# Third Year B.S. (Honors) 2024-2025

Department of Applied Mathematics, University of Dhaka

Course Title: Math Lab III (Matlab), Course No.: AMTH 350

## Assignment 03

**Name:** **Roll:** **Group:**

## Use Matlab to solve each of the following problems.
## Day 01

1. A projectile is fired from with an initial velocity of $250m/s$ at an angle of $\theta = 65°$ relative to the ground. The initial position of the projectile is $(3000, 0, 0)$ and is aimed directly north. Because of a strong wind blowing to the west, the projectile also moves in this direction at a constant speed of $30m/s$. Determine and plot the trajectory of the projectile until it hits the ground. Plot also (in the same figure) the trajectory that the projectile would have had if there was no wind.

2. A safety bumper is placed at the end of a racetrack to stop out-of-control cars. The bumper is designed such that the force that the bumper applies to the car is a function of the velocity $v$ and the displacement $x$ of the front edge of the bumper according to the equation: $F = Kv^3(x+1)^3$, where $K = 30s - kg/m^5$ is a constant. A car with mass $m$ of 1500 kg hits the bumper at a speed of 90 km/h. Determine and plot the velocity of the car as a function of its position for $0 \leq x \leq 3$.

3. Consider a thin wire made of homogeneous material the diameter of which coincides with the x-axis from $x = 0$ to $x = L$. It is assumed that the initial temperature, $f$ of the rod is specified as a function of the distance $x$ from one end of the rod ($x = 0$). The temperature distribution $u(x, t)$ at some later time in the absence of any heat source is then a solution of the one-dimensional, homogeneous heat equation. Suppose the two ends are suddenly placed in contact with ice packs at $0°C$ at time $t = 0$ and this temperature is maintained at all times. Then the temperature distribution of this rod is given by

$$u(x,t) = \Sigma_{n=1}^{\infty} c_n \frac{\sin n\pi x}{L} e^{\frac{-a^2 n^2 \pi^2 t}{L^2}}$$

   (a) Verify that the solution given satisfies the one-dimensional, homogeneous equation.

   (b) Suppose, $f(x) = 10$. Compute the coefficients, $c_n$.

   (c) Assume, $a^2 = 1.71$, and $L = \pi$. Plot the temperature, $u(x, t)$ for first five non-zero values of $n$.

   (d) Express the temperature, $u(x, t)$ as a sum of the terms you obtained in part(b) and hence plot it.

   (e) Plot the temperature at the center of the rod as a function of time.

   (f) What would be the temperature of the rod if it is kept for sufficiently long time?

## Day 02

4. Check the transversality condition for the PDE $u_x + 3y^{\frac{2}{3}}u_y = 2$, subject to the initial condition, $u(x,1) = 1 + x$ and hence solve using the method of characteristics. Plot the solution surface along with characteristic curves.

5. Consider the following heat conduction problem through a thin rod:

$$u_t = a^2 u_{xx};\ 0 < x < 10;\ t > 0$$

$$B.C.:\ u(0,t) = 10;\ u(10,t) = 30;\ t > 0$$

$$I.C.:\ u(x,0) = 0;\ 0 < x < 10$$

The solution of this problem is

$$u(x,t) = 10 + 2x + \frac{20}{\pi}\Sigma_{n=1}^{\infty}\frac{(-1)^n 3 - 1}{n}\sin(\frac{n\pi x}{10})e^{\frac{-a^2 n^2 \pi^2 t}{100}}$$

Consider four rods, each of which is made entirely of each of these four materials, Silver (1.71), Copper (1.14), Aluminum (0.86) and Cast iron (0.12) (bracketed values indicate corresponding diffusivity).

   (a) Create a table demonstrating the temperature, $u(x,t)$ at $t = 1$ for each rod (take 0.5 as spatial increment).

   (b) Repeat part (a) for $t = 10$ and $t = 20$ and hence comment how the diffusivity of material affects the heat conduction.

   (c) Use the first three non-zero terms to visualize the temperature distribution in each rod.

   (d) How long will it take for the centre of each rod to reach a temperature of 15 units?

6. Three sides of a rectangular plate ($a = 5m,\ b = 4m$) are kept at a temperature of $0°C$ and one side is kept at a temperature of $T_1 = 80°C$. The temperature distribution in this plate is given by,

$$T(x,y) = \frac{4T_1}{\pi}\Sigma_{n=1}^{\infty}\frac{sin[(2n-1)\frac{\pi x}{a}]sinh[(2n-1)\frac{\pi y}{a}]}{(2n-1)sinh[(2n-1)\frac{\pi b}{a}]}$$

Plot the temperature distribution in the plate.