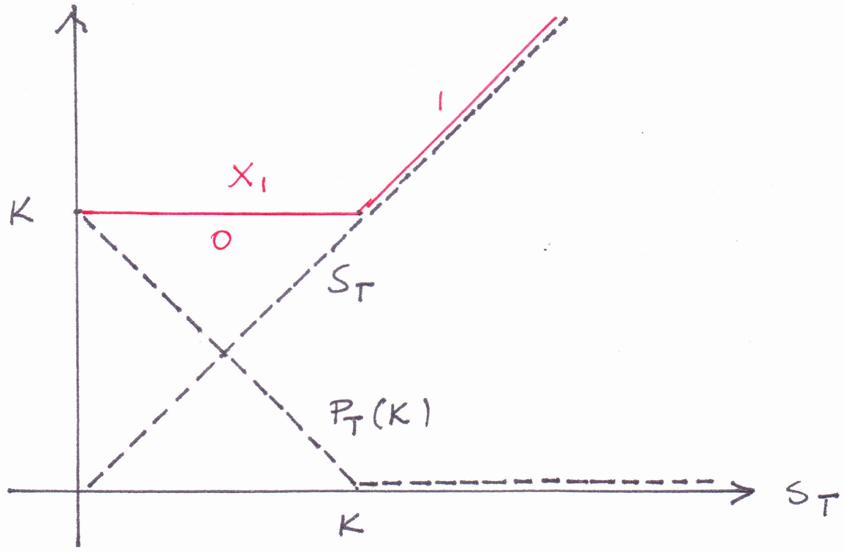
MATH3075/3975 Financial Derivatives

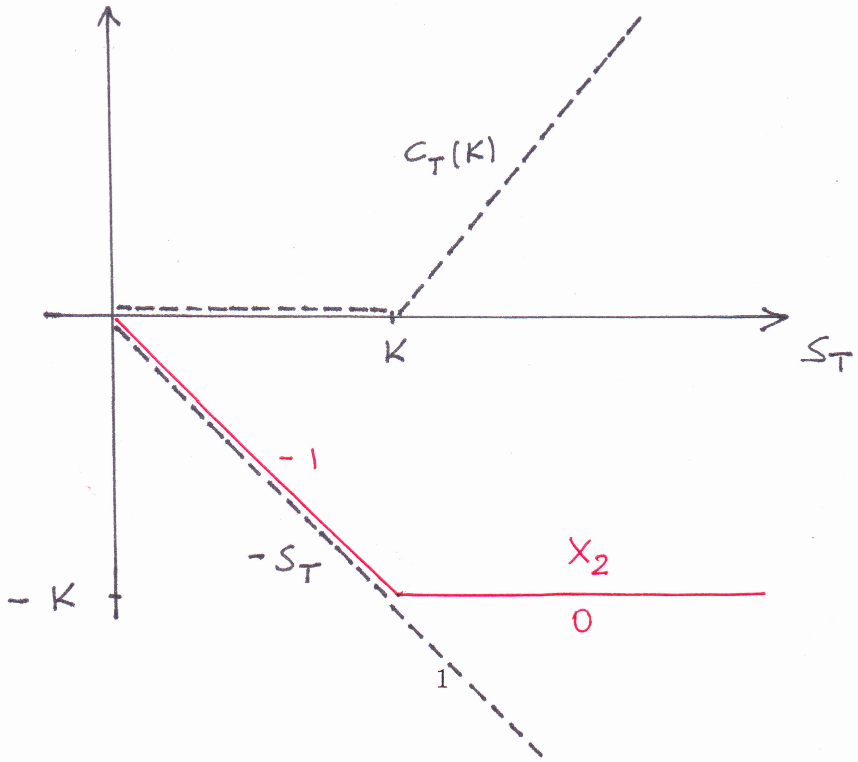
Tutorial 2: Solutions

Exercise 1 It suffices to sketch the graphs of functions representing the portfolio payoff as a function of the terminal price S_T, which may take any nonnegative value.

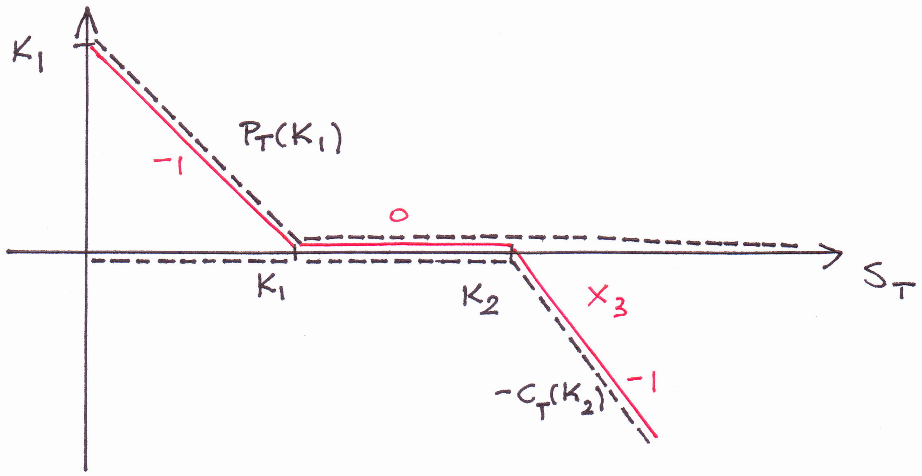
(a) $X_1 = f_1(S_T) = S_T + (K - S_T)^+$ (the initial value should be strictly positive)



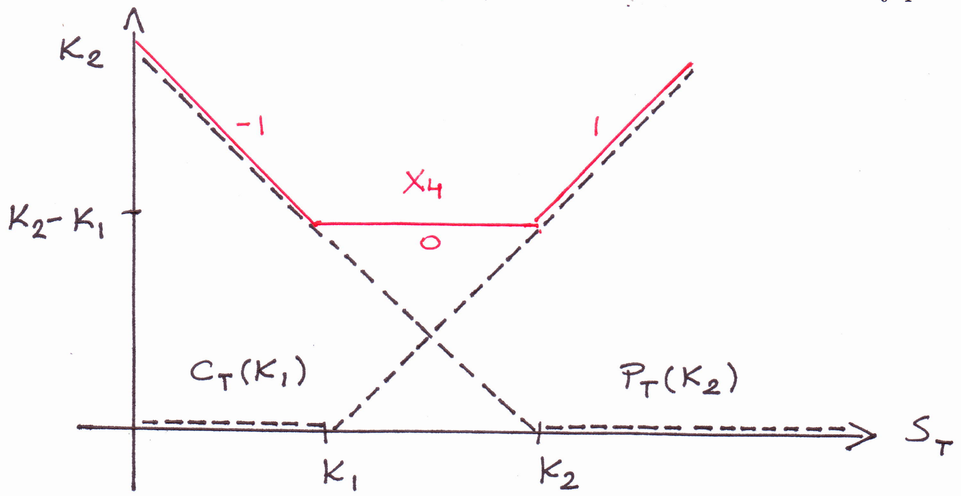
(b) $X_2 = f_2(S_T) = -S_T + (S_T - K)^+$ (the initial value should be strictly negative)



