Multiplayers, Iterative Algorithms, Negative Numbers

Leveraging Adders for a Greater Purpose

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March 5, 2020

Agenda

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   - Project 3
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3. Iterative Algorithms
   - Iterative Algorithms
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Announcements

Project 3

- Project 3's been released, and the name of the game is multipliers.
- You'll find everything you need under the 'week 4' section on the course website
- Today's lecture will give you all the background knowledge that you need to know about implementing a multiplier
- More info to come at the end of lecture

Some Reminders

- As the projects become more involved, we'd like to remind you to reach out and attend office hours if you are struggling.
- Additionally, here's a reminder that all projects can be group projects, but you'll have to let us know **4 days** prior to the project's due date if you'll be working in a group. (Max size of 3)

Multipliers

A Question

How does multiplication work in binary?

Multiplication Table

- Multiplication works the same way in binary as it does in base-10!
- Just like single digit multiplication in base-10 goes up to 9×9, single digit multiplication in binary goes up to 1×1
- Here's the basic multiplication table for binary:

	0	**1**
0	0	0
1	0	1

Multiplication

- Before you get very excited, this is only a small bit of what we're after.
- Now, we know how to multiply one-bit numbers (Yay!)
  - note that `AND` is all you need to represent this operation
- In binary, multiplication is the same as it's always been for us in decimal
- Let's go through an example now

Multiplication

- Again, we will look to the 'long' way of doing this computation for inspiration

$$\begin{array}{r} 1101 \\ \times 1011 \\ \hline \end{array}$$

Multiplication

$$
\begin{array}{r}
1\,1\,0\,1 \\
\times\,1\,0\,1\,1 \\
\hline
1\,1\,0\,1 \\
1\,1\,0\,1\,0 \\
0\,0\,0\,0\,0\,0 \\
1\,0\,1\,1\,0\,0\,0 \\
\hline
1\,0\,0\,0\,1\,1\,1\,1
\end{array}
$$

What can we gather?

- What exactly can we gather from doing this computation out longhand?
  - Binary multiplication is even easier than decimal; we only have two choices: 1 or 0
  - Go from right to left, multiplying out the individual numbers
  - After some analysis, this appears to be repeated additions (which are somewhat similar each time)
  - We will implement multiplication the same way!
  - By leveraging repeated sets of Full Adders, we can easily perform these repeated additions.
- Q: But Aki, what if we have too many carries in a single column for a FA to add???
  - A: We're screwed

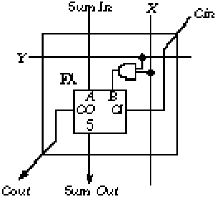
Partial Additions

- Remember how some columns had us adding more than the maximum that a Full Adder could handle?
  - Let's try and address that
- In other words, we want to cut down this big computation into chunks juust small enough so that Full Adders can handle each chunk of it
- Formally, this sort of black magic is called 'Partial Additions'- look to the whiteboard for a demonstration
  - I've also got a video up on the course website explaining this stuff in more detail if you want more clarification

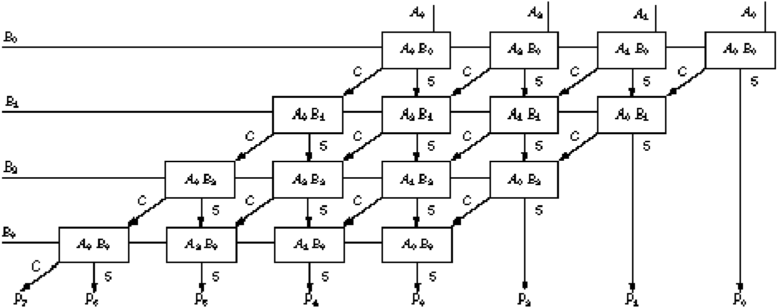
Bringing it all Together

- We know how to multiply one-bit numbers
- We know how to add said numbers together with Full Adders
- We've come up with a clever way to make sure we don't have more carries than we can handle
- Now we just need to implement it (and repeat it a bunch!)

The Actual Circuit



(a) Basic building block



(b) 4 X 4 multiplier structure

Some Additional Notes

- This is what we call a **combinatorial multiplier**.
- There are other ways to implement multipliers

Multiplication with Negatives

- What if we want to multiply negative numbers?
- We're going to use the standard 'Two's Complement', but with a special twist for our application.

Two's Complement

- Say we wanted to represent the number **5** as a negative number.
- First, let's write it binary
  - 0101
- Next, let's flip all the bits
  - 1010
- Finally, let's add 1
  - 1011
- It turns out, this isn't all we need. There's one more nuance to using Two's Complement with multiplication.

Two's Complement

- We will need to extend our multiplier and multiplicand before multiplying
- This is because carries are **POSSIBLE** all the way up till the leftmost most significant bit
- Think of these leading 1's as 'insurance' bits to make sure that Two's Complement kicks in. (Carries will ruin the two's complement business we've got going)
- Think of it this way: both `11` and `1111111111` are both Two's Complement representations of -1

Two's Complement

```
    WRONG !                Sign extended:
      0011 (3)               0000 0011 (3)
    x 1011 (-5)            x 1111 1011 (-5)
    ------                 -----------
      0011                    00000011
     0011                    00000011
    0000                    00000000
 + 0011                    00000011
 ---------                00000011
   0100001               00000011
not -15 in any          00000011
 representation!     + 00000011
                    ------------------
                            1011110001
                              --------
take the least significant 8 bits 11110001 = -15
```

Limitations

- Addition, Subtraction (check out the two's complement video on the course website), and Multiplication- we've done it all.
- What about division? Exponentiation?
- The Answer: **It's harder**
- One Solution: Iterative algorithms

Iterative Algorithms

Iterative Algorithms

- Why would we want to use this fancy sounding method?
- Reusing adders will help us- saving gates at the cost of speed
- This is where we begin bridging the gap between the cave-person computer science we're doing and what's actually in computers these days

Division

- Consider division for a moment- how would we do a classic division?
  - No, I'm not whipping out some long division example for you guys
  - Q: Is it possible to convert this to binary, though?
  - A: We can try!
- Additionally, now that we know how multiplication works, we can also explore the next stage! Exponentiation!
  - Think about how you'd use iterative algorithms and what we've already learned about multiplication in order to efficiently accomplish. (It's a little more than just a few stacked adders)

Division & Exponentiation

- This is a 1-credit class, so we won't be going over this in lecture. More on division and exponentiation will be posted online, but don't expect it on the exam. (It's fun stuff, though!)

Project 3