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Improving diffusion-based recommendation in online rating systems

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Network diffusion processes play an important role in solving the information overload problem. It has been shown that the diffusion-based recommendation methods have the advantage to generate both accurate and diverse recommendation items for online users. Despite that, numerous existing works consider the rating information as link weight or threshold to retain the useful links, few studies use the rating information to evaluate the recommendation results. In this paper, we measure the average rating of the recommended products, finding that diffusion-based recommendation methods have the risk of recommending low-rated products to users. In addition, we use the rating information to improve the network-based recommendation algorithms. The idea is to aggregate the diffusion results on multiple user-item bipartite networks each of which contains only links of certain ratings. By tuning the parameters, we find that the new method can sacrifice slightly the recommendation accuracy for improving the average rating of the recommended products.

Keywords: Recommender systems; online rating systems; network diffusion; recommendation quality.

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1. Introduction

With the rapid progress of information technology, it is undoubtedly that we are in the time of knowledge exploration. Facing huge amounts of information, online users find it difficult to make decisions, which is also known as a problem called information overload.[1] Information filtering[2] is a sort of method to deal with large information flows. Information filtering methods are used for various purposes in different online systems. Users with specific needs can search for what they need directly with keywords in search engine.[3] However, users with ambiguous demands

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need personal recommendation to make proper decisions. Therefore, recommender systems were designed and came into application.

The three components in information recommendation are users, items and recommendation algorithms, of which the last one is the foremost factor. Furthermore, it can determine the performance and preference of recommender systems. In general, recommendation algorithms fall into three categories: collaborative recommendation (also known as collaborative filtering), content-based recommendation and network-based recommendation.[4] Collaborative filtering is perhaps the most frequently-used approach in application[5,6] among them. But beyond that, researchers in different fields have made much progress in recommendation algorithms, to be specific, techniques include content-based analysis,[7] spectral analysis,[8] latent semantic models,[9] matrix factorization[10] and social recommendation.[11,12] See Refs. 13–18 for a current review of various aspects of recommendation in different fields.

In this paper, we concentrate mainly on recommendation algorithms based on bipartite user-item networks in which users connected to the items they have interacted with.[19,20] Recommendation algorithms of this kind aim to predict future links in the networks. Classical physical processes such as random walk[21] and heat conduction[22] have been applied to the bipartite networks to get recommendations for users. See Ref. 18 for a review of the basic ideas in network-based recommendation and ranking algorithms. Many variants and improvements of the originally proposed algorithms have been subsequently published and their scope has been extended to, for example, the link prediction problem[23,24] and the prediction of future trends.[25]

More specifically, mass diffusion[21] and heat conduction[26] have been applied to recommendation algorithms on bipartite user-item networks, respectively. It turns out that the former has high accuracy while the latter has high diversity. Therefore, they can be coupled into a hybrid method and the hybrid method of mass diffusion and heat conduction was proved to work well both in accuracy and diversity.[22] Our algorithm in this paper is based on this hybrid method.

In most network-based recommendation algorithms, bipartite networks link users and items only when the given rating is at least 3. In recent years, some researches make use of rating information into network-based recommendation algorithms. Rather than neglecting negative ratings directly, establishing negative edges may play a positive role.[27,28] Moreover, algorithm weights edges of bipartite network by users' rating on objects were proved to have better performance.[29] To optimize initial resource configuration, items that are more similar to the items the user gives negative ratings to will have less weight.[30] In general, it is beneficial to apply the rating information in the network-based recommendation.

Accuracy, diversity and quality of recommendation are three elements we pursue when we design recommendation algorithms. In practice, an effective recommender system should be able to help users in making decisions. Results from the system should be able to meet users' needs. At the same time, based on a large amount of users' selection records, the recommendation results should match different characteristics of each user. The recommender system should also have the ability to keep inferior results away from users, which can enhance users' experience of the system. As the original research mainly concerned about accuracy and diversity, in this paper, we take quality into consideration. Recommendation quality here is defined as the average rating scores given to a recommended product, and it can be greatly improved at the expense of accuracy by using the original hybrid method on networks formed by links with only the highest rating because the elimination of inferiors means a reduction in the amount of information. Our aim here is to find a way to give consideration to both accuracy and quality.

