$(V, *, \circ)$ over $(F, +, \cdot)$

$a \in F$, $e_+ = 0$, $x \in V$, $e_* = e$ $\quad x * e = x$

$a + 0 = a$

$a \circ x = e$

let $a \neq 0$

$a \circ x = e$

$a^{-1} \circ (a \circ x) = a^{-1} \circ e$

$(a^{-1} \cdot a) \circ x = a^{-1} \circ e$ (associative w.r.t. scaler product)

$1 \circ x = a^{-1} \circ e$ (inverse)

$1 \circ x = e$ ($a \circ e = e$ Thm.)

$x = e$ ($1 \circ x = x$)

let $x \neq e$, suppose $a \neq 0$

$a \circ x = e$

$a^{-1} \circ (a \circ x) = a^{-1} \circ e$

$(a^{-1} \cdot a) \circ x = a^{-1} \circ e$ (associative w.r.t scaler product)

$1 \circ x = e$

$x = e$

#

$\therefore a = 0$

$\therefore a \circ x = e \Rightarrow a = 0$ or $x = e$