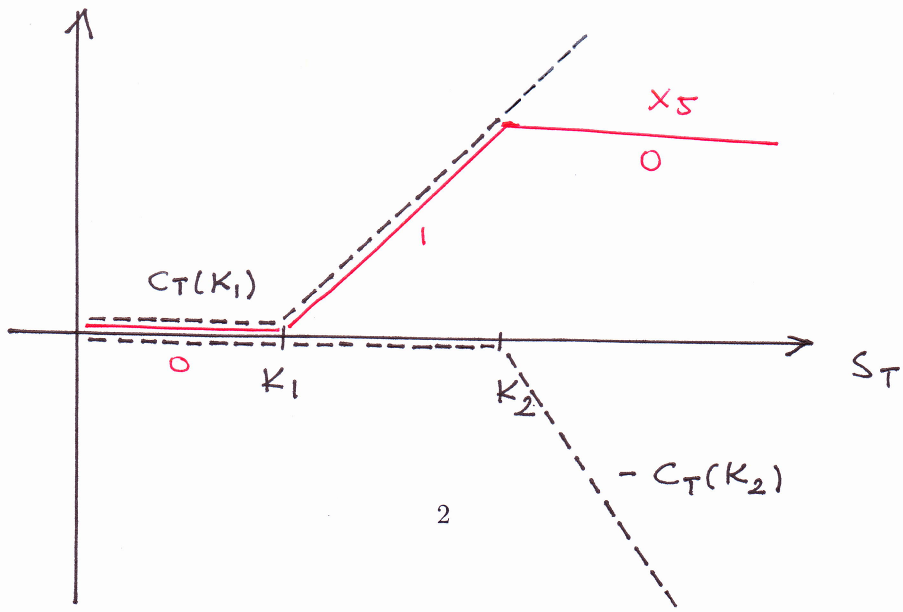
(c) $X_3 = f_3(S_T) = (K_1 - S_T)^+ - (S_T - K_2)^+$ (the sign of the initial value is undetermined)



(d) $X_4 = f_4(S_T) = (K_2 - S_T)^+ + (S_T - K_1)^+$ (the initial value should be strictly positive)



(e) $X_5 = f_5(S_T) = (S_T - K_1)^+ - (S_T - K_2)^+$ (the initial value should be strictly positive)

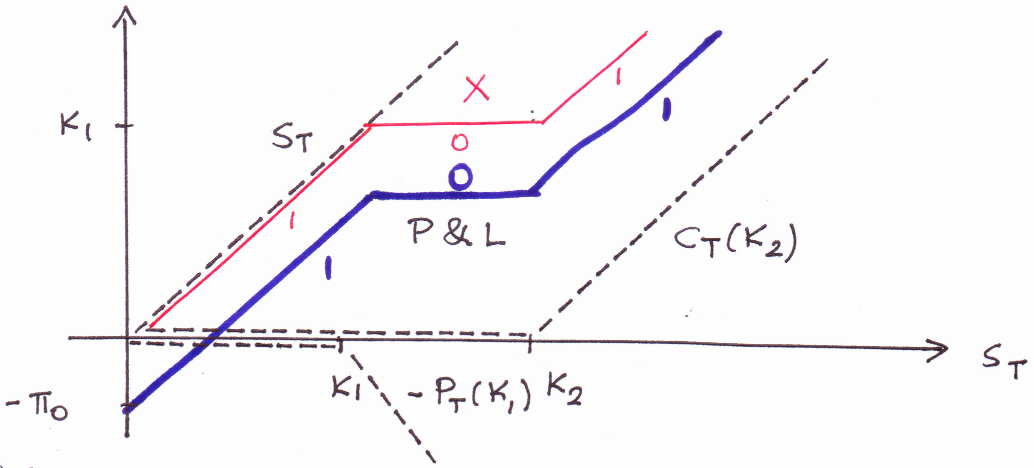


Exercise 2 (a) We note that the portfolio's payoff at time T equals

$$X = S_T - C_T(K_1) + C_T(K_2) = S_T - (S_T - K_1)^+ + (S_T - K_2)^+ \tag{1}$$

where $0 < K_1 < K_2$. Hence the payoff X at time T is strictly positive for every value of $S_T > 0$. Therefore, the price $\pi_0 = S_0 - C_0(K_1) + C_0(K_2)$ paid for the portfolio at time 0 should be strictly positive (but less than S_0 since $X \leq S_T$ for $S_T \geq 0$) and the profits/losses at time T satisfy

$$P\&L = S_T - (S_T - K_1)^+ + (S_T - K_2)^+ - \pi_0. \tag{2}$$



(b) the maximum loss occurs when $S_T = 0$ and equals $-\pi_0$. The maximum profit equals $+\infty$ when $S_T \to +\infty$, that is, the profit is unlimited. The unique break even point is $S_T = \pi_0$.

(c) We assume that S_T may take any nonnegative value. Then profits and losses occur at time T whenever the following inequality holds $\pi_0 = S_0 - C_0(K_1) + C_0(K_2) > 0$, that is, if $S_0 > C_0(K_1) - C_0(K_2)$. Since $K_1 < K_2$, we also expect that $C_0(K_1) > C_0(K_2)$ since $C_T(K_1) \geq C_T(K_2)$ and $C_T(K_1) > C_T(K_2)$ for $S_T > K_1$. Hence, assuming that S_0 is known, we obtain the following conditions $0 < C_0(K_1) - C_0(K_2) < S_0$.

(d) The view of the investor is bullish.

(e) One can perform an analysis similar to (a)–(d), by noting that we now have

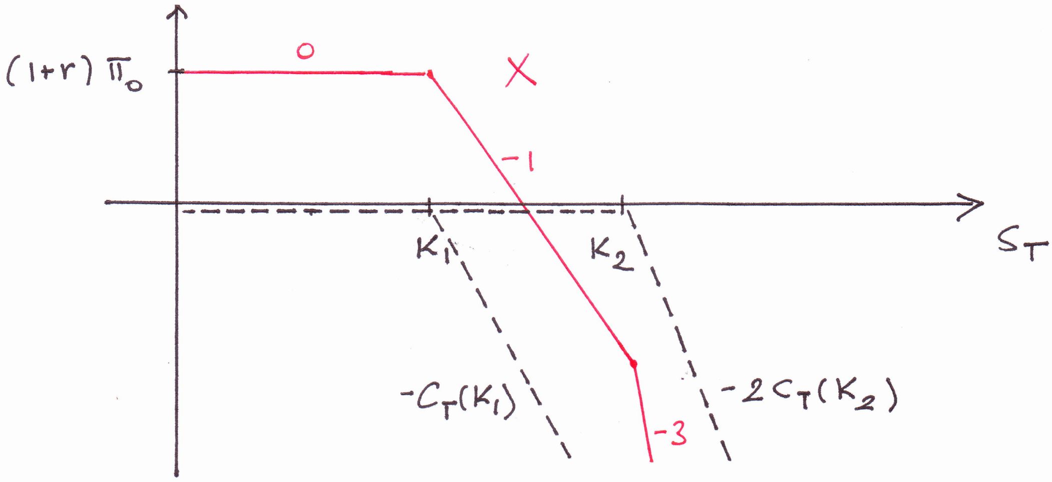
$$X = S_T - 2C_T(K_1) + C_T(K_2) = S_T - 2(S_T - K_1)^+ + (S_T - K_2)^+. \tag{3}$$

