Solution to TPPMark 2019

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Main problem (#4)

Please define a machine that, given an input cyclic tape of unknown size and content, clears the tape (by zero) and halts.
You may assume a minimum length on input tapes.

- **My assumption (most general setting)**
  - Length of input tapes ≥ 1
  - Number of input symbols ≥ 2
    - At least, there must be zero (0).
      - Otherwise, it is impossible to clears.
    - At least, there must be a non-zero symbol (1).
      - Otherwise, it is trivial.

Solution by Yamada-san

(a) Put 1 and move to the right.

(b) Put 1 and move to the right.

(c) Repeatedly put 0 and move to the right until finding 1.

(d) Put 0 and repeatedly move to the left until finding 1.

(e) Move to the left.

(f) If the symbol is 1, then go to (a).

If the symbol is 0, then the tape is zero-cleared.

- **Assumption**
  - Length of tape ≥ 2
    - If the length is 1, the algorithm does not terminate.

My zero-clear algorithm

(a) Put 1 and move to the right.

(b) Put 0 and move to the right.

(c) Repeatedly put 1 and move to the right until finding 1.

(d) Repeatedly put 0 and move to the left until finding 0.

(e) Move to the left.

(f) If the symbol is 1, then go to (a).
If the symbol is 0, then the tape is zero-cleared.

2	4	3	1	5	0	2	1	0	4	0	1	2	3	5

(a)

2	4	1	1	5	0	2	1	0	4	0	1	2	3	5

My zero-clear algorithm

(a) Put 1 and move to the right.

(b) Put 0 and move to the right.

(c) Repeatedly put 1 and move to the right until finding 1.

(d) Repeatedly put 0 and move to the left until finding 0.

(e) Move to the left.

(f) If the symbol is 1, then go to (a).
If the symbol is 0, then the tape is zero-cleared.

2	4	1	1	5	0	2	1	0	4	0	1	2	3	5

(b)

2	4	1	0	5	0	2	1	0	4	0	1	2	3	5

My zero-clear algorithm

(a) Put 1 and move to the right.

(b) Put 0 and move to the right.

(c) Repeatedly put 1 and move to the right until finding 1.

(d) Repeatedly put 0 and move to the left until finding 0.

(e) Move to the left.

(f) If the symbol is 1, then go to (a).
If the symbol is 0, then the tape is zero-cleared.

2	4	1	0	5	0	2	1	0	4	0	1	2	3	5

(c)

2	4	1	0	1	1	1	1	0	4	0	1	2	3	5

My zero-clear algorithm

(a) Put 1 and move to the right.

(b) Put 0 and move to the right.

(c) Repeatedly put 1 and move to the right until finding 1.

(d) Repeatedly put 0 and move to the left until finding 0.

(e) Move to the left.

(f) If the symbol is 1, then go to (a).
If the symbol is 0, then the tape is zero-cleared.

2	4	1	0	1	1	1	1	0	4	0	1	2	3	5

(d)

2	4	1	0	0	0	0	0	0	4	0	1	2	3	5

My zero-clear algorithm

(a) Put 1 and move to the right.

(b) Put 0 and move to the right.

(c) Repeatedly put 1 and move to the right until finding 1.

(d) Repeatedly put 0 and move to the left until finding 0.

(e) Move to the left.

(f) If the symbol is 1, then go to (a).
If the symbol is 0, then the tape is zero-cleared.

2	4	1	0	0	0	0	0	0	4	0	1	2	3	5

(e)

2	4	1	0	0	0	0	0	0	4	0	1	2	3	5

My zero-clear algorithm

(a) Put 1 and move to the right.

(b) Put 0 and move to the right.

(c) Repeatedly put 1 and move to the right until finding 1.

(d) Repeatedly put 0 and move to the left until finding 0.

(e) Move to the left.

(f) If the symbol is 1, then go to (a).
If the symbol is 0, then the tape is zero-cleared.

2	4	1	0	0	0	0	0	0	4	0	1	2	3	5

(f)&(a)

2	4	1	0	0	0	0	0	0	4	0	1	2	3	5

My zero-clear algorithm

(a) Put 1 and move to the right.

(b) Put 0 and move to the right.

(c) Repeatedly put 1 and move to the right until finding 1.

(d) Repeatedly put 0 and move to the left until finding 0.

(e) Move to the left.

(f) If the symbol is 1, then go to (a).

If the symbol is 0, then the tape is zero-cleared.

2	4	1	0	0	0	0	0	0	4	0	1	2	3	5

(b)

2	4	1	0	0	0	0	0	0	4	0	1	2	3	5

My zero-clear algorithm

(a) Put 1 and move to the right.

(b) Put 0 and move to the right.

(c) Repeatedly put 1 and move to the right until finding 1.

(d) Repeatedly put 0 and move to the left until finding 0.

