

Assignment 02

Third Year BS(Honours) 2024-2025

Course Title: Math Lab III Course Code: AMTH 350

Department of Applied Mathematics, University of Dhaka

Name:

Roll:

Group:

1. Find the continued fraction of i) $\frac{576}{391}$, ii) $\sqrt{6}$ and then find successive convergent of the resulting fractions.
2. Write a function to find the GCD of two integers $d = (a, b)$ using the Euclidean algorithm. Then find the linear combination in the form $ax_0 + by_0 = d$.
3. Solve the linear Diophantine equation $23621x + 16376y = 23$ by any method. Then verify your solutions with built-in MATLAB command.
4. Write a MATLAB code to solve the liner congruence

$$ax \equiv b(\text{mod } m)$$

The program should prompt the user to input the values of a, b , and m , determine whether a solution exists, and display the number of incongruent solutions modulo m . If solutions exist, the program should also display all incongruent solutions. Test your program using the following examples:

- i. $18x \equiv 21(\text{mod } 78)$
 - ii. $1107x \equiv 263(\text{mod } 539)$
 - iii. $18x \equiv 30(\text{mod } 42)$
5. Write a MATLAB program to solve a system of linear congruences of the form

$$x \equiv a_1(\text{mod } m_1),$$

$$x \equiv a_2(\text{mod } m_2),$$

$$\vdots$$

$$x \equiv a_r(\text{mod } m_r).$$

In the program, the user should input the vector of remainders $a = [a_1 \ a_2 \ \dots \ a_r]$ and the vector of moduli $m = [m_1 \ m_2 \ \dots \ m_r]$. The program should determine whether a solution exists. If a solution exists, it should display the solution. Use your program to solve the following problems:

- i. $x \equiv 2(\text{mod } 4), x \equiv 6(\text{mod } 8), x \equiv 2(\text{mod } 12), x \equiv 14(\text{mod } 16)$.
 - ii. $x \equiv 1(\text{mod } 4), x \equiv 2(\text{mod } 6), x \equiv 3(\text{mod } 8), x \equiv 0(\text{mod } 10)$.
 - iii. Find an integer that leaves a remainder of 9 when it is divided by either 10 or 11, but it is divisible by 13.
 - iv. We have a number of things, but we do not know how many. If we count them by threes, we have two left over. If we count them by fives, we have three left over. If we count them by sevens, we have two left over. How many things are there.
6. Using MATLAB, solve a system of linear congruences of the form

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \equiv b_1(\text{mod } m),$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \equiv b_2(\text{mod } m),$$

$$\vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n \equiv b_n(\text{mod } m).$$

First, express the system in matrix form as $AX \equiv B \pmod{m}$, where A is $n \times n$ coefficient matrix, X is the column vector of unknowns, and B is the constant vector. Then, solve the system using $X \equiv \bar{A}B \pmod{m}$ where \bar{A} is said to be an inverse of A modulo m . Use your program to solve the following problems:

- i.

$$\begin{aligned} 2x_1 + 3x_2 + x_3 + 5x_4 + 4x_5 + 6x_6 + 7x_7 &\equiv 3 \pmod{11} \\ x_1 + 2x_2 + 3x_3 + x_4 + 5x_5 + 4x_6 + 6x_7 &\equiv 5 \pmod{11} \\ 6x_1 + x_2 + 2x_3 + 3x_4 + x_5 + 5x_6 + 4x_7 &\equiv 7 \pmod{11} \\ 4x_1 + 6x_2 + x_3 + 2x_4 + 3x_5 + x_6 + 5x_7 &\equiv 2 \pmod{11} \\ 5x_1 + 4x_2 + 6x_3 + x_4 + 2x_5 + 3x_6 + x_7 &\equiv 6 \pmod{11} \\ 3x_1 + 5x_2 + 4x_3 + 6x_4 + x_5 + 2x_6 + 3x_7 &\equiv 1 \pmod{11} \\ x_1 + 3x_2 + 5x_3 + 4x_4 + 6x_5 + x_6 + 2x_7 &\equiv 4 \pmod{11} \end{aligned}$$
- ii. Three groups of soldiers—Alpha, Bravo, and Charlie—each have an unknown number of soldiers. One morning, the commanders call their soldiers to form a line to be organized into units, each containing exactly 7 soldiers. In the first round, 2 soldiers from Alpha, 5 from Bravo, and 6 from Charlie march forward to form units, leaving 3 soldiers ungrouped at the end of the line. In the second round, 2 soldiers from Alpha and 1 from Charlie step forward, leaving 4 soldiers ungrouped. In the third round, 1 soldier from Alpha, 2 from Bravo, and 3 from Charlie march into formation, leaving 1 soldier ungrouped. Determine how many soldiers are in each group.

7. Use congruence to schedule a round-robin tournament for n teams and create a table as follows. Also find the total number of games in the tournament. For example,

- i. If $n = 9$ then the table will be like

Round	Team_1	Team_2	...	Team_9
Round_1	9	8		1
Round_2	bye	9	\ddots	2
\vdots				
Round_9	8	7		bye

- ii. If you have 8 football teams name as: Infinity, Asymptote, Eigen, Laplace, Sigma, Vector, Gradient, Prime, then the table will be like

Round	Infinity	Asymptote	...	Prime
Round_1	Gradient	Vector		Laplace
Round_2	Prime	Gradient	\ddots	Infinity
\vdots				
Round_7	Vector	Sigma		Gradient

8. Write a MATLAB code to calculate the check digit of the International Standard Book Number (ISBN). The code first asks whether the ISBN is ISBN-13 or ISBN-10. Based on the selection, the user should then enter the first 12 digits (for ISBN-13) or the first 9 digits (for ISBN-10). Use your program find the check digit for the
 - i. Book *Calculus-Early Transcendentals* by H. Anton which ISBN is 978-1-118-09240
 - ii. Book *A First Course in Differential Equations with Modelling Applications* by Dennis G. Zill which ISBN is 81-219-1667