Q1

1. The following table, provided by Dr. Philip Israelovich of the Federal Reserve Bank, gives the information on capital, labor, and value added of the economics of transportation equipment. (Ashish Sen, and Muni Srivastava, *Regression Analysis*)

Year	Capital	Labor	Value Added
72	1209188	1259142	11150.0
73	1330372	1371795	12853.6
74	1157371	1263084	10450.8
75	1070860	1118226	9318.3
76	1233475	1274345	12097.7
77	1355769	1369877	12844.8
78	1351667	1451595	13309.9
79	1326248	1328683	13402.3
80	1089545	1077207	8571.0
81	1111942	1056231	8739.7
82	988165	947502	8140.0
83	1069651	1057159	10958.4
84	1191677	1169442	10838.9
85	1246536	1195255	10030.5
86	1281262	1171664	10836.5

a. (5%) Consider the model

$$V_t = \alpha K_t^{\beta_1} L_t^{\beta_2} \eta_t ,$$

where the subscript t indicates the year, V_t is value added, K_t is capital, L_t is labor, and η_t is the error term, with $E[\log(\eta_t)] = 0$ and $var[\log(\eta_t)]$ a constant. Assuming the errors are independent across the years, estimate β_1 and β_2.

b. (10%) The model in (a) is said to be of the Cobb-Douglas form. It is easier to interpret if $\beta_1 + \beta_2 = 1$. Estimate β_1 and β_2 under this constraint.

Answer

(a)

根據我們所建立的regression model (詳細資訊如下所示)，我們可以評估模型下的B1約為4.561e-07 ；B2則為6.916e-07

(b)

When B1 + B2 = 1, we estimate B1 = 3.749765918236159 and B2= -2.749765918236159

In [113]:

```
# import packages needed
from sklearn.metrics import mean_squared_error
import pandas as pd
import numpy as np
import statsmodels.api as sm
import scipy
```

In [111]:

```
# prepare the data
# create original dataframe
data_FRB = pd.DataFrame({
    'Year':[i for i in range(72, 87, 1)],
    'Capital':[1209188, 1330372, 1157371, 1070860, 1233475, 1355769, 1351667, 1326248, 1089545, 1111942,
            988165, 1069651, 1191677, 1246536, 1281262],
    'Labor':[1259142, 1371795, 1263084, 1118226, 1274345, 1369877, 1451595, 1328683, 1077207, 1056231, 947502,
           1057159, 1169442, 1195255, 1171664],
    'Value_Added':[11150.0, 12853.6, 10450.8, 9318.3, 12097.7, 12844.8, 13309.9, 13402.3, 8571.0, 8739.7,
             8140.0, 10958.4, 10838.9, 10030.5, 10836.5]
})

# append 'log_Value_Added' as Value_added with log function applied
data_FRB.loc[:, 'log_Value_Added'] = np.log(data_FRB.loc[:, 'Value_Added'])

# divide data into X and y
X = data_FRB.iloc[:, 1:-2]
y = data_FRB.iloc[:, -1]
```

In [112]:

```
# train with multiple regression
```

```
# create a statsmodel
# create statsmodels model
X = sm.add_constant(X)
model = sm.regression.linear_model.OLS(y, X).fit()

print(model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:        log_Value_Added   R-squared:                       0.823
Model:                            OLS   Adj. R-squared:                  0.794
Method:                 Least Squares   F-statistic:                     27.91
Date:                Thu, 09 Apr 2020   Prob (F-statistic):           3.07e-05
Time:                        21:02:00   Log-Likelihood:                 19.449
No. Observations:                  15   AIC:                            -32.90
Df Residuals:                      12   BIC:                            -30.77
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          7.9019      0.208     37.941      0.000       7.448       8.356
Capital     4.561e-07   4.13e-07      1.106      0.291   -4.43e-07    1.35e-06
Labor       6.916e-07   3.41e-07      2.028      0.065   -5.16e-08    1.43e-06
==============================================================================
Omnibus:                        7.893   Durbin-Watson:                   2.002
Prob(Omnibus):                  0.019   Jarque-Bera (JB):                4.398
Skew:                           1.140   Prob(JB):                        0.111
Kurtosis:                       4.356   Cond. No.                     1.87e+07
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.87e+07. This might indicate that there are
strong multicollinearity or other numerical problems.
```

```
C:\Users\ricardo\Anaconda3\lib\site-packages\scipy\stats\stats.py:1535: UserWarning: kurtosistest only valid for n>=20 ... continuing anyway, n=15
  "anyway, n=%i" % int(n))
```

