Problem 2

1)

TOA Radiation balance: Absorbed SW = Surface LW emission + Atmospheric LW emission

$\alpha\, j^* / 4 = \Gamma_a\, \sigma\, T_s^4 + (1 - \Gamma_a)\, \sigma\, T_a^4$

Surface Radiation balance: Absorbed SW = Surface LW emission – Atmospheric LW emission

$\alpha\, j^* / 4 = \sigma\, T_s^4 - (1 - \Gamma_a)\, \sigma\, T_a^4$

Therefore, $T_s = (\alpha\, j^* / (2\, \sigma\, (1 + \Gamma_a)))^{1/4} = (0.55 * 1366 / (2 * 5.67*10^{-8} * (1 + 0.15)))^{1/4} =$ 275.5 K

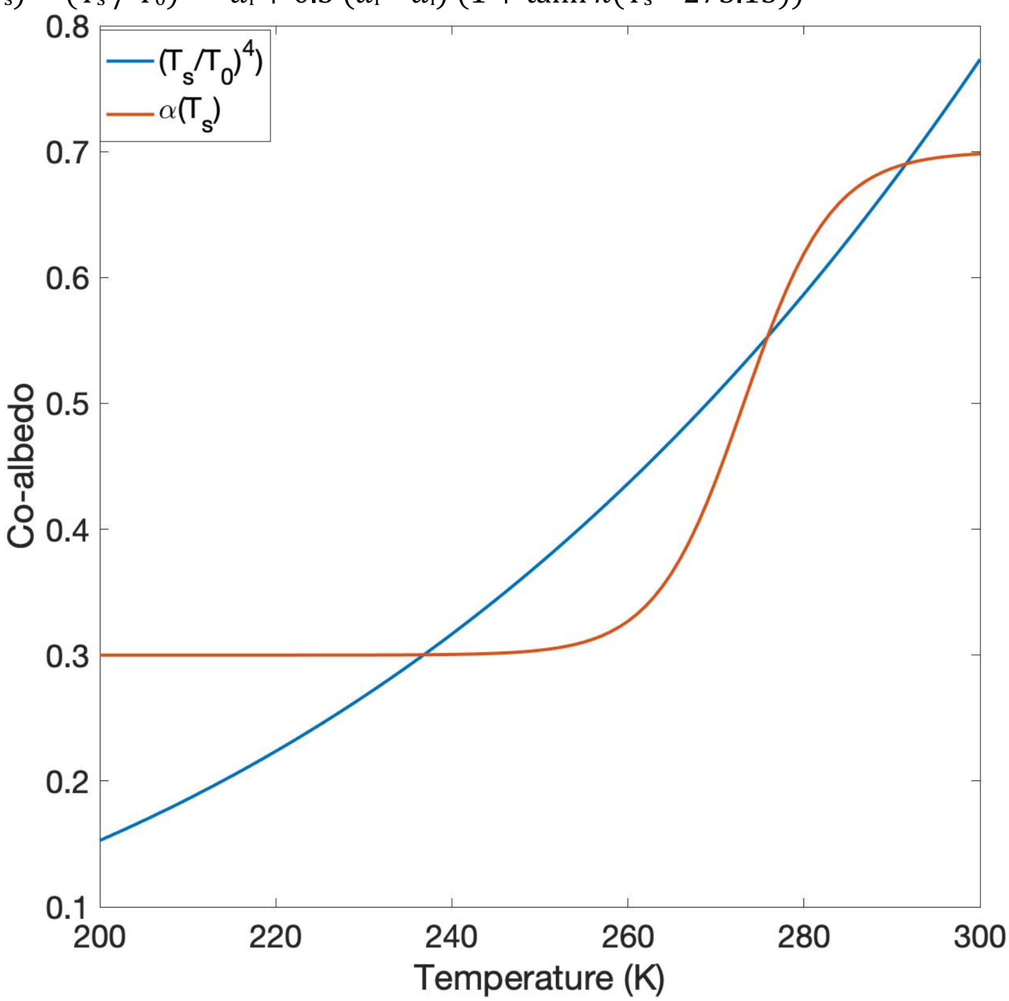
$T_a = (1/2)^{1/4}\, T_s = 231.7$ K

2)

$T_s = (\alpha(T_s)\, j^* / (2\, \sigma\, (1 + \Gamma_a)))^{1/4} = T_0\, \alpha(T_s)^{1/4}$

constant $T_0 = (j^* / (2\, \sigma\, (1 + \Gamma_a)))^{1/4} = 319.9$ K is the equilibrium surface temperature when $\alpha = 1$.

$\alpha(T_s) = (T_s / T_0)^4 = \alpha_i + 0.5\, (\alpha_f - \alpha_i)\, (1 + \tanh \kappa(T_s - 273.15))$



There are 3 solutions: $T_1 = 236.8$ K, $T_2 = 275.9$ K, and $T_3 = 291.6$ K.

The corresponding co-albedos are $\alpha_1 = 0.30$, $\alpha_2 = 0.55$, and $\alpha_3 = 0.69$.

3)

$T_s = T_0\, \alpha^{1/4}$, so the variation of surface temperature is $(1 \pm 5\%)^{1/4} - 1 \approx 1.25\%$.
When $\alpha > (T_s / T_0)^4$, surface temperature increases; when $\alpha < (T_s / T_0)^4$, surface temperature decreases.
Therefore, if α increases by 5%, then surface temperature increases from the equilibrium.
For T_1 and T_3, this temperature increase leads to $\alpha(T_s) < (T_s / T_0)^4$, so temperature decreases back to the original equilibrium.
For T_2, this temperature increase leads to $\alpha(T_s) > (T_s / T_0)^4$, so temperature will keep rising until reaching the new equilibrium T_3.
Similarly, if α decreases by 5%, equilibria T_1 and T_3 will go back to the starting point in the end while T_2 will shift to the new equilibrium T_1.
This implies that T_1 and T_3 are stable equilibria while T_2 is an unstable equilibrium.