question 3

(a)

```
clc; clear;
disp("(a)")
```

```
(a)
```

```
syms n x L a cn t

u(x,t) = cn * sin(n*sym(pi)*x / L) * exp(-(a^2 * n^2 * sym(pi)^2 * t)/L^2);
ut = diff(u(x,t), t);
uxx = diff(diff(u(x,t), x), x);

disp("Ut = ")
```

```
Ut =
```

```
disp(ut)
```

$$-\frac{a^2\,\mathrm{cn}\,n^2\,\pi^2\,\mathrm{e}^{-\frac{a^2 n^2 t \pi^2}{L^2}}\,\sin\left(\frac{\pi\,n\,x}{L}\right)}{L^2}$$

```
disp("Uxx = ")
```

```
Uxx =
```

```
disp(uxx)
```

$$-\frac{\mathrm{cn}\,n^2\,\pi^2\,\mathrm{e}^{-\frac{a^2 n^2 t \pi^2}{L^2}}\,\sin\left(\frac{\pi\,n\,x}{L}\right)}{L^2}$$

```
if ut == a^2 * uxx
    disp("Heat Eqn is satisfied")
end
```

```
Heat Eqn is satisfied
```

(b)

```
fprintf("\n(b)\n")
```

```
(b)
```

```
clc; clear;
syms n x L f(x)

f(x) = 10;
cn = (2 / L) * int(f(x) * sin(n*pi*x / L), x, 0, L);

disp("value of cn is ")
```

```
value of cn is
```

```
disp(cn)
```

$$\frac{40 \sin\left(\frac{\pi n}{2}\right)^2}{n \pi}$$

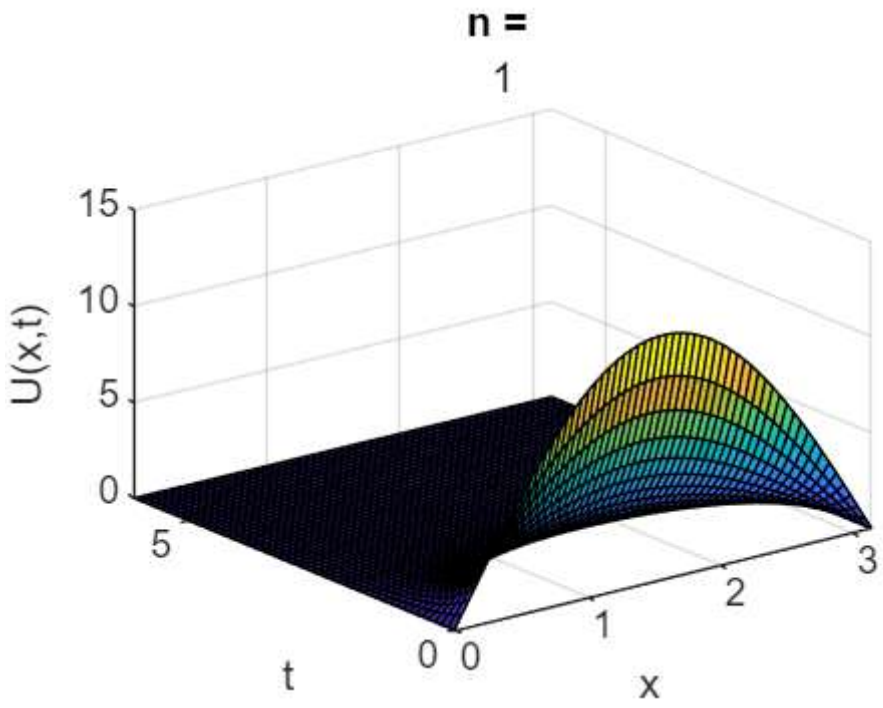
(c)