Exercise 3 We consider the portfolio with the payoff

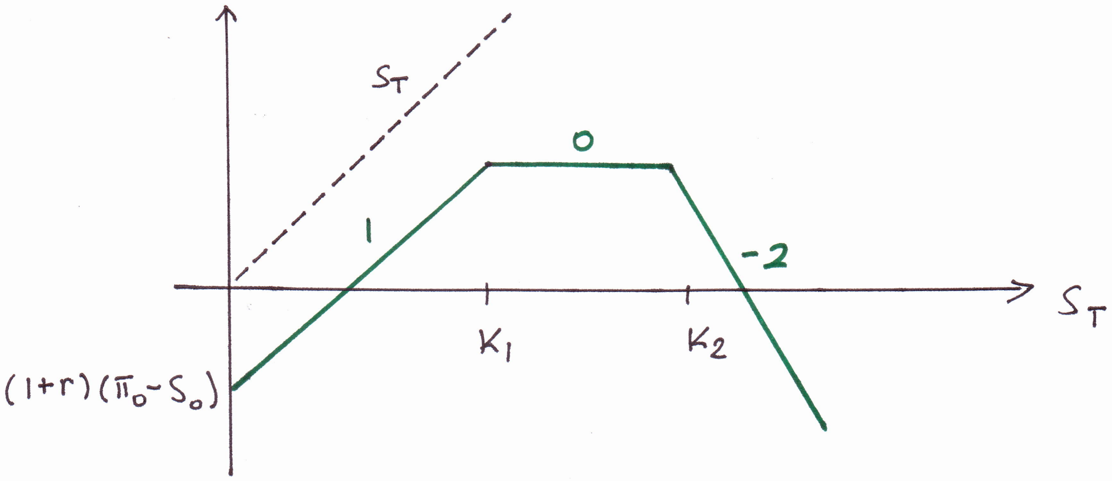
$$X = -C_T(K_1) - 2C_T(K_2) = -(S_T - K_1)^+ - 2(S_T - K_2)^+ \tag{4}$$

with the strictly positive price $\pi_0 = C_0(K_1) + 2C_0(K_2) > 0$ received at time 0, which becomes $(1+r)\pi_0$ at time T. Hence the profits/losses at time T are given by

$$P\&L = (1+r)\pi_0 - C_T(K_1) - 2C_T(K_2) = (1+r)\pi_0 - (S_T - K_1)^+ - 2(S_T - K_2)^+. \tag{5}$$



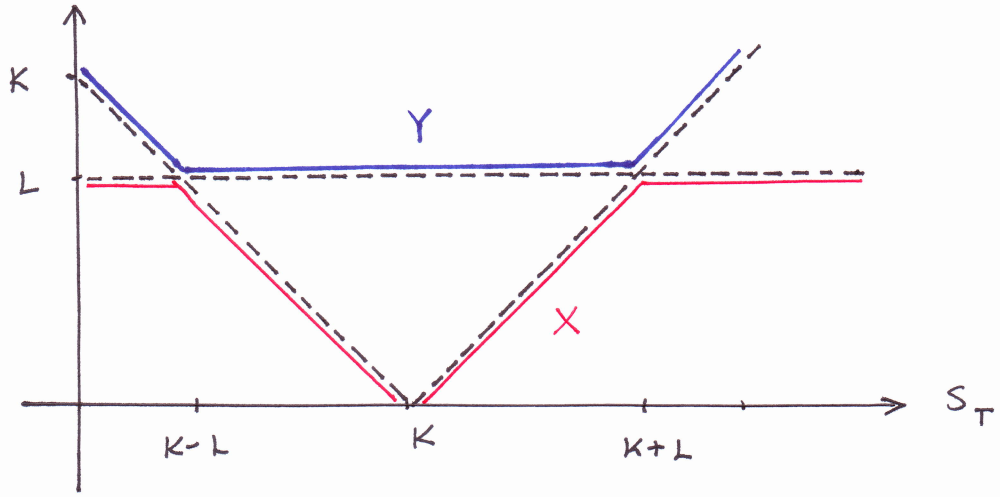
(a) Suppose that at time 0 we buy/short α shares of stock S (notice that $\alpha > 0$ means that we buy shares and $\alpha < 0$ means that we sell short shares). Then the initial and terminal values of the combined portfolio become $-\alpha S_0 + C_0(K_1) + 2C_0(K_2)$ and $\alpha S_T - C_T(K_1) - 2C_T(K_2)$, respectively. If an investor wishes to make profits when S_T is in the interval (K_1, K_2) then he/she should take $\alpha = 1$, that is, buy one share of the stock S.



(b) If we buy one share of the stock S at time 0, then the maximum profit at time T will be $K_1 + (1+r)\pi_0 - (1+r)S_0$ when $S_T \in (K_1, K_2)$. The loss in unlimited when S_T tends to infinity. Break even points are $S_T = (1+r)(S_0 - \pi_0)$ and $S_T = 0.5(K_1 + 2K_2 + (1+r)(\pi_0 - S_0))$.

(c) Since the maximum profit equals $K_1 + (1+r)\pi_0 - (1+r)S_0$, it is strictly positive whenever $S_0 < \pi_0 + (1+r)^{-1}K_1$.

Exercise 4 Recall that $0 < L < K$ are arbitrary real numbers. It is easy to sketch the graph of the payoffs $X = \min\big(|S_T - K|, L\big)$ and $Y = \max\big(|S_T - K|, L\big)$ as a function of the stock price S_T.



(a) The decomposition of X in terms of long/short positions in standard call options combined with a constant payoff L is

$$X = L - C_T(K-L) + 2C_T(K) - C_T(K+L).$$

Notice that other decompositions of the payoff X are possible (for instance, if we include in our portfolio long/short positions in the put option $P_T(K)$ and the stock S_T).

(b) From the law of one price, we deduce that the price of X at time $t \in [0,T]$ satisfies

$$\pi_t(X) = LB(t,T) - C_t(K-L) + 2C_t(K) - C_t(K+L).$$

(c) The method is analogous to the case of the payoff X. The decomposition of Y in terms of long/short positions in standard call and put options reads

$$Y = P_T(K) + C_T(K-L) - C_T(K) + C_T(K+L)$$

so that the price of Y at time $t \in [0,T]$ satisfies

$$\pi_t(Y) = P_t(K) + C_t(K-L) - C_t(K) + C_t(K+L).$$

Exercise 5 (a) It suffices to generalise the approach used in Exercise 4 by first mimicking the payoff function on the interval $[K_0, K_1] = [0, K_1]$. Then the 'initial portfolio' should be adjusted by a suitable number of long/short positions of call options with strikes $K_1, K_2, \dots, K_{n-1}$ to match the slope of the payoff on each interval $[K_i, K_{i+1}]$ for $i = 1, 2, \dots, n-1$. As soon as the portfolio in the first step is chosen, the solution to the second step (that is, when we consider the interval $[K_1, \infty)$) is unique.

(b) It suffices to apply the law of one price to your solution to part (a).

(c) Since the payoff X is already known at time T, its price at time $t \in [T, U]$ equals $\pi_t(X) = B(t,U)g(S_T)$. Hence before time T, the price satisfies, for every $t \in [0,T]$,

$$\pi_t(X) = \pi_t\big(B(T,U)g(S_T)\big) = B(T,U)\pi_t\big(g(S_T)\big)$$

where the second equality holds only under the assumption that $B(T,U)$ is deterministic (for instance, $B(T,U) = e^{-r(U-T)}$ if the continuously compounded short-term rate is constant). Notice that our approach hinges on the 'backward induction' argument.

