Theoretical Foundations of Applied Statistics

Tme Series Analysis

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March/April, 2023

Non Stationary Time Series

- A non-stationary series is a set of data in which the statistical properties such as mean, variance, and autocorrelation change over time. This means that the distribution of data changes over time, which makes it difficult to make predictions using traditional statistical methods.

- Non-stationary series can be caused by a variety of factors, including trends, seasonality, and cycles.
- E.g., a time series that shows an upward/downward trend over time is non-stationary because the mean of the series increases as time goes on.
- Similarly, a time series that exhibits seasonal patterns is also non-stationary because the mean and variance change over time.

Figure: Non Stationary Time Series

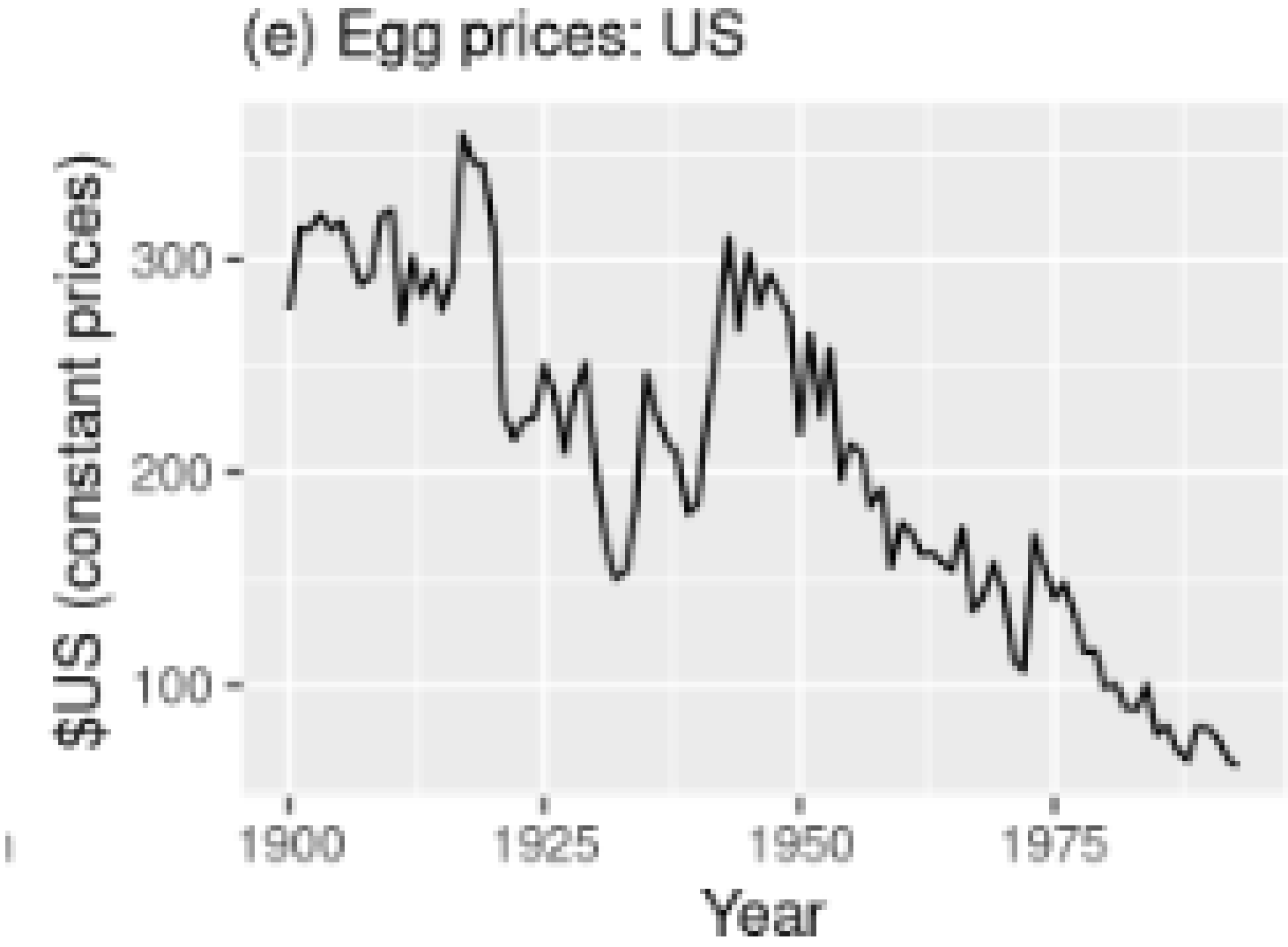

Figure: Non Stationary Time Series

Time Series models

WHITE NOISE PROCESS

- Before we see the models, lets define the ***white noise process***.
- If a variable is white noise, then each element has an identical, independent, mean-zero distribution.
- Each period's observation in a white-noise time series is a complete "surprise":

- Nothing in the previous history of the series gives us a clue whether the new value will be positive or negative, large or small.
- Formally, we say that ε is a white-noise process if

$$E(\varepsilon_t) = 0, \forall_t$$

$$Var(\varepsilon_t) = \sigma^2, \forall_t$$

$$Cov(\varepsilon_t, \varepsilon_{t-s}) = 0, \forall_{s \neq 0}$$

Autoregressive Models

- Autoregressive (AR) model is a type of time series model that predicts future values of a variable based on its past values. It is a linear regression model where the predictors are lagged values of the same variable.
- The AR model assumes that the future values of the variable are a linear combination of its past values and an error term.

Cont...

- The order of the AR model is determined by the number of lagged values used as predictors. For example, an AR(1) model uses only one lagged value as a predictor, while an AR(2) model uses two lagged values.

- The general form of an AR(p) model can be written as:

$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + \ldots a_p y_{t-p} + \varepsilon_t$$

Where; $y_t =$ is the variable of interest at time t, $a_0 =$ is the constant, a_1 to $a_p =$ are the coefficients of the lagged values, $\varepsilon_t =$ Is the error term at time t, and p $=$ Is the order of the model.

Moving Average Process

- Moving Average (MA) model is a type of time series model that predicts future values of a variable based on the errors of its past predictions.
- It is a linear regression model where the predictors are the lagged errors of the same variable.

- The MA model assumes that the future values of the variable are a linear combination of the error terms and a constant term.
- The order of the MA model is determined by the number of lagged errors used as predictors.
- For example, an MA(1) model uses only one lagged error as a predictor, while an MA(2) model uses two lagged errors.

- The general form of an MA(q) model can be written as:

$$y_t = c + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \ldots + \beta_q \varepsilon_{t-q}$$

Where; $y_t =$ is the variable of interest at time t, $c =$ is the constant, β_1 to β_q are the coefficients of the lagged errors, and q is the order of the model.