2. Recommendation Algorithms

Bipartite networks can be used to model online rating systems, where links only exist between users and items which have interactions. For example, we link a user and an item when the user has rated the item. In the following statement, there are I item nodes and U user nodes in the online rating system. We represent a bipartite network with an adjacency matrix A_n, which consists of links with rating no less than n. The rating is generally range from 1 to 5 in most online rating systems. 1 means the user does not like the item at all while 5 means the user likes the item very much. In other words, the higher the rating, the more the user likes the item. Every element $a_{i\alpha}$ in the adjacency matrix A_n represents relationship between user i and item α. $a_{i\alpha} = 1$ means user i has a rating no less than n on item α. On the contrary, $a_{i\alpha} = 0$ means user i has no interaction with item α. k_i denotes the total number of items user i have linked with, in other words we called it degree of user node i. k_α denotes the total number of users item α have linked with, in other words we called it degree of item node α. We use Latin letters to denote user nodes, and Greek letters to denote item nodes.

The original hybrid recommendation algorithm.[22] We first briefly explain the original hybrid method (HY), which is the hybrid of mass diffusion (MD) method and heat conduction (HC) method. For every target user, we initially assign items that the target user has selected as 1 while what he has not selected as 0. Then we evaluate the similarity of the target user to other users by the hybrid of mass diffusion and heat conduction on the bipartite network. The similarity of target user i to user j can be written as

$$f_{ij} = \sum_{\alpha=1}^{I} \frac{a_{i\alpha} a_{j\alpha}}{k_\alpha^\lambda k_j^{1-\lambda}}, \tag{1}$$

where λ weighs the proportion of two methods. $\lambda = 0$ and $\lambda = 1$ denote heat conduction and mass diffusion, respectively. Then we find recommended items for the target user i by similar process from users to items on the bipartite network. The grade of recommendation for item α to target user i can be written as

$$f_{i\beta} = \sum_{j=1}^{U} \frac{a_{j\beta} f_{ij}}{k_j^\lambda k_\beta^{1-\lambda}}, \tag{2}$$

To provide a recommendation list for the target user i, we sort the value $f_{i\beta}$ in descending order and get the top L items except what the user has bought as a recommendation list for the target user i.

The HYSR recommendation algorithm. The original hybrid method was proved to be accurate and diversified. However, this method can recommend items of poor quality to users because it weighs every link equally even it has a low rating score. To eliminate the risk of recommending inferior items, one straightforward idea is to use only the links with the highest rating to make recommendations. Conceivably, we lose much accuracy while the quality is promoted because we eliminate substantial information.

Hybrid method with structure reduction (HYSR) is conceived on the basis of the original hybrid method.[22] In this method, we use all the links in the network to find the similarity between users. Whereas when we use the similarity to find the grade of recommendation, we use only the links with the highest rating. The whole process is illustrated in Fig. 1. Generally speaking, we use all the information to find relatively accurate similarity between users, while exclude inferiors from recommendation lists to promote quality of recommendation results.

The HYNA recommendation algorithm. HYSR method is a tradeoff between accuracy and quality, see the detail discussion in Sec. 5. Then we realize that it may