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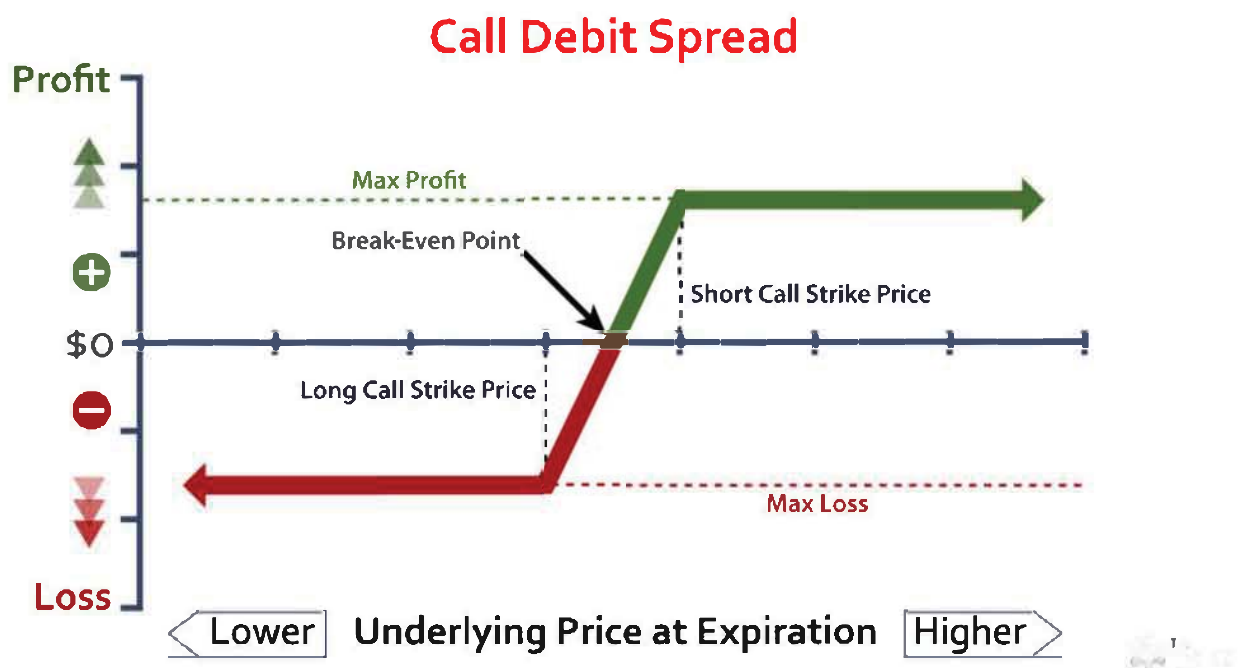
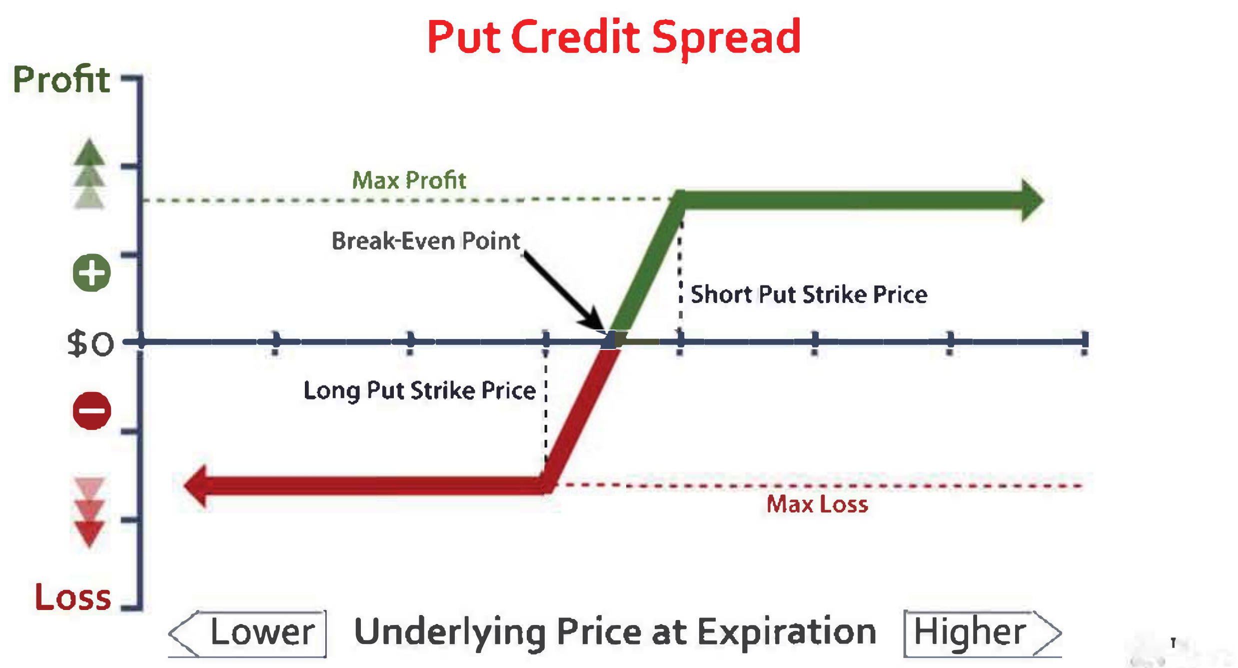
Basic Options Strategies