(e) Move to the left.

(f) If the symbol is 1, then go to (a).
If the symbol is 0, then the tape is zero-cleared.

2	4	1	0	0	0	0	0	0	4	0	1	2	3	5

(c)

2	4	1	0	1	1	1	1	1	1	1	1	2	3	5

My zero-clear algorithm

(a) Put 1 and move to the right.

(b) Put 0 and move to the right.

(c) Repeatedly put 1 and move to the right until finding 1.

(d) Repeatedly put 0 and move to the left until finding 0.

(e) Move to the left.

(f) If the symbol is 1, then go to (a).
If the symbol is 0, then the tape is zero-cleared.

2	4	1	0	1	1	1	1	1	1	1	1	2	3	5

(d)

2	4	1	0	0	0	0	0	0	0	0	0	2	3	5

My zero-clear algorithm

(a) Put 1 and move to the right.

(b) Put 0 and move to the right.

(c) Repeatedly put 1 and move to the right until finding 1.

(d) Repeatedly put 0 and move to the left until finding 0.

(e) Move to the left.

(f) If the symbol is 1, then go to (a).
If the symbol is 0, then the tape is zero-cleared.

2	4	1	0	0	0	0	0	0	0	0	0	2	3	5

(e)

2	4	1	0	0	0	0	0	0	0	0	0	2	3	5

My zero-clear algorithm

(a) Put 1 and move to the right.

(b) Put 0 and move to the right.

(c) Repeatedly put 1 and move to the right until finding 1.

(d) Repeatedly put 0 and move to the left until finding 0.

(e) Move to the left.

(f) If the symbol is 1, then go to (a).
If the symbol is 0, then the tape is zero-cleared.

2	4	1	0	0	0	0	0	0	0	0	0	2	3	5

(f)&(a)

2	4	1	0	0	0	0	0	0	0	0	0	2	3	5

My zero-clear algorithm

(a) Put 1 and move to the right.

(b) Put 0 and move to the right.

(c) Repeatedly put 1 and move to the right until finding 1.

(d) Repeatedly put 0 and move to the left until finding 0.

(e) Move to the left.

(f) If the symbol is 1, then go to (a).

If the symbol is 0, then the tape is zero-cleared.

2	4	1	0	0	0	0	0	0	0	0	0	2	3	5

(b)

2	4	1	0	0	0	0	0	0	0	0	0	2	3	5

My zero-clear algorithm

(a) Put 1 and move to the right.

(b) Put 0 and move to the right.

(c) Repeatedly put 1 and move to the right until finding 1.

(d) Repeatedly put 0 and move to the left until finding 0.

(e) Move to the left.

(f) If the symbol is 1, then go to (a).
If the symbol is 0, then the tape is zero-cleared.

2	4	1	0	0	0	0	0	0	0	0	0	2	3	5

(c)

1	1	1	0	1	1	1	1	1	1	1	1	1	1	1

My zero-clear algorithm

(a) Put 1 and move to the right.

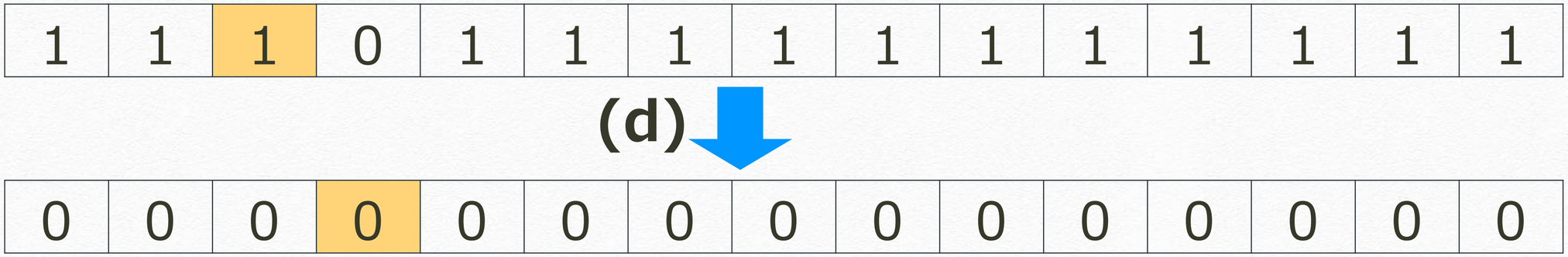
(b) Put 0 and move to the right.

(c) Repeatedly put 1 and move to the right until finding 1.

(d) Repeatedly put 0 and move to the left until finding 0.

(e) Move to the left.

(f) If the symbol is 1, then go to (a).
If the symbol is 0, then the tape is zero-cleared.



My zero-clear algorithm

(a) Put 1 and move to the right.

