An Investigation on the Relationship Between Mass and Terminal Velocity

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February 10, 2020

February 19, 2020

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Abstract

This investigation explored the relationship between terminal velocity and mass. This was done by dropping stacks of coffee filters with their masses varied by the size of the stacks (between 1-10 coffee filters) and measuring their terminal velocity. It was found that the terminal velocity (v_{term}) increased with the square root of mass ($\sqrt{M}$) and that the two quantities were related by the equation $v_{term} = k\sqrt{M}$ where the constant $k = 1.06$ m/s/g$^{1/2}$.

Purpose

The purpose of the experiment was to determine the relationship between terminal velocity and mass for an object dropped from rest at a fixed height.

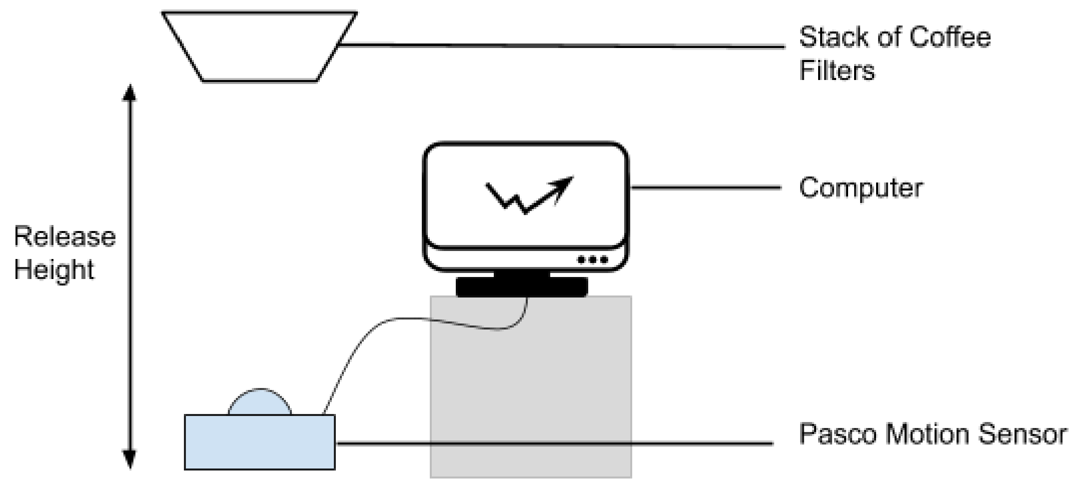
Hypothesis

I hypothesize that the terminal velocity of the coffee filters increases with mass, since the force of air resistance will increase at a slower rate with mass compared to the force of gravity.

Materials

- 10 coffee filters
- Meter stick
- Pasco Pasport motion sensor
- Computer with DataStudio software installed
- Mass balance (± 0.01 g)

Figure 1: Diagram of experimental setup



Procedure

1. The sensor was put into cart mode and placed on the ground facing directly up as shown in Figure 1.
2. The stack of 10 coffee filters was weighed on the mass balance, and its mass in grams was recorded.
3. The stack of coffee filters was released at a height of approximately 2 meters over the motion sensor—the meter stick was used to roughly determine the release height. As the coffee filters fell, its position over time was plotted in DataStudio. Linear regression was performed on the straight section of the generated position-time graph to determine a slope that represented the object's terminal velocity. This was repeated for three trials.
4. A stack of one less coffee filter was weighed, released from the same height, and its terminal velocity measured as described in steps 2-3. This was done until there was only one coffee filter being released.

<u>Data</u>

The data obtained in the experiment is summarized below in Table 1. Note that the average terminal velocity is the arithmetic mean of the three terminal velocities measured for a particular mass. A sample calculation for the average terminal velocity when the mass is 9.08 g (first row of table) is shown below:

$$v_{avg} = (v_1 + v_2 + v_3) / 3$$
$$v_{avg} = (3.37 + 3.19 + 3.12) / 3$$
$$v_{avg} = 3.23 \text{ m/s}$$

The uncertainty of each average terminal velocity value is calculated using the expression, $\pm(v_{greatest} - v_{least}) / 2$, where $v_{greatest}$ is the greatest terminal velocity measured for a particular mass and v_{least} is the lowest terminal velocity measured for that mass. A sample calculation for the uncertainty of the first average terminal velocity value is shown below:

$$\text{uncertainty} = \pm (v_{greatest} - v_{least}) / 2$$
$$\text{uncertainty} = \pm (3.37 - 3.12) / 2$$
$$\text{uncertainty} = \pm 0.125 \text{ m/s}$$

Thus the uncertainty is ± 0.125 m/s.