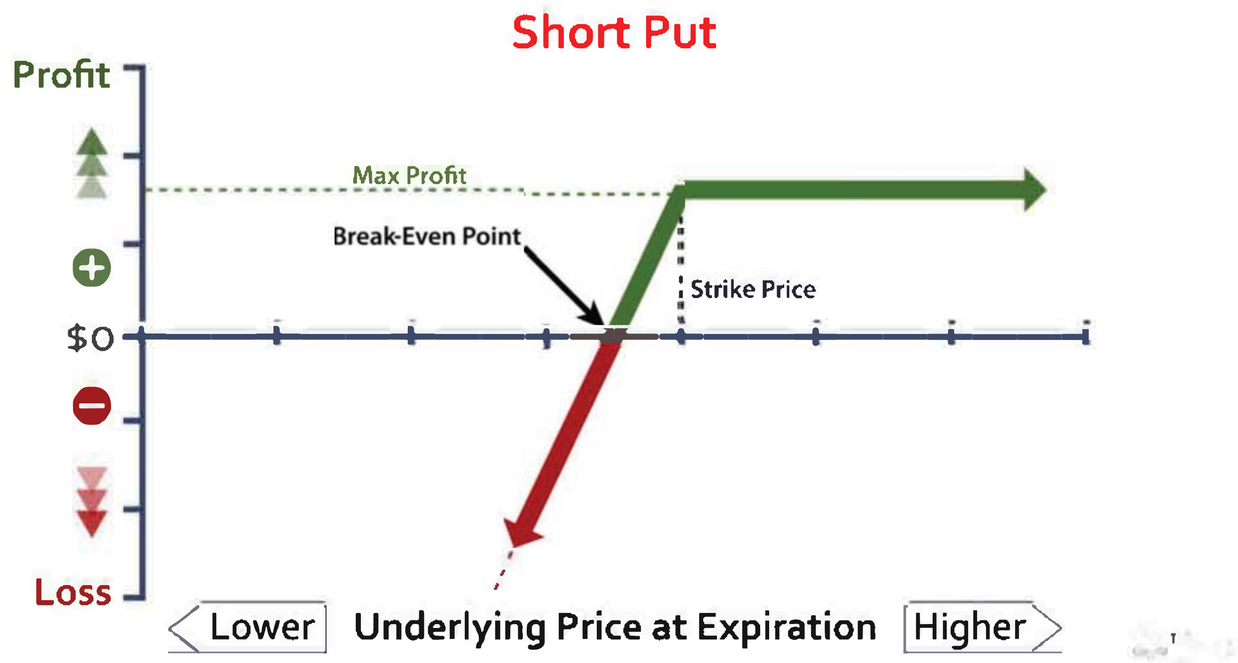
Bullish Strategies



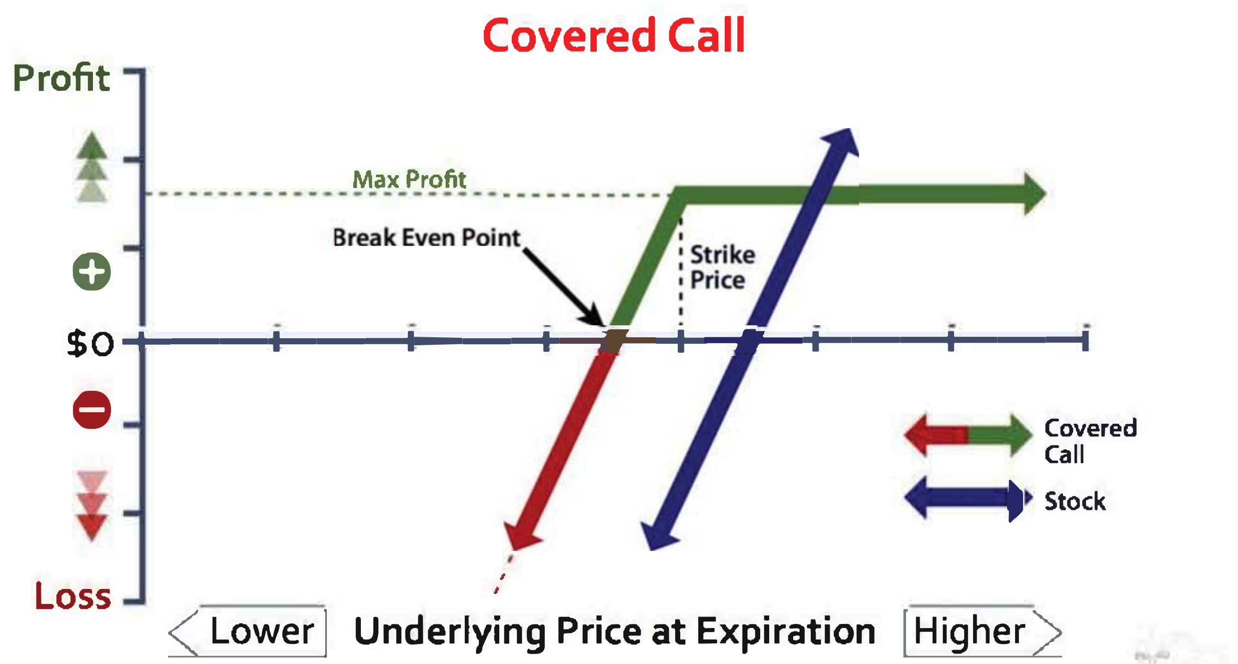
Call Debit Spread



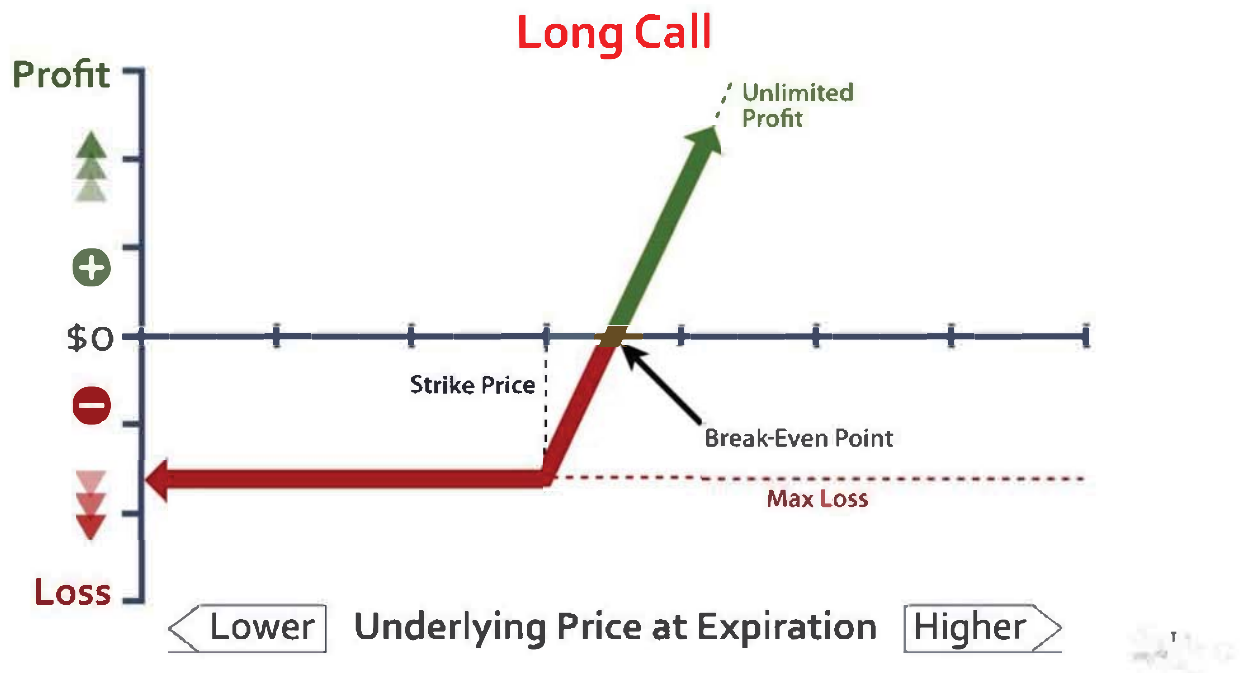
Put Credit Spread



Short Put