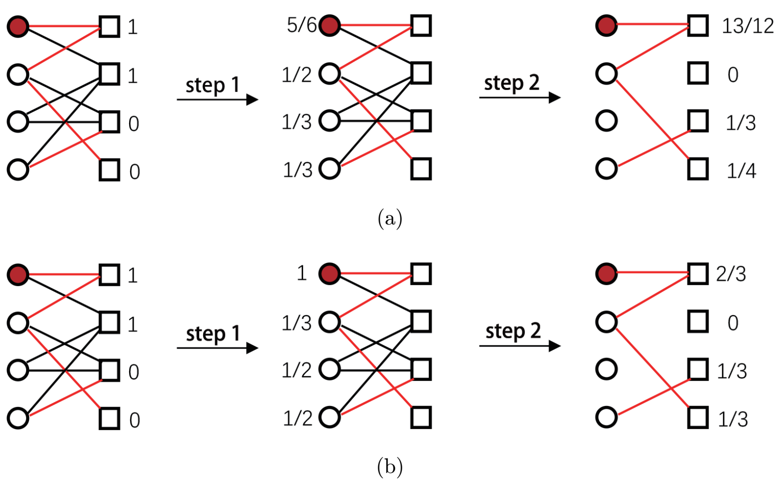

Fig. 1. (Color online) This figure shows the recommendation process of the HYSR method. Figure 1(a) shows mass diffusion method with structure reduction and Fig. 1(b) shows heat conduction method with structure reduction. In both rows the process has three stages. In the first stage (the left column) we know what items the target user has had interaction with. In the second stage (the middle column) we know how similar the target user is to other users. In the third stage (the right column) we know the possibilities of items that the target user may like to have interaction with. Red lines represent links with the highest rating. In step one which from the first stage to the second, all links work equally. However, in step two which from the second stage to the third, only links with the highest rating are retained. As in step 2, we do not use all links, we call this as structure reduction.

be dogmatic to remove a large amount of links in step two. The hybrid method with network aggregation (HYNA) contains more possibilities so that we can find whether there are measures satisfactory to both accuracy and quality. The so-called network aggregation is to integrate results of several networks. The final grade of recommendation can be written as

$$S = 1^{\theta} S_{\geq 1} + 2^{\theta} S_{\geq 2} + 3^{\theta} S_{\geq 3} + 4^{\theta} S_{\geq 4} + 5^{\theta} S_{\geq 5}. \tag{3}$$

In this equation, $S_{\geq n}$ is a set of the final value of Eq. (2) when we use links whose rating is no less than n in step two. When $\theta \to -\infty$, the HYNA method degenerates to the original HY method. Similarly, when $\theta \to +\infty$, the HYNA method transforms into the HYSR method. The higher θ is, the more the links with the highest rating affect the result. The lower θ is, the recommendation algorithm accordingly tends to weigh every link more equally. We get the final recommendation list by sorting the results of Eq. (3) in descending order and excluding the items the user has had interaction with at the initial stage.

3. Data

In this paper, we use three benchmark datasets of online rating systems, i.e. MovieLens, Netflix and Rate Your Music (RYM), to validate our methods.

MovieLens data[a] contains 1682 movies, 943 users and 10^5 ratings in total. Ratings are indicated by integer scale from 1 to 5, and the higher the better. To meet different needs, we have different ways to construct bipartite networks. Using n to show how much information retained in network, we represent all ratings of n or more as a link between a individual user and item. For example, the resulting number of links is 82 520 when $n = 3$. Netflix data[b] contains 3000 users, 3000 movies and 197 248 ratings in total. Rate Your Music (RYM) data[c] contains 33 786 users and 5381 albums and 676 449 ratings in total.We map the rating from [1,10] to [1,5]. We process Netflix and RYM data in the same way with MovieLens data.

4. Evaluation

There are two kinds of evaluating indicators, one is for recommendation accuracy, and the other is for recommendation quality.

4.1. *Indicators for recommendation accuracy*

To evaluate accuracy of a recommender algorithm, we randomly divide all the data into two sets: a training set E^T containing 90% links and a probe set E^P containing the rest. We run our algorithm using E^T data only. Accuracy could be quantified by comparing the recommending results and the real links in E^P data.

[a] Data is available at http://www.grouplens.org/.
[b] Original data is available at http://www.netflixprize.com/.
[c] https://rateyourmusic.com/.

Ranking score (RS) was defined as the average rank of items in recommendation lists which the items simultaneously link with the target user in E^P. If $r_{i\alpha}$ represents rank of item α in the recommendation list of user i, ranking score of the user-item pair is $\mathrm{RS}_{i\alpha} = r_{i\alpha}/I$. Ranking score of this algorithm can be calculated by averaging ranking score of all user-item pairs that have linked in E^P, and can be written as

$$\mathrm{RS} = \frac{1}{|E^P|} \sum_{(i,\alpha)\in E^P} \mathrm{RS}_{i\alpha}. \tag{4}$$

The lower the value, the better the algorithm on accuracy.

Real online recommender systems usually show only limited amount of recommendation results to users, actually users can see only the top part of the recommendation list. We thus use another indicator to measure precision because of the importance of the top-L area in users' recommendation list.