In [152]:

```
def sm_model(B1):
    return np.sum( ((X_1 * B1 + X_2 * B2) - y) ** 2 )

sm_model = sm_model(1)
```

In [165]:

```
y = data_FRB.iloc[:, -1].to_numpy()
X_1 = data_FRB.iloc[:, 1].to_numpy()
X_2 = data_FRB.iloc[:, 2].to_numpy()

def sm_model(B1):
    return np.sum( ((X_1 * B1 + X_2 * (1-B1)) - y) ** 2 )

# optimize the parameter with scipy
res = scipy.optimize.minimize(sm_model, x0=[0])
print('When B1 + B2 = 1, we estimate B1 = ', res.x[0], 'and B2=', (1- res.x[0]))
```

```
When B1 + B2 = 1, we estimate B1 =  3.749765918236159 and B2= -2.749765918236159
```

Q2

2. Find the proper libraries/packages in your coding environment to perform the LASSO and Ridge regressions on the ORL face dataset (use the same gender labels created in HW03).
   a. (10%) Select the lambda associated with the minimal MSE fit and compare the results with that of your stepwise regression in HW03.
   b. (5%) Plot the chosen pixels from LASSO regression on a 46×56 canvas.

Answer

(a) 根據我的程式碼，Lambda在越小時，MSE同有下降的趨勢，並在Lambda=0.0時達到最小，MSE=1.8042175649346816e-08；使用stepwise regression得到MSE=0.014399127818860755為；相比較發現LASSO regression方法的預測效率較佳

(b) 如本題最下方程式碼所產生的圖所示。

In [1]:

```
# import packages needed
from sklearn.metrics import mean_squared_error
from sklearn import linear_model
import cv2
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import statsmodels.api as sm
```

In [2]:

```
# prepare the data
ORL_data = pd.read_csv('./data/ORL_data.csv')
# divide into X and y
X = ORL_data.iloc[:, :-1]
y = ORL_data.iloc[:, -1].tolist()
```

In [6]:

```
# implement with Lasso regression (with sklearn package)

lambda_list = np.arange(0, 10.1, 0.1)
mse_list = []

for i, j in enumerate(lambda_list):
    reg_lasso = linear_model.Lasso(alpha=j)
    reg_lasso.fit(X, y)
    predict_list_original = reg_lasso.predict(X).tolist()
    mse_list.append(mean_squared_error(y, predict_list_original))

minimum_value = np.min(mse_list)
minimum_index_list = [i for i, j in enumerate(mse_list) if j == minimum_value]
# print(mse_list)
# print(minimum_index_list)
print('MSE is lowest when lambda=', lambda_list[minimum_index_list[0]],
      ', while MSE is ', mse_list[minimum_index_list[0]])
```

```
C:\Users\ricardo\AppData\Roaming\Python\Python37\site-packages\ipykernel_launcher.py:8: UserWarning: With alpha=0, this algorit
hm does not converge well. You are advised to use the LinearRegression estimator

C:\Users\ricardo\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_descent.py:476: UserWarning: Coordinate descent w
ith no regularization may lead to unexpected results and is discouraged.
  positive)
C:\Users\ricardo\Anaconda3\lib\site-packages\sklearn\linear_model\_coordinate_descent.py:476: ConvergenceWarning: Objective did
not converge. You might want to increase the number of iterations. Duality gap: 0.025962355545366744, tolerance: 0.004375
  positive)
```

```
MSE is lowest when lambda= 0.0 , while MSE is  1.8042175649346816e-08
```

In [7]:

```
# implement with Ridge regression (with sklearn package)
lambda_list = np.arange(0, 10.1, 0.1)
mse_list = []

for i, j in enumerate(lambda_list):
    reg_ridge = linear_model.Ridge(alpha=j)
    reg_ridge.fit(X, y)
    predict_list_original = reg_ridge.predict(X).tolist()
    mse_list.append(mean_squared_error(y, predict_list_original))
```

```
minimum_value = np.min(mse_list)
minimum_index_list = [i for i, j in enumerate(mse_list) if j == minimum_value]
# print(mse_list)
# print(minimum_index_list)
print('MSE is lowest when lambda=', lambda_list[minimum_index_list[0]],
      ', while MSE is ', mse_list[minimum_index_list[0]])
```

```
C:\Users\ricardo\Anaconda3\lib\site-packages\sklearn\linear_model\_ridge.py:188: LinAlgWarning: Ill-conditioned matrix (rcond=6.203
87e-19): result may not be accurate.
  overwrite_a=False)
```

```
MSE is lowest when lambda= 0.0 , while MSE is  3.033838863451482e-29
```