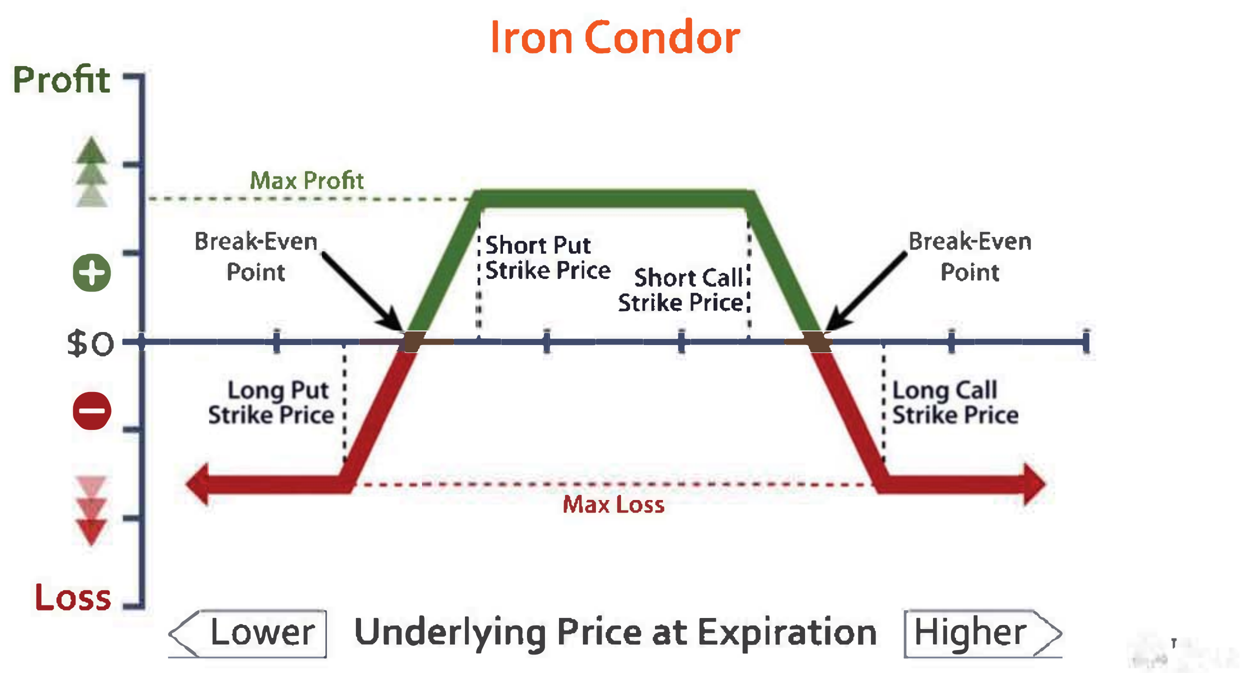


Covered Call

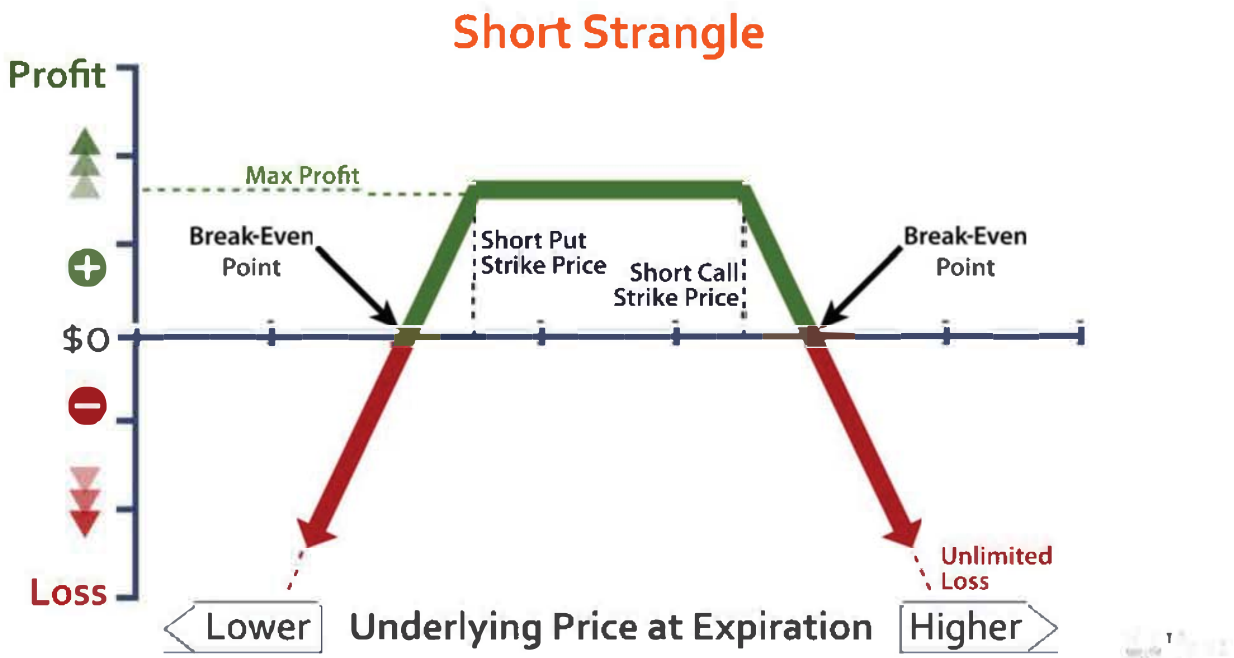


Long Call

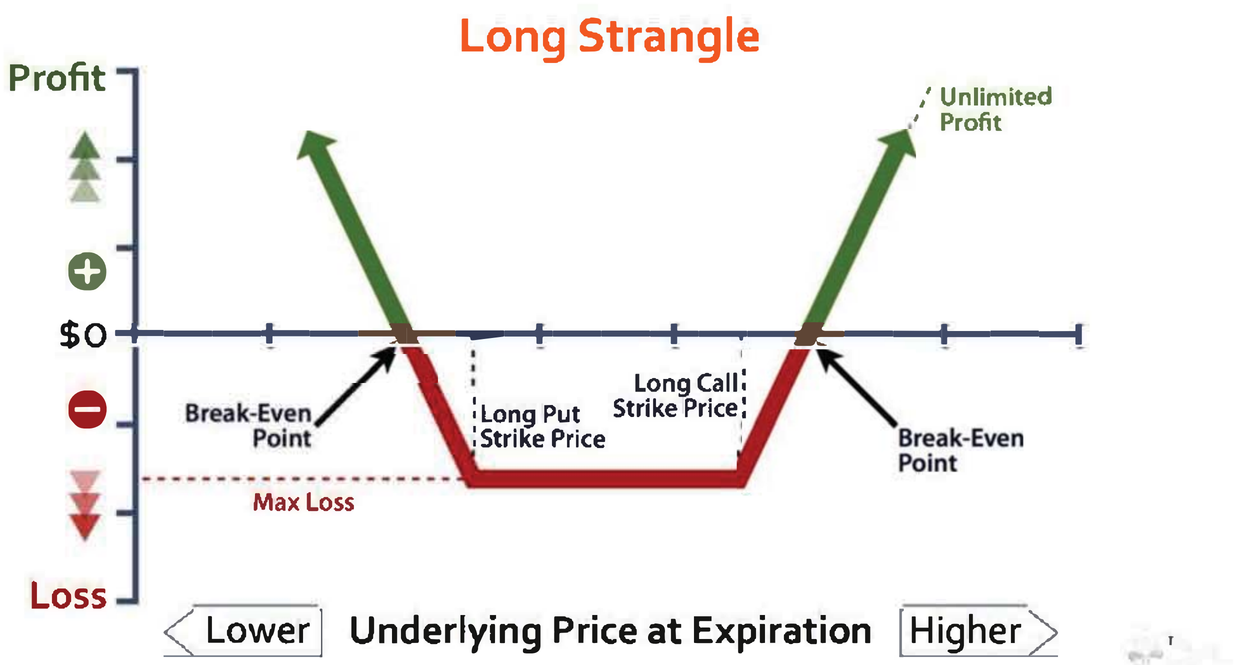
Neutral Strategies



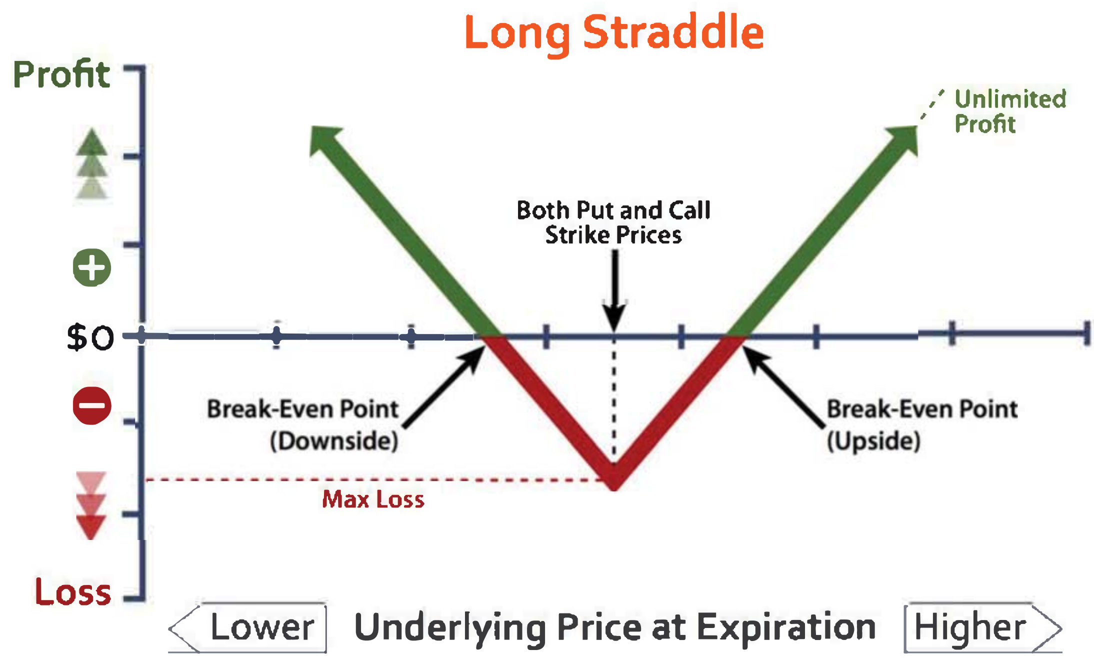