For a certain user i, precision is expressed as

$$P_i(L) = \frac{d_i(L)}{L}, \tag{5}$$

where $d_i(L)$ shows the number of items linked to user i in E^P and simultaneously in the top-L recommendation list for user i. In some of the discussions that follow, we set $L = 20$. Hence, $P_i(L)$ indicates the proportion of successfully recommendations for user i in the top-L recommendation list. $P(L)$, the average $P_i(L)$ of all users, could be used to express precision of the system.

4.2. *Indicators for recommendation quality*

In this paper, we define quality of certain item as the average rating of the item. Quality of item α can be written as

$$Q_\alpha = \frac{\sum_{i=1}^{I} s_{i\alpha}}{\sum_{i=1}^{I} a_{i\alpha}}, \tag{6}$$

where $s_{i\alpha}$ denotes the rating score of user i to item α.

Mean quality (MQ) measures average quality of recommendation results. We use the average quality of items in the top-L recommendation lists of all users to measure mean quality of a recommendation algorithm. Mean quality of user i could be written as

$$\mathrm{MQ}_i = \frac{\sum_{\alpha\in O_L^i} Q_\alpha}{L}, \tag{7}$$

where O_L^i is a set including all the items in the top-L recommendation list for user i. Therefore, the average MQ_i of all users could be used to measure quality of the recommendation algorithm.

In addition to MQ, we also focus on extreme situations of recommendation results. Then we pay attention to the fraction of low-quality items in the top-L recommendation list (FLQ for short). In the analysis in Sec. 5 we define an item as a low-quality item if its quality is less than 3. For user i, the fraction of low-quality items can be written as

$$\mathrm{FLQ}_i = \frac{l_i(L)}{L}, \tag{8}$$

where top-L recommendation list of user i has $l_i(L)$ low-quality items. FLQ, the average FLQ_i of all users, thus be another indicator measuring quality of the recommendation algorithm. Note that the lower FLQ means the better quality because systems of high quality do not allow many inferiors in top part of recommendation lists.

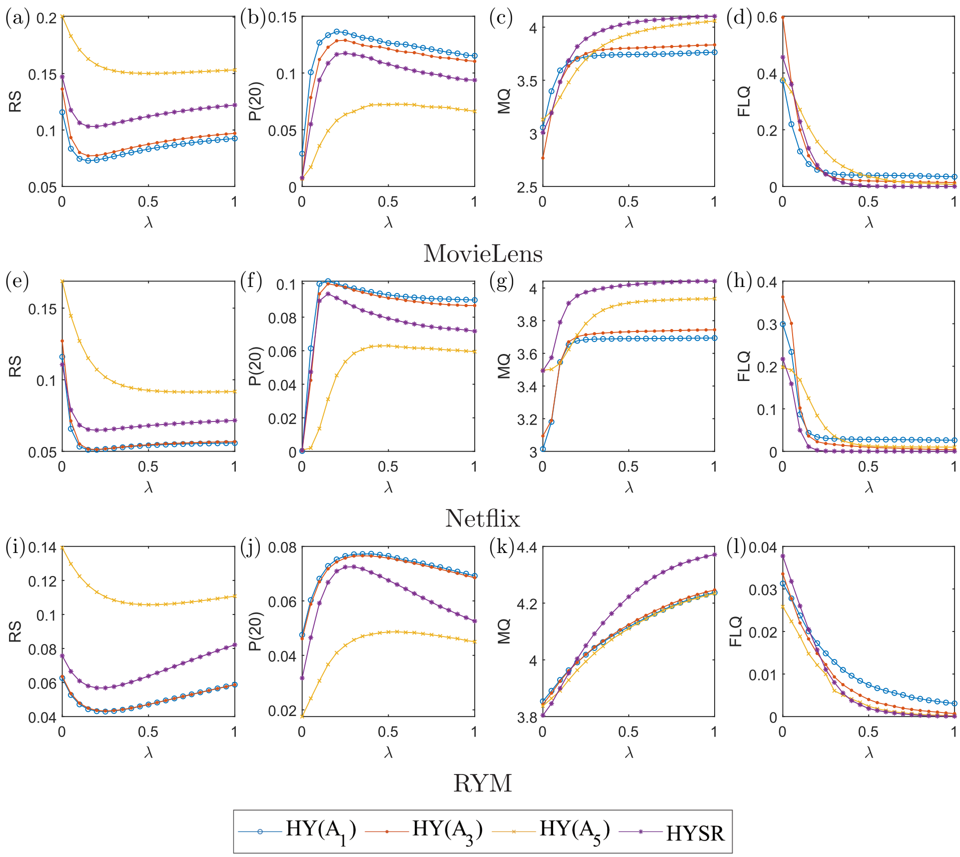

Fig. 2. (Color online) This figure shows accuracy and quality of HY method, respectively, on A_1, A_3, A_5, and the HYSR method by four indicators. Results in (a)–(d) are obtained on MovieLens data, and (e)–(h) show results of Netflix data, and (i)–(l) show results of RYM data.

