HW5 ReadMe

June 2021

Introduction

This code calculates the linear finite element solution to the problem

$$\nabla^2 u = -f \text{ On } \Omega$$
$$\text{And } u = 0 \text{ On } \partial\Omega = 0$$

Where, f is a given forcing function and $\Omega = [-1\ 1] \times [-1\ 1]$.
For the sake of demonstration we choose the two forcing functions:

$$f1 = \sin(\pi x)\sin(\pi y)$$
$$f2 = (1 - x^2)(1 - y^2)$$

The connectivity and coordinate matrices are provided in the folder meshes/meshes.
The finite element solution is given by:

$$U_h = \sum_{i=1}^{n_p} \alpha_i N_i$$

Where α_i are the coefficients and N_i are the basis functions for the mesh provided.

Running the code and Output of the code

To run the code, simply type **make** in the terminal which is opened in the proper path. This compiles the program from **makefile.txt**. The output of the routine is the file **OutputFile.txt**, which stores the values of α_i's in a column vector. Along with **OutputFile.txt**, the program also prints the following:

1. Number of Nodes and Elements in the meshing
2. The global mesh size, h_G
3. The Apriori Error estimate in L^2 norm, $||u - u_h||_{L^2(\Omega)}$
4. The Apriori Error estimate for U_h in the energy norm, $||\nabla(u - u_h)||_{L^2(\Omega)}$
5. The Apriori Error estimate for πu in the energy norm, $||\nabla(u - \pi u)||_{L^2(\Omega)}$

6. The value of the Functional,

$$I[v] = \int_\Omega \left(\frac{\nabla v \cdot \nabla v}{2} - fv \right) \mathrm{d}\Omega$$

For $v = u, u_h$ and πu.

Modifying the code

The mesh size can be changed by changing the arguments of the **ReadMesh** function, in **HW5.cpp**. Upto 6 different mesh sizes have been provided.

To change the forcing functions from $f1$ to $f2$ or vice-versa changes in the following routines must be made: **HW5.cpp**, **ComputeGaussQuad.cpp**, **ComputeGaussQuad_AprioriError.cpp**, **ComputeGaussQuad_EnergyNorm_Uh.cpp**, **ComputeGaussQuad_EnergyNorm_PiU.cpp** and **ComputeGaussQuad_Functional.cpp**.

The Header file **HeaderFile.h** contains the libraries and global constants used, including the weights and nodes for Gaussian quadrature. Any additional function declarations must be added to this file, with the necessary compile information added to **makefile.txt**.