Iron Condor



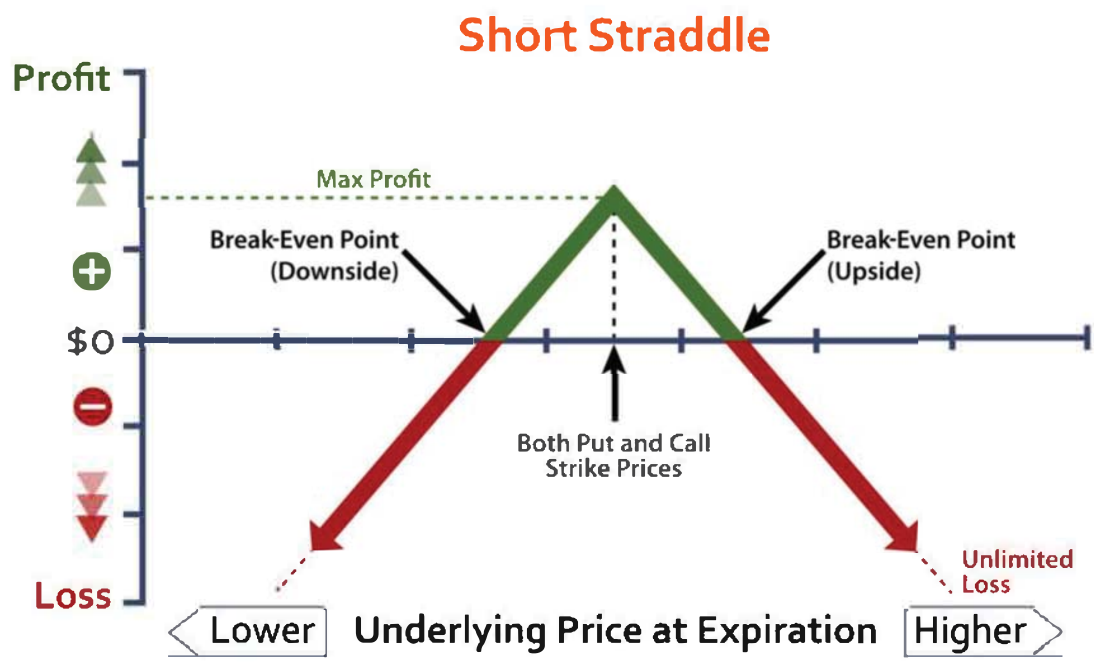
Short Strangle



Long Strangle

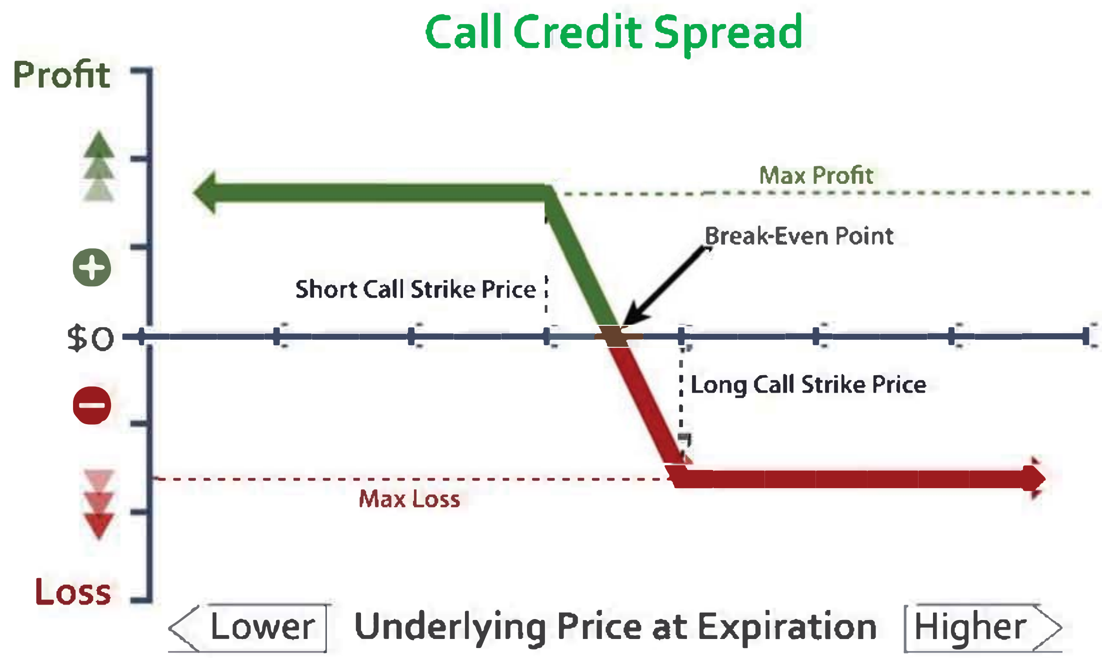


Long Straddle

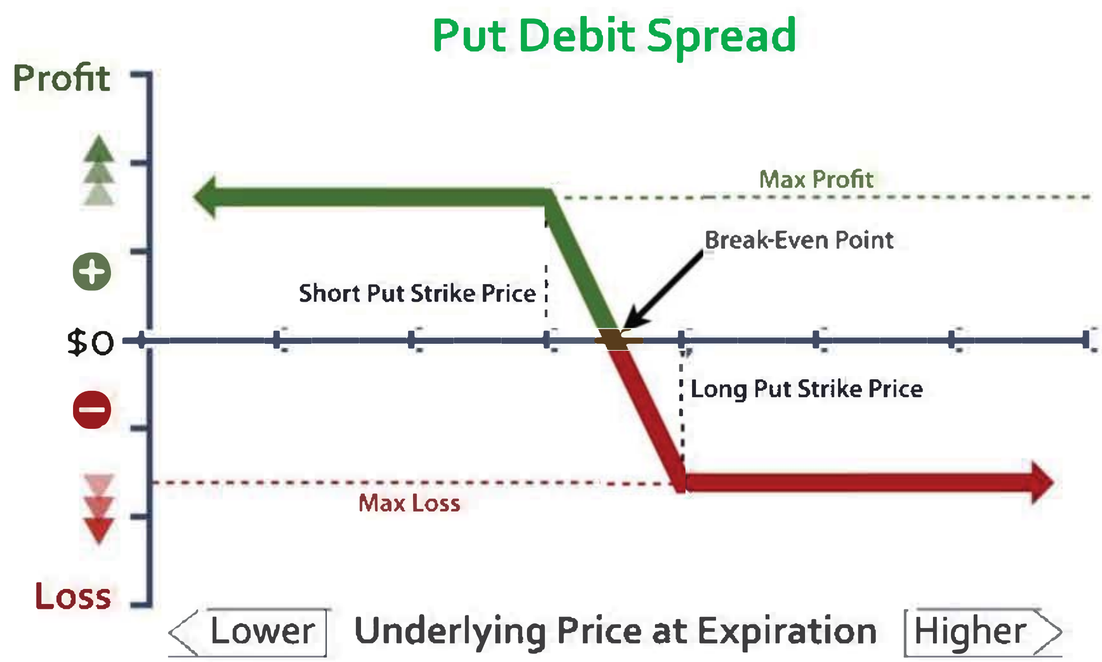


