MODULE 2.

TWO DIMENSIONAL RANDOM VARIABLES AND RANDOM PROCESSES.

- Two dimensional Random variables
- Joint distributions
- Marginal and conditional distributions
- Covariance
- Correlation and Regression.
- Classification of Random processes
- Definitions
- Strictly stationary processes
- Wide sense stationary processes
- Poisson Process
- Markov Process
- Markov chain.
- Transition Probabilities.

INTRODUCTION:

Two dimensional Random Variable:

Let s be the sample space. Let $x = x(s)$, and $y = y(s)$ be The Two functions each real number to each outcomes $s \in S$. Then (x, y) is a two dimensional random variable.

Types of 2D Random Variable:

Two types;

(i) 2D discrete random variable.

(ii) 2D continuous random variable

2D discrete Random variable.

If the possible values (x, y) are finite or countably infinite, then (x, y) is called 2D discrete random variable.

2D continuous Random variable:

If (x, y) can assume all values in a specified region R in the (x, y) plane.

<u>Joint Probability Distributions of 2D random variable x and y.</u>

We define the probability of the joint event $\{X \le x, Y \le y\}$, which is a function of a numbers x and y, by joint

<u>Properties of the Joint Distribution:</u>

* $F_{xy}(-\infty, \infty) = 0$
* $F_{xy}(-\infty, y) = 0$ and $F_{xy}(x, -\infty) = 0$.
* $0 \le F_{xy}(x, y) \le 1$
*

If x, y is a 2Dimensional discrete random variable, such that $P(X = x_i, Y = y_j) = P_{ij}$ is called the joint probability function (or) Joint Probability Mass Function of (x, y) providing the following conditions are satisfied.

(i) $P_{ij} \geq 0$

(ii) $\sum_{i} \sum_{y} P_{ij} = 1$ (Total probability is equal to 1).

Marginal probability mass function of x.

If the joint probability distribution of two random variables x and y is given, then the marginal probability function of x is given by .P.

Note:

Marginal Probability Mass Function of y:

If the joint probability distribution of two random variable x, and y is given then the marginal probability function of y is given by.

$$P_y(y_j) = P[y = y_j) =$$

NOTE:

The set $\{ y_j, P_{*j} \}$ is called the marginal distribution of y.

Conditional Probability Distribution:

CASE (i):

Probability of x_i given y_j is

$$P\left\{X = x_i / Y = y_j\right\} = \frac{P[X = x_i, Y = y_j]}{P[Y = y_j]}$$

CASE (ii):

The conditional probability of y_j given x_i is,

$$P\left\{Y = y_j / X = x_i\right\} = \frac{P[X = x_i, Y = y_j]}{P[X = x_i]}$$

Independent Random Variables

Two random variables x and y are said to be independent if.

- For discrete cases:

  $\Rightarrow P_{ij} = P_{i*} \cdot P_{*j}$

- For continuous Case:

  $\Rightarrow f(x, y) = f(x) \cdot f(y).$

PROBLEMS:

7) The joint probability mass function of x, y is given by $P(x, y) = K(2x + 3y)$, $x = 0, 1, 2$, & $y = 1, 2, 3$. Find all the marginal and conditional distributions. Also find the marginal distribution $(X+Y)$ and $P(X+Y > 3)$.

Solution:

$P(x,y) = K(2x+3y)$ $x = 0,1,2$ & $y = 1,2,3$

$x \backslash y$	1	2	3
0	$3K$ $P(0,1)$	$6K$ $P(0,2)$	$9K$ $P(0,3)$
1	$5K$ $P(1,1)$	$8K$ $P(1,2)$	$11K$ $P(1,3)$
2	$7K$ $P(2,1)$	$10K$ $P(2,2)$	$13K$ $P(2,3)$

(i) To find K:

WKT: $\sum_i \sum_j P_{ij} = 1$

$3K + 6K + 9K + 5K + 8K + 11K + 7K + 10K + 13K = 1.$

$72K = 1$

$\boxed{K = \frac{1}{72}}$

Sub $K = \frac{1}{72}$ in table:

	1	2	3	Marginal distribution of x.
0	$3/72$ $P(0,1)$	$6/72$ $P(0,2)$	$9/72$ $P(0,3)$	$P(X=0) = 18/72 = P_{0*}$ Sum of 1st row
1	$5/72$ $P(1,1)$	$8/72$ $P(1,2)$	$11/72$ $P(1,3)$	$P(X=1) = 24/72 = P_{1*}$
2	$7/72$ $P(2,1)$	$10/72$ $P(2,2)$	$13/72$ $P(2,3)$	$P(X=2) = 30/72 = P_{2*}$
Marginal distribution of Y.	$P(Y=1) = 15/72$ P_{*1} (sum of 1st column)	$P(Y=2) = 24/72 = P_{*2}$	$P(Y=3) = 33/72 = P_{*3}$	1

Marginal distribution of X:

$P(X=0) = \frac{18}{72}$, $P(X=1) = \frac{24}{72}$, $P(X=2) = \frac{30}{72}$.

Marginal distribution of y:

$P(y=1) = \frac{15}{72}$, $P(y=2) = \frac{24}{72}$, $P(y=3) = \frac{33}{72}$.

The conditional distribution of x is given y:

(ie) $P(X = x_i / y = y_i)$

CASE (i): given $\boxed{y=1}$

$P(x=0/y=1) = P(0,1)/P(y=1) = \frac{3/72}{15/72} = \frac{1}{5}$

$P(x=1/y=1) = P(1,1)/P(y=1) = \frac{5/72}{15/72} = 1/3$

$P(x=2/y=1) = P(2,1)/P(y=1) = \frac{7/72}{15/72} = 7/15$

CASE (ii) $\boxed{y=2}$

$P(x=0/y=2) = P(0,2)/P(y=2) = \frac{6/72}{24/72} = \frac{1}{4}$

$P(x=1/y=2) = P(1,2)/P(y=2) = \frac{8/72}{24/72} = \frac{1}{3}$

$P(x=2/y=2) = P(2,2)/P(y=2) = \frac{10/72}{24/72} = \frac{5}{12}$.

CASE (iii) $\boxed{y=3}$

$P(x=0/y=3) = P(0,3)/P(y=3) = \frac{9/72}{33/72} = 3/11$

$P(x=1/y=3) = P(1,3)/P(y=3) = \frac{11/72}{33/72} = 1/3$.

$P(x=2/y=3) = P(2,3)/P(y=3) = \frac{13/72}{33/72} = \frac{13}{33}$

$\because$ always $P(x,y) \longrightarrow$ 1st x value, 2nd y value

Conditional Distribution of y given x is

$$P[y = y_i / x = x_i]$$

CASE (i) given $\boxed{x=0}$:

$P(y=1/x=0) = P(0,1) / P(x=0) = \frac{3/72}{18/72} = 1/6$

$P(y=2/x=0) = P(0,2) / P(x=0) = \frac{6/72}{18/72} = 1/3$

$P(y=3/x=0) = P(0,3) / P(x=0) = \frac{9/72}{18/72} = 1/2$

CASE (ii) given $\boxed{x=1}$:

$P(y=1/x=1) = P(1,1)/P(x=1) = \frac{5/72}{24/72} = 5/24.$

$P(y=2/x=1) = P(1/2)/P(x=1) = \frac{8/72}{24/72} = 1/3.$

$P(y=3/x=1) = P(1,3)/P(x=1) = \frac{11/72}{24/72} = 11/24.$

CASE (iii) given $\boxed{x=2}$:

$P(y=1/x=2) = P(2,1)/P(x=2) = \frac{7/72}{30/72} = 7/30$

$P(y=2/x=2) = P(2,2)/P(x=2) = \frac{10/72}{30/72} = 1/3.$

$P(y=3/x=2) = P(2,3)/P(x=2) = \frac{13/72}{30/72} = 13/30.$

Probability Distribution of $x+y$:

$x+y$		Probability.
1	$P(0,1)$	$3/72$
2	$P(0,2) + P(1,1)$	$\frac{6}{72} + \frac{5}{72} = \frac{11}{72}$
3.	$P(0,3) + P(1,2)\ P(2,1)$	$\frac{9}{72} + \frac{8}{72} + \frac{7}{72} = \frac{24}{72}$

5	$P(2,3)$	$13/72$
	Total	1

$P(X+Y>3) = P(X+Y=4) + P(X+Y=5)$.