5. Results

The HYSR recommendation algorithm. We first use the original HY method on different networks to make recommendations. As shown in first and second columns of Fig. 2, for the four curves, it is similar that how accuracy changing affected by the hybrid parameter λ. In addition, their λs for optimal accuracy are near to each other. Accuracy is decreased sharply using the original HY method on A_5, because we discard a large amount of links. Similarly, the HY method on A_1 shows the highest accuracy because A_1 retains the most information. As shown in third and forth columns of Fig. 2, for the four curves recommendation quality increases with λ. In addition, the larger λ is, the slower the growth is. Compared to HY method on A_1, the quality of HY method on A_5 is decreased on a short interval, while increased on the other large interval. HY method on A_3 is most used in former researches. It could

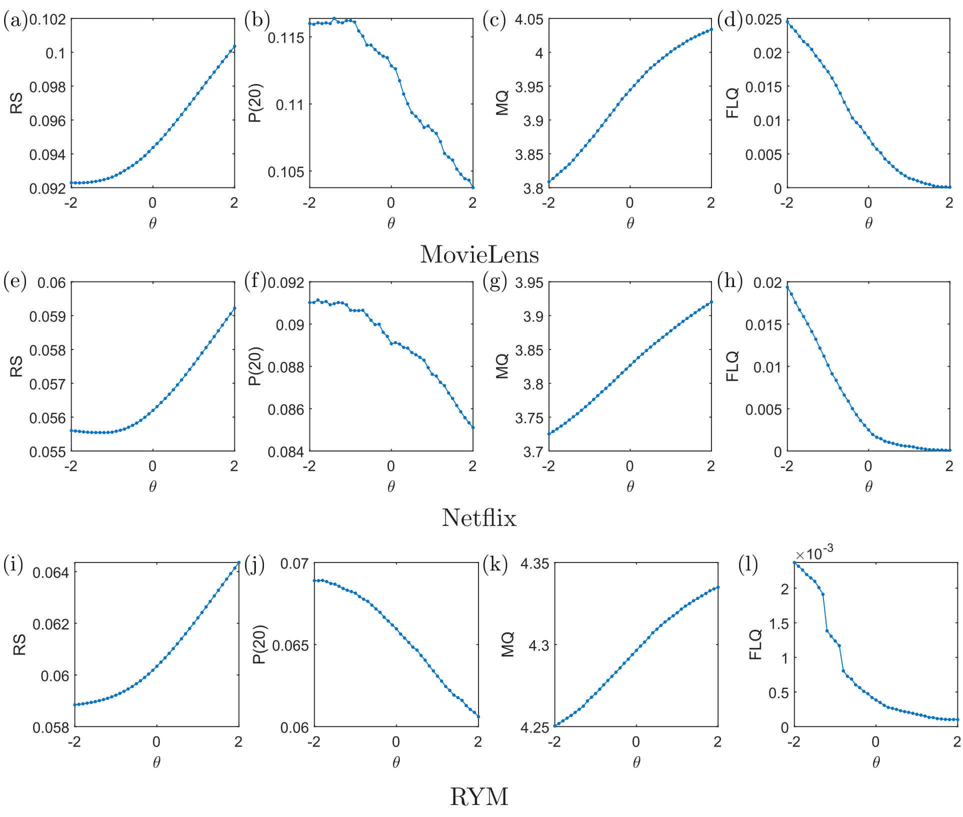

Fig. 3. (Color online) This figure shows a typical case of the HYNA method that $\lambda = 1$ to observe effect of the parameter θ in accuracy and quality. Results in (a)–(d) are obtained on MovieLens data, and (e)–(h) show results of Netflix data, and (i)–(l) show results of RYM data. The value of θ can range from negative infinity to positive infinity. To observe the way θ influences accuracy and quality of HYNA, we show results of an interval $[-2, 2]$ and the step is 0.1 by four indicators in this figure.

be found in Fig. 2 that we can obtain medium accuracy and quality when we use HY method on A_3.