In [11]:

```
# To compare with stepwise regression, we select LASSO regression with lambda = 0, 0.001, and 1
# We use statsmodels package this time.
# Reference:
# Documentation of penalized (regularized) regression in statsmodels (Elastic Net)
# https://www.statsmodels.org/dev/generated/statsmodels.regression.linear_model.OLS.fit_regularized.html

lambda_list = [0, 0.001, 1]
L1_wt = 1 # 1 when LASSO fit, 0 when ridge fit

# create statsmodels model
model = sm.regression.linear_model.OLS(y, X)

for i, j in enumerate(lambda_list):
    result_LASSO = model.fit_regularized(L1_wt=L1_wt, alpha=float(j))
    predict_list_original = result_LASSO.fittedvalues.tolist()
    mse_value = mean_squared_error(y, predict_list_original)
    print('MSE when lambda=', j,
      'is: ', mse_value)
```

```
MSE when lambda= 0 is:  0.03492422008163612
MSE when lambda= 0.001 is:  0.034903909423569794
MSE when lambda= 1 is:  0.06668417375325184
```

In [14]:

```
# create stepwise regression function

def stepwise_selection(X, y, initial_list=[], threshold_in=0.01, threshold_out=0.05, verbose=True):
    """ Perform a forward-backward feature selection
    Reference: # https://www.twblogs.net/a/5c13a86fbd9eee5e40bb7431
    based on p-value from statsmodels.api.OLS
    Arguments:
        X - pandas.DataFrame with candidate features
        y - list-like with the target
        initial_list - list of features to start with (column names of X)
        threshold_in - include a feature if its p-value < threshold_in
        threshold_out - exclude a feature if its p-value > threshold_out
        verbose - whether to print the sequence of inclusions and exclusions
    Returns: list of selected features
    Always set threshold_in < threshold_out to avoid infinite looping.
    """
    included = list(initial_list)

    while True:
        changed = False
        # forward step
        excluded = list(set(X.columns) - set(included))
        new_pval = pd.Series(index=excluded)
        for new_column in excluded:
            model = sm.OLS(y, sm.add_constant(pd.DataFrame(X[included + [new_column]]))).fit()
            new_pval[new_column] = model.pvalues[new_column]
        best_pval = new_pval.min()
        if best_pval < threshold_in:
            best_feature = new_pval.idxmin()
            included.append(best_feature)
            changed = True
            if verbose:
                print('Add  {:30} with p-value {:.6}'.format(best_feature, best_pval))
```

```
        # backward step
        model = sm.OLS(y, sm.add_constant(pd.DataFrame(X[included].values))).fit()
        print('R_square = ', model.rsquared)
        # use all coefs except intercept
        pvalues = model.pvalues.iloc[1:]
        worst_pval = pvalues.max() # null if pvalues is empty
        if worst_pval > threshold_out:
            changed = True
            worst_feature = pvalues.idxmax()
            included.remove(included[worst_feature])
            if verbose:
                print('Drop {:30} with p-value {:.6}'.format(worst_feature, worst_pval))
        if not changed:
            break
    return included
```