Short Straddle

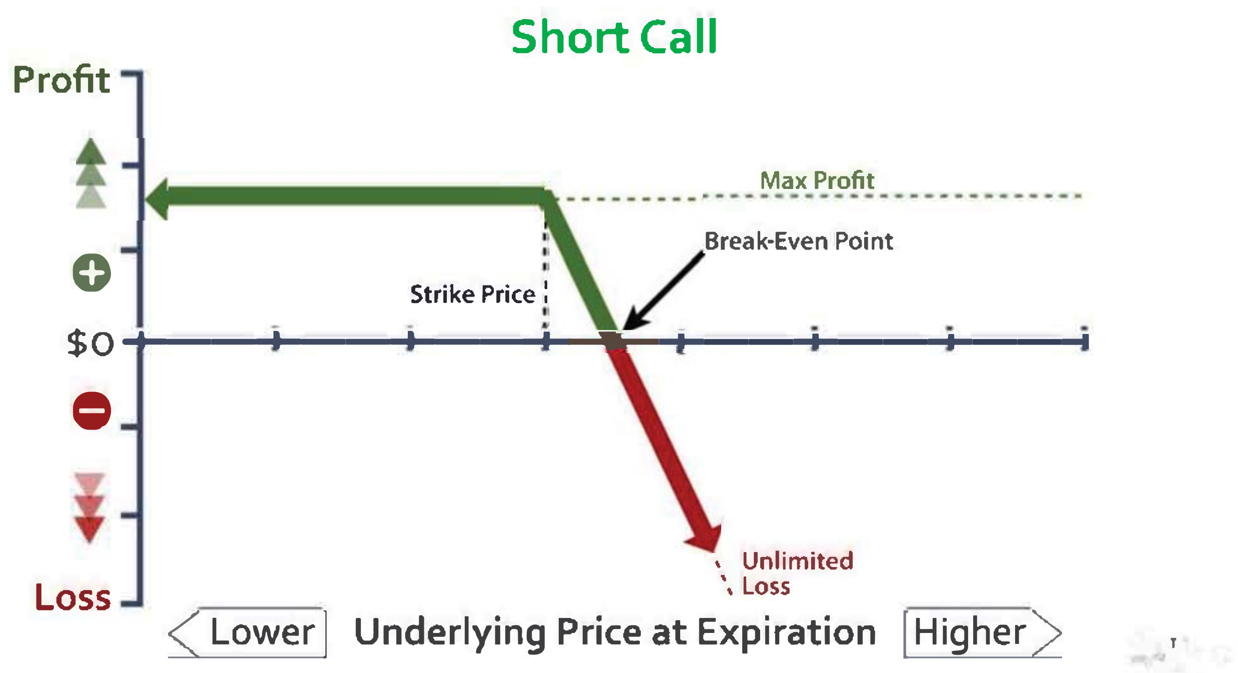
Bearish Strategies



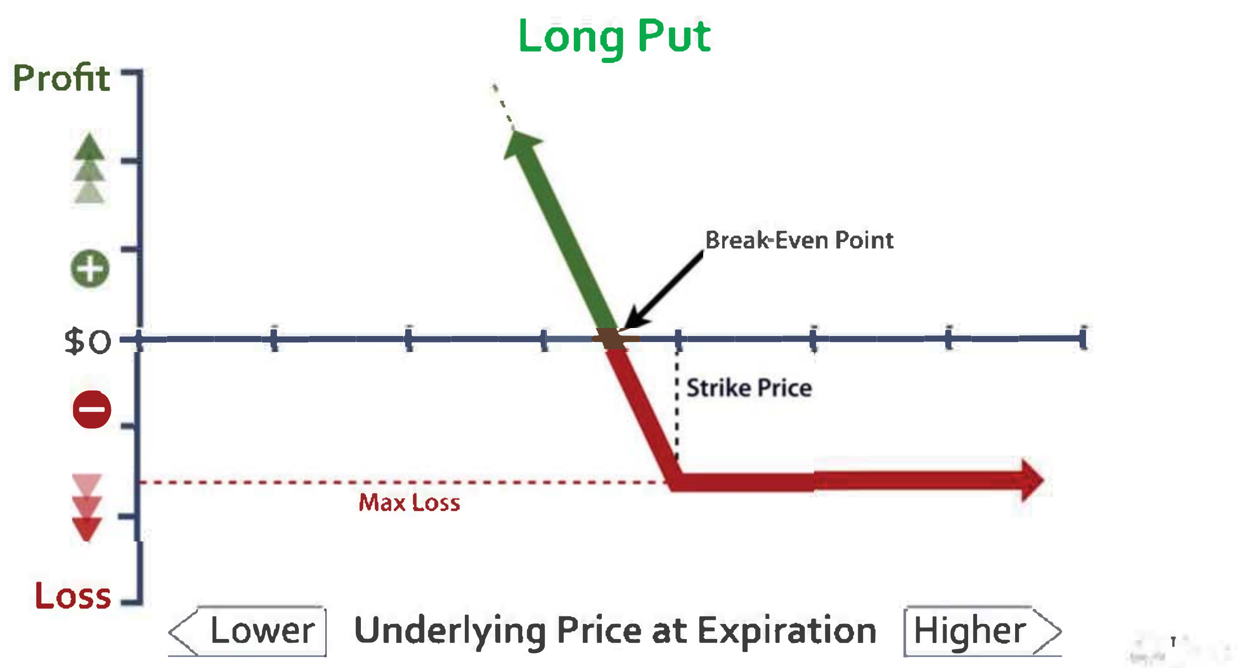
Call Credit Spread



Put Debit Spread



Short Call



Long Put