We then compare performance of the HYSR method with the original HY methods. The HYSR method, making use of all links in step one and neglecting links with low ratings in step two, obtains accuracy between HY method on A_5 and on A_1. However, HYSR method shows higher quality than HY method in the most interval of the parameter λ. When compared with HY method on A_3, the HYSR method has better quality and worse accuracy.

In conclusion, we increase quality of recommendation results at the expense of accuracy by using HYSR method. Consequently, HYSR is a tradeoff method between accuracy and quality compared with the original HY method. Although we have achieved the aim that increases quality of recommendation algorithm by using HYSR method, the accompanying decrease of accuracy is due to the fact that HYSR neglected too much information in the user-item bipartite network. To find a way that accuracy and quality can be promoted at the same time, we come up with the HYNA method (see Sec. 2 for the method description).

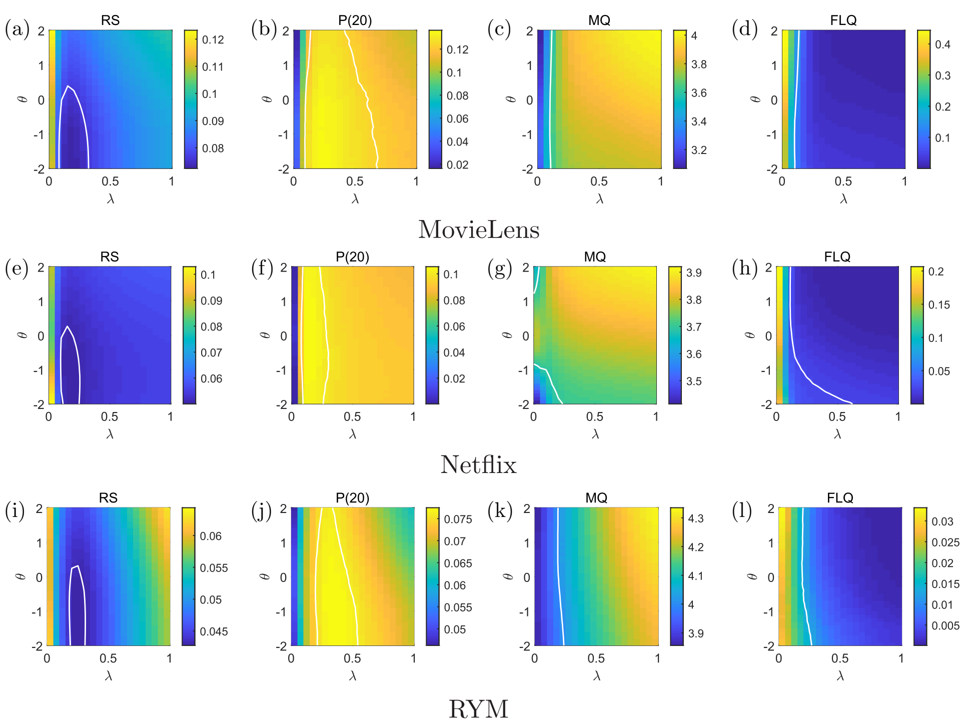

Fig. 4. (Color online) This figure shows changes of the four indicators with different parameters (λ, θ). Results in (a)–(d) are obtained on MovieLens data, and (e)–(h) show results of Netflix data, and (i)–(l) show results of RYM data. Curves on them show the optimal value of the original HY method on A_3. The parameters within these curves yield the results that are better than the HY method.