The overall uncertainty for average terminal velocity is the arithmetic mean of all the uncertainties for every average terminal velocity value. This was determined to be ± 0.075 m/s as shown below:

$$\text{overall uncertainty} = \pm\ (0.125 + 0.04 + 0.195 + 0.08 + 0.03 + 0.105 + 0.025 + 0.055 + 0.055 + 0.04) / 10$$
$$= \pm\ 0.08 \text{ m/s}$$

Table 1: Terminal velocity for stacks of coffee filters with varying masses

Number of Coffee Filtlers	Mass in g (± 0.01 g)	Terminal Velocity (m/s)			
		Trial 1	Trial 2	Trial 3	Average (± 0.08 m/s)
10	9.08	3.37	3.19	3.12	3.23
9	8.17	3.27	3.23	3.19	3.23
8	7.26	2.75	3.04	3.14	2.98
7	6.37	2.93	3.04	2.88	2.95
6	5.47	2.77	2.72	2.78	2.76
5	4.54	2.44	2.59	2.65	2.56
4	3.63	2.36	2.32	2.31	2.33
3	2.71	2.05	1.98	1.94	1.99
2	1.80	1.67	1.69	1.58	1.65
1	0.89	1.22	1.14	1.15	1.17

Analysis and Discussion

Table 1 shows that terminal velocity decreased as the mass decreased. A plot of the average terminal velocity versus mass from the data in Table 1 is drawn in Graph 1. The shape of this graph (rising with decreasing slope) suggests that the terminal velocity is some root of the mass.

Further analysis of the data collected in Table 1 can be done to determine the specific relationship between terminal velocity and mass. Table 2 (on next page) shows the values of average terminal velocity and the square root mass as obtained from Table 1.

Table 2: Terminal Velocity and Square Root of Mass for Various Released Coffee Filter Stacks

Number of Coffee Filters	Mass in g (± 0.01 g)	Square Root of Mass (g)	Average Terminal Velocity in m/s (± 0.06 m/s)
10	9.08	3.01	3.23
9	8.17	2.86	3.23
8	7.26	2.69	2.98
7	6.37	2.52	2.95
6	5.47	2.34	2.76
5	4.54	2.13	2.56
4	3.63	1.91	2.33
3	2.71	1.65	1.99
2	1.80	1.34	1.65
1	0.89	0.94	1.17

A plot of the terminal velocity versus the square root of mass is shown in Graph 2. Graph 2 shows a linear relationship, though there are outliers in the top right corner. The points coloured red lie significantly below the line of best fit. The rightmost of these two points could be explained by there not being enough distance for the 10 coffee filters to reach its terminal velocity. Excluding these points, the linear relationship could be represented by the equation $v = k\sqrt{m}$, where k is the slope of the linear plot in Graph 3. The calculation of the slope of the best fit line of this plot is shown below:

$$\begin{aligned} m &= (y_2 - y_1) / (x_2 - x_1) \\ &= (3.30 - 1.30) / (2.88 - 1.00) \\ &= (2.00) / (1.88) \\ &= 1.06 \text{ m/s/g}^{1/2} \end{aligned}$$

The slope being 1.06 m/s/$g^{1/2}$ shows that the equation relating terminal velocity (v_{term}) and mass (M) is $v_{term} = 1.06 \text{ m/s/g}^{1/2} \sqrt{M}$.

Conclusion

Our investigation shows that the mass of an object affects its terminal velocity. Terminal velocity increases with mass, consistent with my hypothesis. Moreover, the data shows that the terminal velocity (v_{term}) is related to an object's mass (M) by the equation $v_{term} = k \sqrt{M}$ where the constant k is 1.06 m/s/$g^{1/2}$. We have a high level of confidence that an object's terminal velocity is proportional to the root of its mass since the linear relationship between terminal velocity and the root of mass is strong, i.e., when the relationship is graphed, the points fall close to the best fit line. For this same reason, we believe the precision of our proportionality constant, k, is good; however, some uncertainty has likely arisen from drawing the best fit line of the v_{term} versus $M^{1/2}$ graph by hand rather than computing the equation of that line from the given data points. Future experiments should consider the use of a computer algebra system to find the constant of proportionality, k.

A factor that may have affected the validity of these results was that the coffee filters were dropped from a height of 2 meters. This may not have been a sufficient height for the dropped coffee filters to reach terminal velocity. This error would affect larger stacks of coffee filters in particular since they take a greater distance to reach terminal velocity. This error could be minimized by dropping the filters from a greater height of 3 or 4 meters. Another issue was that the motion sensor was unable to capture the full motion profile of the coffee filters as they was dropped. Often it is only the last part of the motion that was captured. This prevented us from accurately assessing the part of the motion graph that represented the object falling at terminal velocity. This could be remedied by getting higher quality motion detecting equipment.