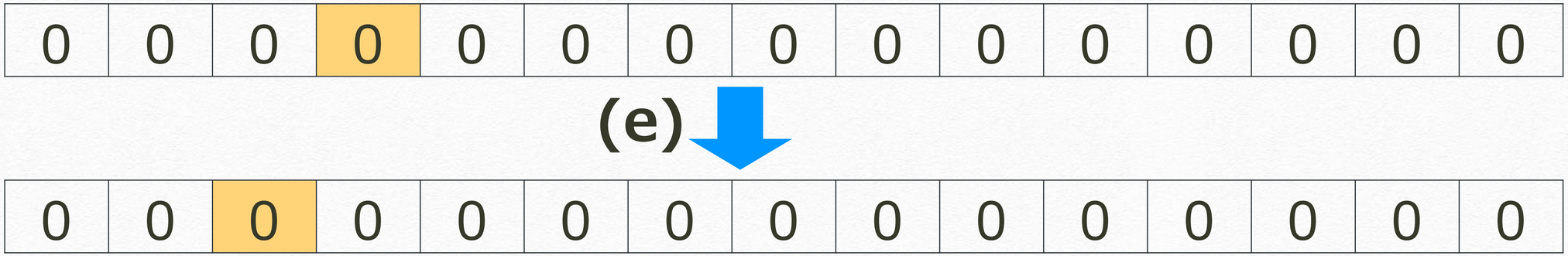
(b) Put 0 and move to the right.

(c) Repeatedly put 1 and move to the right until finding 1.

(d) Repeatedly put 0 and move to the left until finding 0.

(e) Move to the left.

(f) If the symbol is 1, then go to (a).
If the symbol is 0, then the tape is zero-cleared.



My zero-clear algorithm

(a) Put 1 and move to the right.

(b) Put 0 and move to the right.

(c) Repeatedly put 1 and move to the right until finding 1.

(d) Repeatedly put 0 and move to the left until finding 0.

(e) Move to the left.

(f) If the symbol is 1, then go to (a).
If the symbol is 0, then the tape is zero-cleared.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

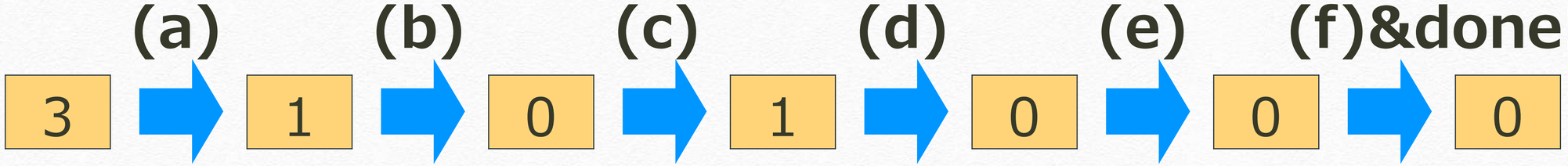
(f)& done!

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

My zero-clear algorithm

(a) Put 1 and move to the right.
(b) Put 0 and move to the right.
(c) Repeatedly put 1 and move to the right until finding 1.
(d) Repeatedly put 0 and move to the left until finding 0.
(e) Move to the left.
(f) If the symbol is 1, then go to (a).
If the symbol is 0, then the tape is zero-cleared.

- **It works even when the tape length is 1.**



Transition for Cyclic TM

(a) Put 1 and move to the right.
(b) Put 0 and move to the right.
(c) Repeatedly put 1
and move to the right until finding 1.
(d) Repeatedly put 0
and move to the left until finding 0.
(e) Move to the left.
(f) If the symbol is 1,
then go to (a).
If the symbol is 0,
then the tape is zero-cleared.

$$\delta(q_{in}, s) = (q_{pr}, 1, \rightarrow)$$
$$\delta(q_{pr}, s) = (q_{cl}, 0, \rightarrow)$$
$$\delta(q_{cl}, s) = \begin{cases} (q_{bk}, 0, \leftarrow) & s = 1 \\ (q_{cl}, 1, \rightarrow) & s \neq 1 \end{cases}$$
$$\delta(q_{bk}, s) = \begin{cases} (q_{ck}, 0, \leftarrow) & s = 0 \\ (q_{bk}, 0, \leftarrow) & s = 1 \end{cases}$$
$$\delta(q_{ck}, s) = \begin{cases} (q_{fn}, 0, \downarrow) & s = 0 \\ (q_{pr}, 1, \rightarrow) & s = 1 \end{cases}$$

q_{in} : Initial
q_{fn} : Final

Proving termination

Key lemma

Any tape t with $\#_1(t) \geq 2$ of the form

at state q_{cl}

goes to a tape t' with $\#_1(t') < \#_1(t)$ of the form

at state q_{cl}

where $\#_1(t)$ stands for the number of "1" in a tape t.