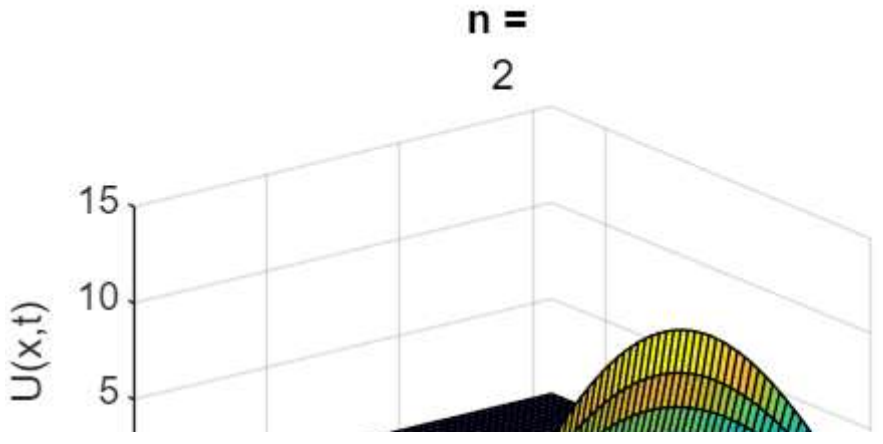
```
fprintf("\n(c)\n")
```

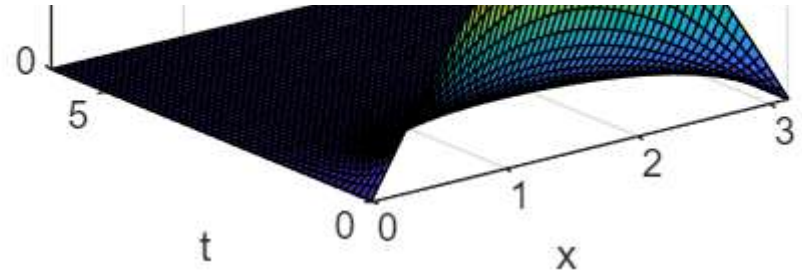
```
(c)
```

```
L = pi; a = sqrt(1.71);
syms n x t
u(x,t,n) = cn * sin(n*pi*x / L) * exp(-(a^2 * n^2 * pi^2 * t)/L^2);
x = linspace(0, pi, 60);
t = linspace(0, 6, 50);
[X,T] = meshgrid(x,t);
arr = zeros(length(t),length(x));

for n = 1:5
    arr = arr + eval(u(X,T,n));
    figure;
    surf(X, T, arr)
    title("n = ", n)
    xlabel("x")
    ylabel("t")
    zlabel("U(x,t)")
end
```