$= \frac{21}{72} + \frac{13}{72} = \boxed{\frac{34}{72}}$

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(2). The 2D R.V. (X,Y) has the JDF $f(x,y) = \frac{x+2y}{27}$

(ie) $\left(P(x,y) = \frac{x+2y}{27}\right)$. $x=0,1,2$. $y=0,1,2$. Find the conditional distribution of Y given X and X given Y and also find the marginal distribution function of X & Y.

Sol:

X / Y	0	1	2	Marginal distribution of X
0	0 $P(0,0)$	$2/27$ $P(0,1)$	$4/27$ $P(0,2)$	$P(X=0) = 6/27$
1	$1/27$ $P(1,0)$	$3/27$ $P(1,1)$	$5/27$ $P(1,2)$	$P(X=1) = 9/27$
2	$2/27$ $P(2,0)$	$4/27$ $P(2,1)$	$6/27$ $P(2,2)$	$P(X=2) = 12/27$
Marginal distribution of Y	$P(Y=0)$ $3/27$	$P(Y=1)$ $9/27$	$P(Y=2)$ $15/27$	1

Marginal distribution of x:

$P(X=0) = \frac{6}{27}$, $P(X=1) = \frac{9}{27}$, $P(X=2) = \frac{12}{27}$.

marginal distribution of y:

$P(Y=0) = \frac{3}{27}$, $P(Y=1) = \frac{9}{27}$, $P(Y=2) = \frac{15}{27}$.

The conditional distribution of x and y when x given y is

$P(X = x_i / Y = y_i)$

CASE (i): $y=0$, $x=0,1,2$

$P(X=0/Y=0) = \dfrac{P(0,0)}{P(y=0)} = 0$

$P(X=1/Y=0) = \dfrac{P(1,0)}{P(y=0)} = \dfrac{1}{27} \times \dfrac{27}{3} = \dfrac{1}{3}$

$P(X=2/Y=0) = \dfrac{P(2,0)}{P(y=0)} = \dfrac{2}{27} \times \dfrac{27}{3} = \dfrac{2}{3}.$

case (ii) $Y=1, \; X=0,1,2.$

$P(X=0/Y=1) = \dfrac{P(0,1)}{P(y=1)} = \dfrac{2}{27} \times \dfrac{27}{9} = \dfrac{2}{9}$

$P(X=1/Y=1) = \dfrac{P(1,1)}{P(y=1)} = \dfrac{3}{27} \times \dfrac{27}{9} = \dfrac{1}{3}$

$P(X=2/Y=1) = \dfrac{P(2,1)}{P(Y=1)} = \dfrac{4}{27} \times \dfrac{27}{9} = \dfrac{4}{9}$

case (iii): $Y=2, \; X=0,1,2.$

$P(X=0/Y=2) = \dfrac{P(0,2)}{P(y=2)} = \dfrac{4}{27} \times \dfrac{27}{15} = \dfrac{4}{15}$

$P(X=1/Y=2) = \dfrac{P(1,2)}{P(Y=2)} = \dfrac{5}{27} \times \dfrac{27}{15} = \dfrac{1}{3}$

$P(X=2/Y=2) = \dfrac{P(2,2)}{P(Y=2)} = \dfrac{6}{27} \times \dfrac{27}{15} \Rightarrow \dfrac{2}{5}$

The conditional distribution of y given x is

$$P(Y=y_i / X=x_i) = \frac{P[x_i, y_j]}{P[X=x_i]}.$$

case (i): $X=0.$

$P(Y=0/X=0) = \dfrac{P(0,0)}{P(X=0)} = 0$

$P(Y=1/X=0) = \dfrac{P(0,1)}{P(X=0)} = \dfrac{2}{27} \times \dfrac{27}{6} = \dfrac{1}{3}$

$P(Y=2/X=0) = \dfrac{P(0,2)}{P(X=0)} = \dfrac{4}{27} \times \dfrac{27}{6} = \dfrac{2}{3}$

case (ii): $X=1$

$P(X=1/Y=0) = 1/9, \quad P(X=1/Y=1) = 3/27 / 9/27 = 1/3$

$P(X=2/Y=2) = 5/27 / 9/27 = 5/9.$

case (iii) $X=2,$

$P(X=2/Y=0) = 2/27 / 12/27 = 1/6$

$P(X=2/Y=1) = 4/27 / 12/27 = 1/3.$

$P(X=2/Y=2) = 6/27 / 12/27 = 1/2$