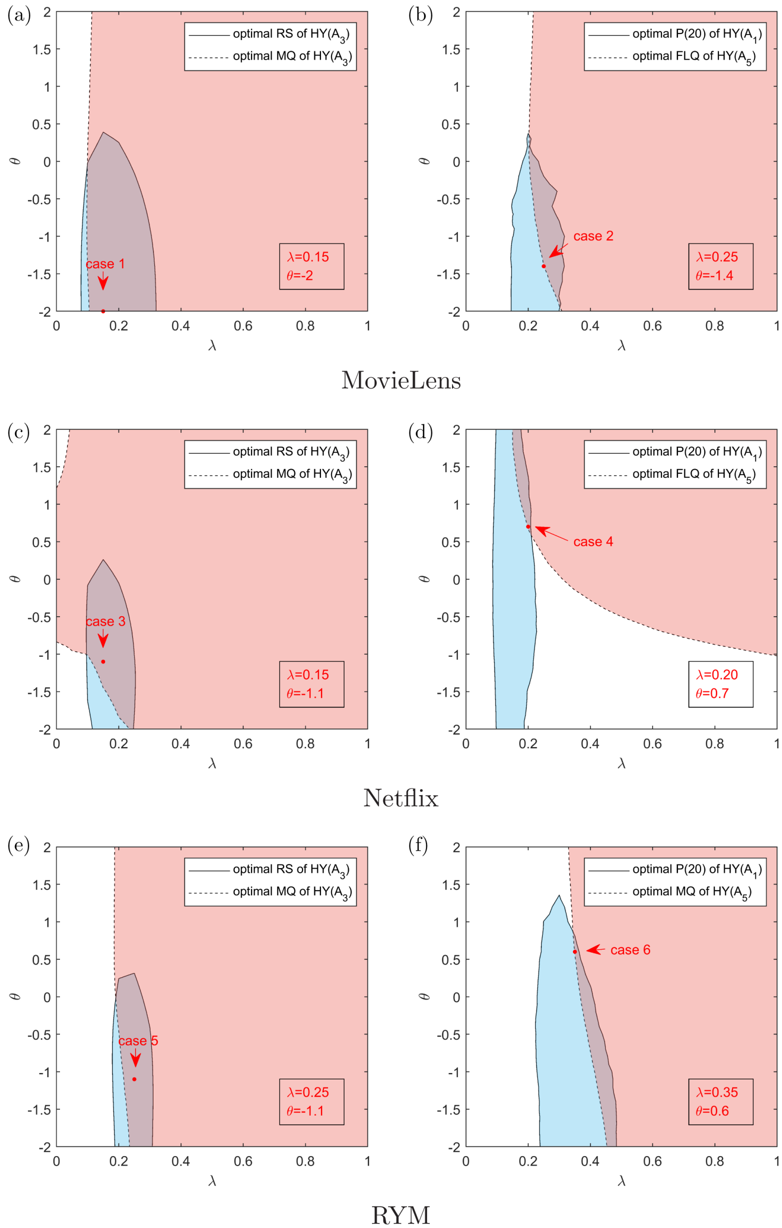

Fig. 5. (Color online) This figure compares HYNA method with different parameters to the best accurate HY method on networks with different amount of information. Results in (a) and (b) are obtained on MovieLens data, and (c) and (d) on Netflix data, and (e) and (f) show results of RYM data. The HYNA method with parameters in the red area has better quality while the blue area indicates better accuracy. It turns out an overlapping area that with these parameters the HYNA method performs better both in accuracy and quality than the HY method on corresponding network. The red dots in this figure are examples to see how much accuracy and quality HYNA can promote. Parameters of red dots are at the bottom right in each figure. The improvement rates of the red dots are shown in Table 1.

The HYNA recommendation algorithm. Figure 3 shows how accuracy and quality of HYNA are changed by the parameter θ. As can be seen in Fig. 3, the accuracy is cut down and quality built up with the increase of θ. This is because the HYNA method with a larger θ attaches more importance to links with high rating score rather than treating every links equally.

Figure 4 is a heat map to show how the accuracy and quality of HYNA controlled by the two parameters. The solid curve draws out the optimal result of the commonly used HY method (that is the HY method on A_3). Here the optimal result is defined by the θ that can minimize RS. Therefore, we can find pairs of parameters that make the HYNA method better than the original HY method in each indicator. For example, in Figs. 4(a), 4(e) and 4(i), parameters below the solid curve can make the HYNA method better than HY on A_3 in accuracy measured by RS. If the parameters with better accuracy and better quality have overlapping parts, then the HYNA method with these parameters is a way that simultaneously improves the accuracy and quality of the commonly used HY method. As HYSR is a specific case of HYNA, we do not compare directly HYSR and HYNA.

