Statistical Inference Course Project (Part 1)

8/26/2020

Part 1

A. Overview

This is a document for Coursera Statistical Inference Course Final Project. This project will investigate the exponential distribution in R and compare it with the Central Limit Theorem. Given that lambda = 0.2 for all of the simulations. Part 1 of the project will investigate the distribution of averages of 40 exponentials over a thousand simulations.

A.1 Simulations

Using pre-defined parameters

```
lambda <- 0.2
n <- 40
sims <- 1:1000
set.seed(123)
```

Check for missing dependencies and load necessary R packages

```
if(!require(ggplot2)){install.packages('ggplot2')}; library(ggplot2)
```

```
## Loading required package: ggplot2
```

```
## Warning: package 'ggplot2' was built under R version 3.6.2
```

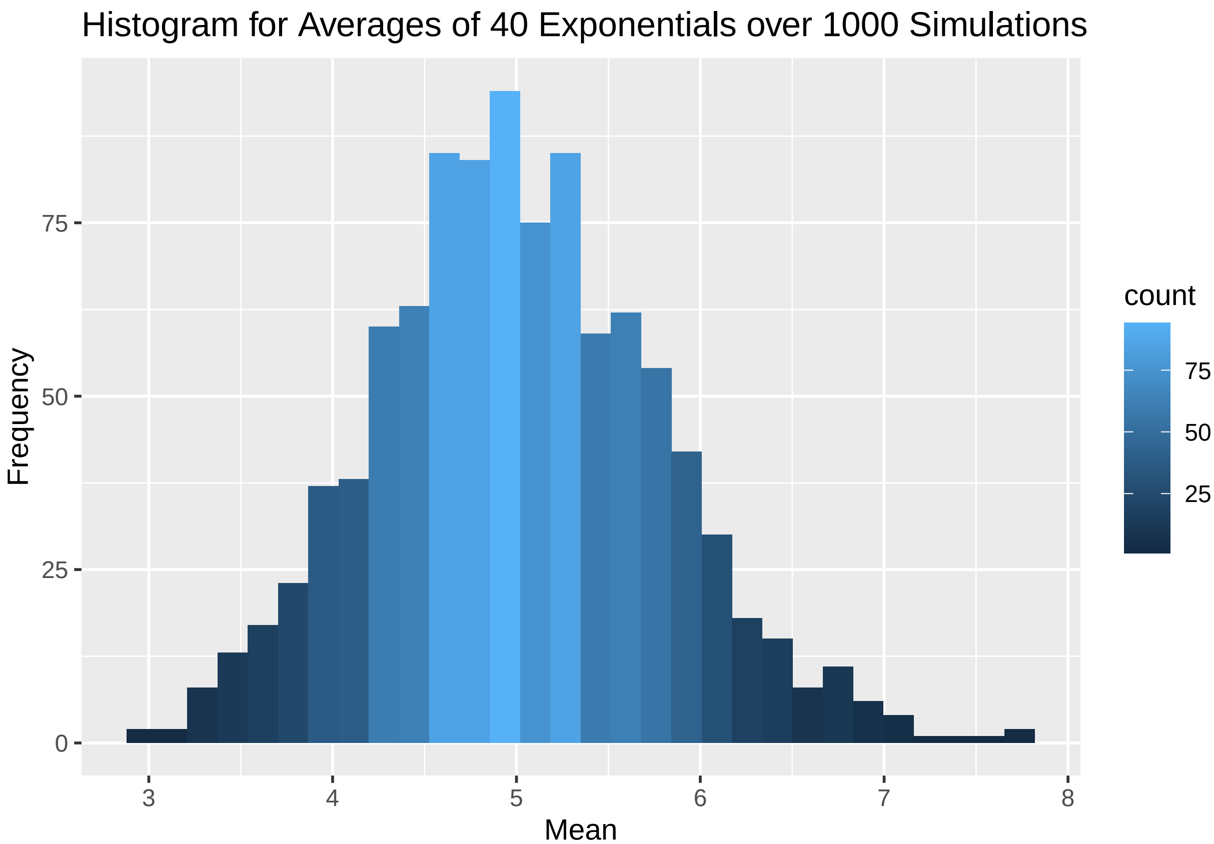
Simulate the population

```
population <- data.frame(x=sapply(sims, function(x) {mean(rexp(n, lambda))}))
```

Plotting the histogram

```
hist.pop <- ggplot(population, aes(x=x)) +
  geom_histogram(aes(y=..count.., fill=..count..)) +
  labs(title="Histogram for Averages of 40 Exponentials over 1000 Simulations", y="Frequency", x="Mean"
hist.pop
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



A.2 Sample Mean versus Theoretical Mean

As we can see below, both sample mean and theoretical mean are very close.

Tabulating the Sample Mean & Theoretical Mean

```
sample.mean <- mean(population$x)
theoretical.mean <- 1/lambda
cbind(sample.mean, theoretical.mean)
```

```
##      sample.mean theoretical.mean
## [1,]    5.011911                5
```

Checking 95% confidence interval for Sample Mean

```
t.test(population$x)[4]
```

```
## $conf.int
## [1] 4.963824 5.059998
## attr(,"conf.level")
## [1] 0.95
```

At 95% confidence interval, the sampled mean is between 4.9638242 and 5.0599984.

A.3 Sample Variance Vs Theoretical Variance

As we can see below both Sample Variance and Theoretical Variance are very close.