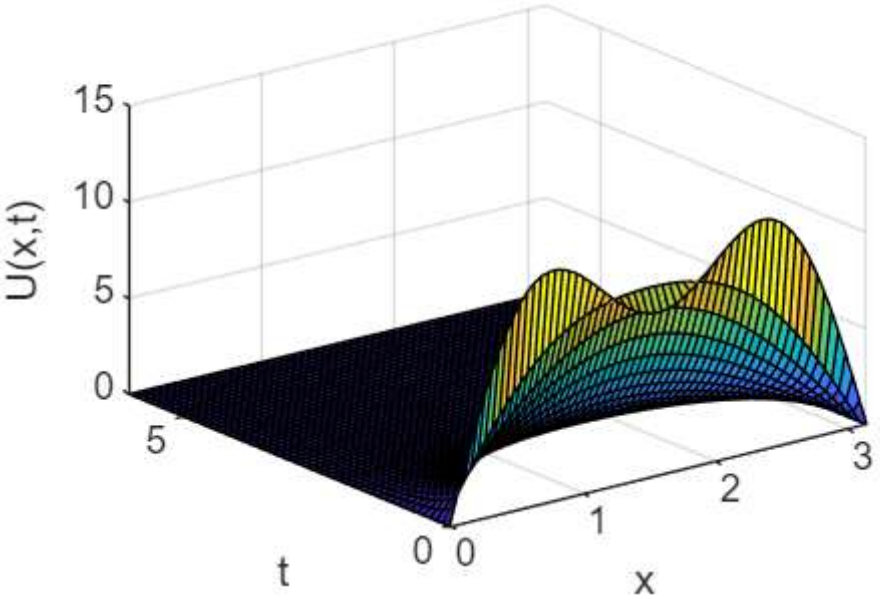






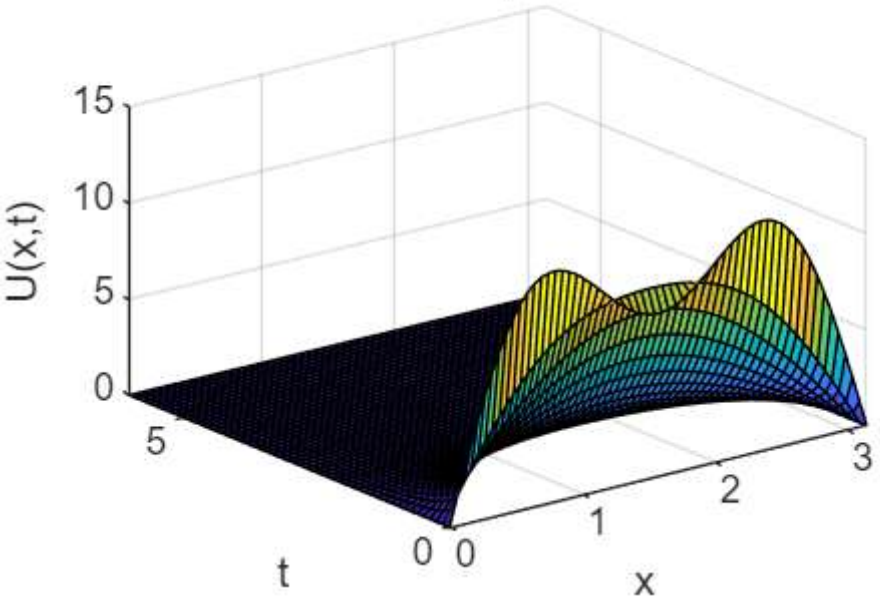
n =

3



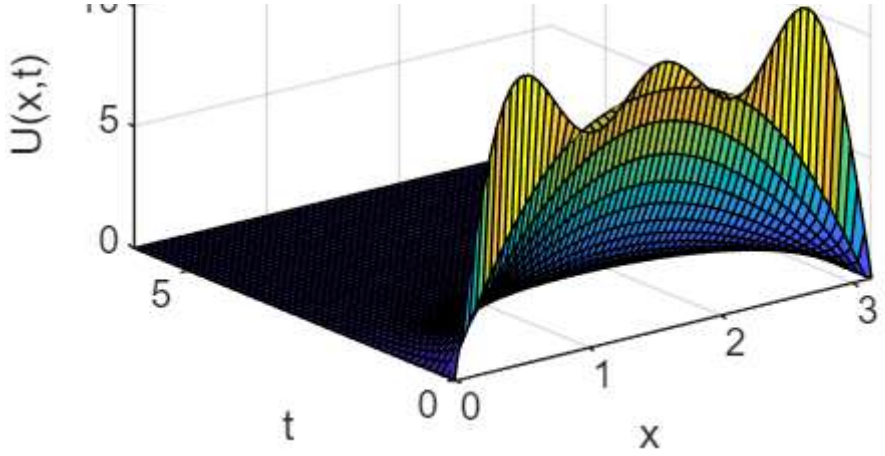
n =

4



n =

5

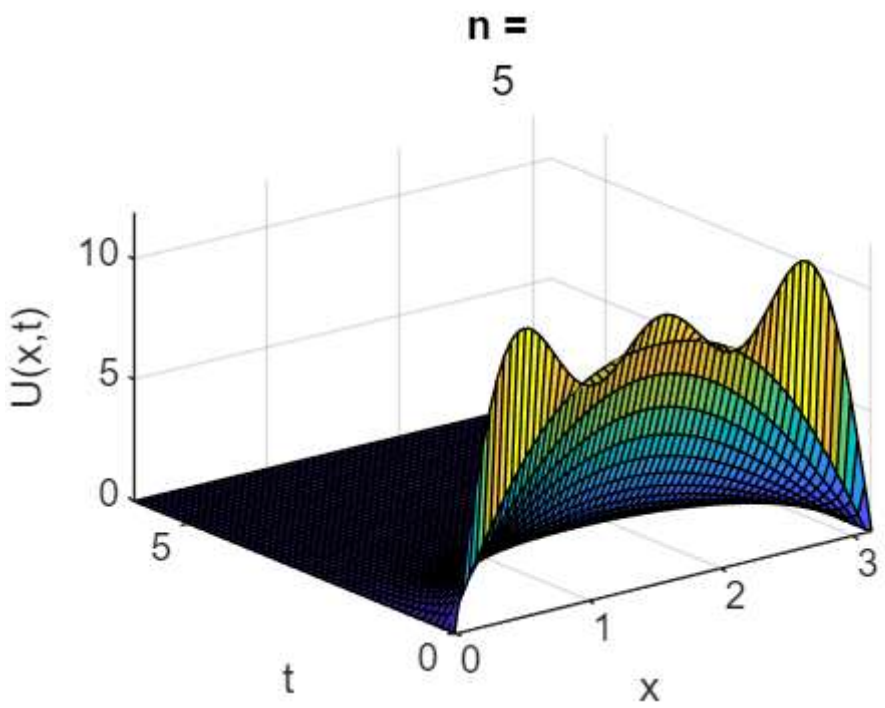


(d)

```
fprintf("\n(d)\n")
```

```
(d)
```

```
surf(X, T, arr)
title("n = ", n)
xlabel("x")
ylabel("t")
zlabel("U(x,t)")
```



(e)

```
fprintf("\n(e)\n")
```

```
(e)
```

```
plot(t, eval(u(pi/2, t, 1)))
title("Temperature vs Time graph at midpoint of rod")
xlabel("t")
ylabel("U")
```