Desirable area of parameters is indicated in Figs. 5(a), 5(c) and 5(e). Compared to the most used HY method, HYNA method with parameters in the blue area owns more accuracy, while in the red area has higher quality. There is a distinctly overlapping area in the parameter space. In order to show the degree of improvement, we select a pair of parameters from the area with which the HYNA can achieve the lowest RS. The improvement rates of accuracy and quality are shown in Table 1. In brief, the results above indicate that with certain parameters the HYNA method can be better both in accuracy and quality than the most used HY method.

For a more complete analysis, we compare the results with the accuracy of HY on A_1 and the quality of HY on A_5. As a result, the overlapping area is shrunk because the accuracy of HY on A_1 and the quality of HY on A_5 almost mean the best accuracy or quality that the HY method can reach without considering the other. There is an overlapping area, as shown in Figs. 5(b), 5(d) and 5(f), where $P(20)$ stands for accuracy and FLQ stands for quality. For RYM data, the parameters with which HYNA has better accuracy than HY(A_1) and better quality than HY(A_5) are

Table 1. The values and improvement rates of six cases with different parameters that marked by red dots in Fig. 5. The values of indicators are outside brackets, while values in brackets show improvement rates. Note that all the improvement rates are calculated with the baseline to the accuracy of HY(A_3).

Data	Cases	RS	$P(20)$	MQ	FLQ
MovieLens	Case 1	0.072(6.17%)	0.134(9.19%)	3.692(1.59%)	0.071(34.67%)
	Case 2	0.075(2.95%)	0.135(9.60%)	3.764(3.57%)	0.035(67.87%)
Netflix	Case 3	0.050(2.35%)	0.104(5.09%)	3.715(0.33%)	0.026(−16.20%)
	Case 4	0.053(−2.31%)	0.102(2.90%)	3.810(2.88%)	0.010(54.86%)
RYM	Case 5	0.043(1.30%)	0.077(1.63%)	4.038(0.47%)	0.011(11.54%)
	Case 6	0.045(−3.73%)	0.077(1.21%)	4.111(2.28%)	0.005(60.23%)

two separated areas when FLQ stands for quality, whereas they have an overlapping area when MQ represents quality. The overlapping area signifies that HYNA in the regime of the parameter space is superior to HY in terms of accuracy and quality. In order to see the degree of improvement, we similarly select a pair of parameters from the area, respectively, and calculate improvement rates compared to the most used HY method (that is the HY method on A_3). The values and improvement rates of accuracy and quality are shown in Table 1.

6. Conclusion

The online user-item bipartite networks are in general sparse. Therefore, in order to accurately predict users future interest, the recommendation algorithms should be provided with sufficient historical data. However, if one wants to recommend more highly rated products, the common way is to exclude low rating links, which largely decreases the number of links in the user-item network and consequently cause lower recommendation accuracy due to the network sparsity. This causes the trade-off between the recommendation accuracy and the average rating of the recommended products. In our method, we propose an improved recommendation method with a well-designed link weighting scheme. The method can maintain the network sparsity (without losing accuracy) and give more weights to the high rating links (improve quality of recommended items). The method has two parameters. We identify the regime of the parameter space in which the method can have high average rating of the recommended items while keeping comparable recommendation accuracy compared to the classic threshold link deleting method.

In this paper, we propose two methods on the basis of the original hybrid method of mass diffusion method and heat conduction method (HY). To quantify the quality of a recommendation algorithm, we define products' quality as its average rating and come up with two indicators that expressed as MQ and FLQ. The results generated on MovieLens, Netflix and RYM data sets show that the quality of the hybrid method with structure reduction (HYSR) increases at the expense of accuracy. The hybrid method with network aggregation (HYNA) that has two tunable parameters is an improvement based on HYSR. The performance of HYNA could be enhanced by adjusting parameters. The tests on three datasets show that we can find parameters to promote both accuracy and quality. In other words, we finally formulate a recommendation algorithm that improve quality without losing accuracy.

Our work is a key to reduce the possibility that the inferiors can be recommended to users in the original HY method. Our work creates several possibilities for future research. For example, we can define quality of items by an iterative ranking method.[31] Moreover, the HYNA can achieve different effects by adjusting parameters. The optimal area in results can be determined by two or more factors we concern. In future research we can add other factors that we find important to recommendation to identify a new optimal parameters area.

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