```r
sample.variance <- var(population$x)
theoretical.variance <- ((1/lambda)^2)/n
cbind(sample.variance, theoretical.variance)
```

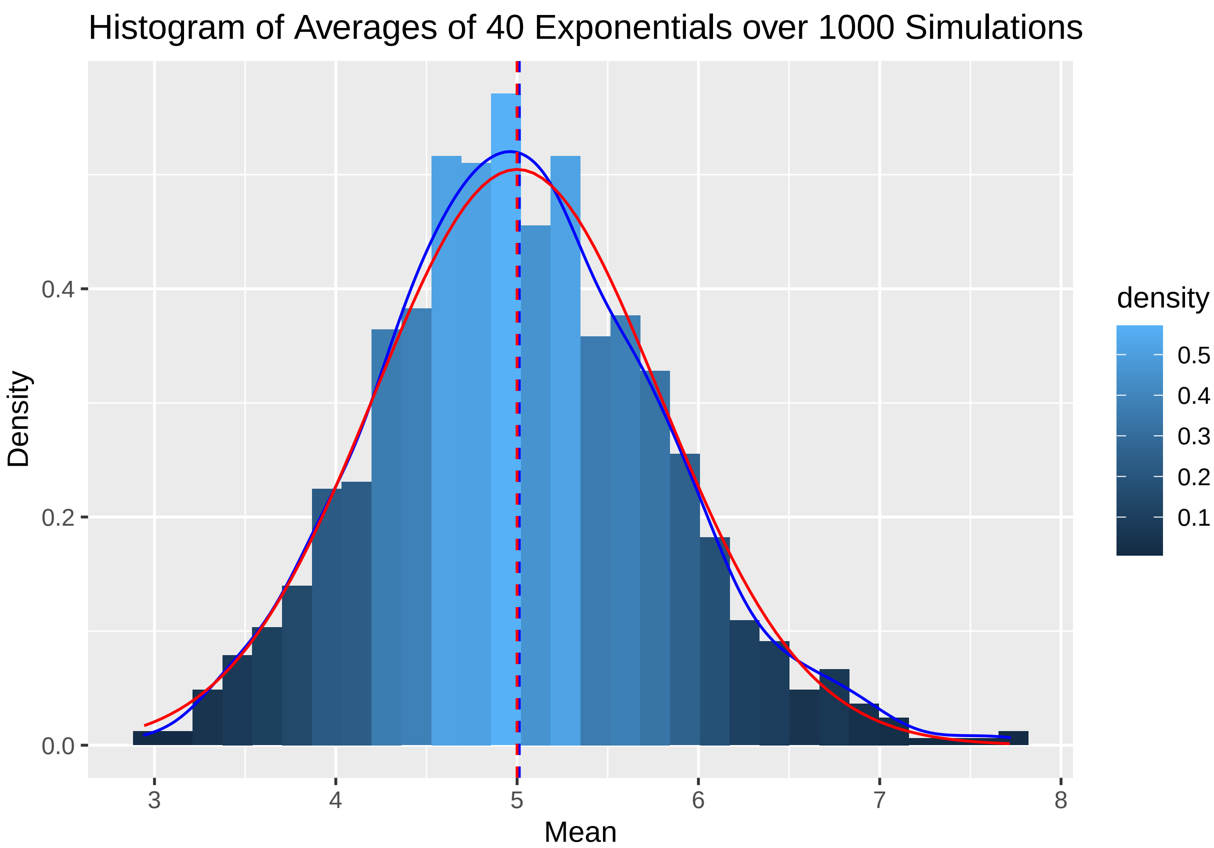
```
##      sample.variance theoretical.variance
## [1,]       0.6004928                0.625
```

A.4 Distribution

Plotting Sample Mean & Varience vs Theoretical Mean & Varience

```r
gg <- ggplot(population, aes(x=x)) +
  geom_histogram(aes(y=..density.., fill=..density..)) +
  labs(title="Histogram of Averages of 40 Exponentials over 1000 Simulations", y="Density", x="Mean") +
  geom_density(colour="blue") +
  geom_vline(xintercept=sample.mean, colour="blue", linetype="dashed") +
  stat_function(fun=dnorm,args=list( mean=1/lambda, sd=sqrt(theoretical.variance)),color = "red") +
  geom_vline(xintercept=theoretical.mean, colour="red", linetype="dashed")
gg
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



As we can see, the Sampled mean for 40 exponentials simulated 1000 times are very close to the Theoretical mean for a normal distribution.

Please note the assumptions is we are sampling without replacement and set.seed is at 123.