MATH3075/3975
Financial Derivatives

School of Mathematics and Statistics
University of Sydney

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Tutorial sheet 3

Background: Section 2.1 – Elementary Market Model.

Exercise 1 What is the price at time 0 of a contingent claim represented by the payoff $h(S_1) = S_1$? Give at least two explanations.

Exercise 2 Give a proof of the put-call parity relationship in the elementary market model.

Exercise 3 Compute the hedging strategies for the European call and the European put in Examples 2.1.1 and 2.1.2 from the course notes.

Exercise 4 Consider the elementary market model with the following parameters: $r = \frac{1}{4}$, $S_0 = 1$, $u = 3$, $d = \frac{1}{3}$, $p = \frac{4}{5}$. Compute the price of the **digital call** option with strike price K and the payoff function given by

$$h(S_1) = \begin{cases} 1, & \text{if } S_1 \geq K, \\ 0, & \text{otherwise.} \end{cases}$$

Exercise 5 Prove that the condition $d < 1 + r < u$ implies that there is no arbitrage in the elementary market model.

Exercise 6 Consider a single-period two-state market model $\mathcal{M} = (B, S)$ with the two dates: 0 and 1. Assume that the stock price S_0 at time 0 is equal to \$27 per share, and that the price per share will rise to either \$28 or \$31 at the end of a period, that is, at time 1, with probabilities $\frac{3}{4}$ and $\frac{1}{4}$ respectively. Assume that the one-period simple interest rate r equals 10%. We consider call and put options written on the stock S, with the strike price $K = \$28.5$ and the expiry date $T = 1$.

(a) Construct unique replicating strategies for these options as vectors $(\phi^0, \phi^1) \in \mathbb{R}^2$ such that $V_1(\phi^0, \phi^1) = \phi^0 B_1 + \phi^1 S_1$. Note that $V_1(\phi^0, \phi^1) = V_1(x, \phi)$ where $x = \phi^0 + \phi^1 S_0$ and $\phi = \phi^1$.

(b) Compute arbitrage prices of call and put options through replicating strategies.

(c) Check that the put-call parity relationship holds.

(d) Find the unique risk-neutral probability $\widetilde{\mathbb{P}}$ for the market model $\mathcal{M}$ and recompute the arbitrage prices of call and put options using the risk-neutral valuation formula.

(e) How will the replicating portfolios and arbitrage prices of the call and put options change if we assume that the interest rate r equals 5%?

Exercise 7 (MATH3975) Under the assumptions of Section 2.1, show that there exists a random variable Z such that the price x of a contingent claim $h(S_1)$ can be computed using the equality $x = \mathbb{E}_{\mathbb{P}}(Zh(S_1))$ where the expectation is taken under the original probability measure $\mathbb{P}$. A random variable Z is then called a **pricing kernel** (notice that Z does not depend on the choice of a payoff function h).
Hint: Use the fact that the probability measures $\mathbb{P}$ and $\widetilde{\mathbb{P}}$ are equivalent.

Exercise 8 (MATH3975) The goal is to examine a real-world application of the elementary market model with $d = u^{-1}$ and $r = 0$. We consider actively traded near-the-money call options on JPM (JPMorgan Chase & Co.) with maturity 18 September 2020. Recall that an option is at-the-money (ATM) when its strike is closest to underlying price (among all the available strikes).

We use the following table of mid-prices of European call and put options from 1 September 2020:

Call $C_0(K)$	**Strike** K	**Put** $P_0(K)$
\$3.95	\$98	\$2.19
\$3.65	\$99	\$2.45
\$3.12	\$100	\$2.91
\$2.65	\$101	\$3.42
\$2.23	\$102	\$4.02

(a) Assume that $S_0 = \$100.23$ and consider the ATM call option with strike $K = \$100$. Using the market quote for the option, find the value of u which makes the theoretical arbitrage price of the call computed within the setup of the elementary market model coincide with the market quote. We then say that the model is *calibrated* to market data. Generally speaking, the model *calibration* involves finding values of parameters such that the model is able to reproduce (as close as possible) the prices of the "calibration instruments" observed in the market.

(b) Compute the theoretical prices of near-the-money ITM and OTM call options using the calibrated elementary market model and compare them with their market quotes given in the table.

(c) Compute the model prices of all near-the-money put options and compare them with market quotes for put options given in the table.