In [ ]:

```
# implement stepwise regression model
included_features = stepwise_selection(X, y)
print('resulting features:')
print(included_features)
print('Number of included feincluded_featurees is: ', len(included_features))
```

```
C:\Users\ricardo\AppData\Roaming\Python\Python37\site-packages\ipykernel_launcher.py:23: DeprecationWarning: The default dtyp
e for empty Series will be 'object' instead of 'float64' in a future version. Specify a dtype explicitly to silence this warning.
```

```
Add  pixel_1470                     with p-value 3.91841e-14
R_square =  0.13404309865692476
Add  pixel_2488                     with p-value 3.38413e-16
R_square =  0.2678723779962152
Add  pixel_2271                     with p-value 6.02768e-11
R_square =  0.34298316156352904
Add  pixel_1722                     with p-value 7.004e-08
R_square =  0.38964653438298524
Add  pixel_555                      with p-value 1.31884e-07
R_square =  0.4313340061910136
Add  pixel_1474                     with p-value 5.65748e-06
R_square =  0.46041103180240095
Add  pixel_1386                     with p-value 8.20976e-06
R_square =  0.48713763178062175
Add  pixel_1476                     with p-value 6.706e-07
R_square =  0.5185697594190422
Add  pixel_1051                     with p-value 6.99533e-06
R_square =  0.5428944164631441
Add  pixel_1561                     with p-value 1.92879e-05
R_square =  0.5638799029382524
Add  pixel_202                      with p-value 1.4955e-05
R_square =  0.5844727398029121
Add  pixel_133                      with p-value 1.23609e-05
R_square =  0.6045148272306424
Add  pixel_186                      with p-value 6.96418e-12
R_square =  0.6499422370848498
Add  pixel_1687                     with p-value 3.21537e-06
R_square =  0.6691369901454154
Add  pixel_319                      with p-value 3.01682e-06
R_square =  0.6874248986626645
Add  pixel_2393                     with p-value 7.13528e-05
R_square =  0.7000535030407766
Add  pixel_1984                     with p-value 3.73409e-06
R_square =  0.7164130615202478
Add  pixel_1325                     with p-value 6.84276e-05
R_square =  0.7279861021848284
Add  pixel_343                      with p-value 9.94106e-05
R_square =  0.7386297350875746
Add  pixel_325                      with p-value 0.000562402
R_square =  0.7467169625438768
Add  pixel_1730                     with p-value 0.000406945
R_square =  0.7549664661083192
Add  pixel_546                      with p-value 0.000298487
R_square =  0.7633328875127459
Add  pixel_380                      with p-value 4.57692e-05
R_square =  0.7735769523121608
Add  pixel_183                      with p-value 0.000308109
```

```
R_square =  0.7813138125011371
Add  pixel_95                      with p-value 3.63205e-05
R_square =  0.7910747573914318
Drop                               12 with p-value 0.606526
Add  pixel_1180                    with p-value 3.86642e-05
R_square =  0.8001947167154811
Add  pixel_2158                    with p-value 9.85121e-05
R_square =  0.8081656771085574
Add  pixel_2208                    with p-value 3.67526e-05
R_square =  0.8167620724289808
Add  pixel_2168                    with p-value 0.000133239
R_square =  0.8238401408803954
Add  pixel_1935                    with p-value 0.000485927
R_square =  0.8295471736784147
Add  pixel_2117                    with p-value 0.000222031
R_square =  0.8357387833553851
Add  pixel_430                     with p-value 0.000142728
R_square =  0.8420789131635588
Drop                               17 with p-value 0.0709408
Add  pixel_2516                    with p-value 0.000718295
R_square =  0.8455552264931512
Add  pixel_1260                    with p-value 0.00163652
R_square =  0.8496787374970376
Add  pixel_1809                    with p-value 0.00187537
R_square =  0.853602684916661
Add  pixel_1565                    with p-value 0.000980758
R_square =  0.8579015069074752
Add  pixel_845                     with p-value 0.00346772
R_square =  0.861202327963152
Add  pixel_1366                    with p-value 0.00615045
R_square =  0.8640466240370939
Add  pixel_414                     with p-value 0.00446665
R_square =  0.8670529819217331
Add  pixel_2121                    with p-value 0.00397132
R_square =  0.8700777953085174
Add  pixel_1936                    with p-value 0.00453467
R_square =  0.8729569119610687
```

In [9]:

```
# create new_dataset with feature selected
included_features = ['pixel_1470', 'pixel_2488', 'pixel_2271', 'pixel_1722', 'pixel_555', 'pixel_1474', \
 'pixel_1386', 'pixel_1476', 'pixel_1051', 'pixel_1561', 'pixel_202', 'pixel_133',  \
 'pixel_1687', 'pixel_319', 'pixel_2393', 'pixel_1984', 'pixel_1325', 'pixel_325', 'pixel_1730', \
 'pixel_546', 'pixel_380', 'pixel_183', 'pixel_95', 'pixel_1180', 'pixel_2158', 'pixel_2208', 'pixel_2168', \
 'pixel_1935', 'pixel_2117', 'pixel_430', 'pixel_2516', 'pixel_1260 ', 'pixel_1809', 'pixel_1565', 'pixel_845', \
 'pixel_1366', 'pixel_414', 'pixel_2121', 'pixel_1936']


# prepare the data
X = ORL_data[ORL_data.columns & included_features]
# included_featuresde into X and y
y = ORL_data.iloc[:, -1].tolist()
# calculate mse with features selected by stepwise regression
reg = linear_model.LinearRegression()
reg.fit(X, y)
predict_list = reg.predict(X)
mse_value = mean_squared_error(y, predict_list)
print('MSE=', mse_value)
```

```
MSE= 0.014399127818860755
```

In [13]:

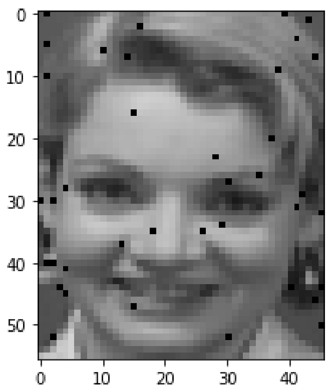
```
# Implement LASSO model when setting lambda=1
result_LASSO = model.fit_regularized(L1_wt=1, alpha=1)
predict_list_original = result_LASSO.fittedvalues.tolist()
mse_value = mean_squared_error(y, predict_list_original)
print('MSE when lambda=', j, 'is: ', mse_value)

# interpret the result of LASSO
param_data_LASSO = pd.DataFrame({
    'Pixel':result_LASSO.params.index.tolist(),
    'Params':result_LASSO.params.values.tolist()
```

```
})

param_data_LASSO = param_data_LASSO.sort_values(by=['Params'], ascending=False)
chosen_vars_LASSO = param_data_LASSO.iloc[0:54, 0].values.tolist()
```

MSE when lambda= 1 is: 0.06668417375325184

In [14]:

```
# Interpret and plot the result
pixel_list_number = []
pixel_list_x = []
pixel_list_y = []
# transform back to pixel
for i, j in enumerate(chosen_vars_LASSO):
    value = int(j[6:])
    row = value / 46 - 1
    column = value % 46 - 1
    pixel_list_number.append(value)
    pixel_list_x.append(row)
    pixel_list_y.append(column)
    file = "./data/ORL_Faces_Classified/0/1_1.png"
    sample = cv2.imread(file, cv2.IMREAD_GRAYSCALE)
    for i, j in enumerate(pixel_list_x):
        x_value = int(j - 1)
        y_value = int(pixel_list_y[i] - 1)
        sample[x_value, y_value] = 0
# cv2.imwrite( "./sample.png", sample)
imgplot = plt.imshow(sample, cmap='gray', vmin=0, vmax=255)
plt.show()
```



Q3

3. (15%) Code a PCA function in R/Python without using the available packages/libraries. The input parameters of this function are the data matrix **X** and a Boolean flag "isCorrMX." The Boolean flag allows the user to choose if the correlation matrix is used when set TRUE, otherwise, the covariance matrix would be decomposed. You can start with the function of Spectral Decomposition or Singular Value Decomposition. Necessary outputs are:
   a. the loading matrix;
   b. the eigenvalue value vector;
   c. the score matrix, i.e., the matrix of principal components;
   d. the scree plot where eigenvalues are shown as bars and cumulative variance explained is drawn as a line (similar to the one on p. 33 of DA04).

   (5%) Demonstrate your PCA function using the AutoMPG dataset. By comparing the results of "isCorrMX == TRUE" and "isCorrMX == FALSE", do you think PCA is scale-invariant?

   *Directly applying the existed PCA library/package in your function gets no points in this exercise.

Answer

(a)

如下方程式碼所示

(b)

我們在做PCA的時候，是找向量來maximize variance，但我們的X只有做center沒有做標準化，所以會被各個variable的scale影響，所以是 scale variant 的，其受 scale 影響的情況並可從下方程式馬所output的圖形得知。

In [30]:

```
from sklearn.decomposition import PCA
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
```

In [105]:

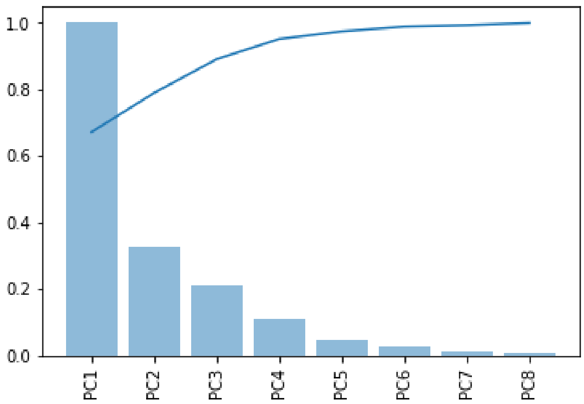
```
def PCA_decomposition(X, isCorrMX):
    # Reference: https://machinelearningmastery.com/calculate-principal-component-analysis-scratch-python/
    # calculate mean of the array
    X_mean = np.mean(X.T, axis=1)
    # center the values
    X_center = X - X_mean
    # check if use Correlation Matrix
    if isCorrMX == True:
        X_covariance = np.corrcoef(X_center.T)
    else:
        X_covariance = np.cov(X_center.T)
    # calculate eigenvalues and eigenvectors
    # https://docs.scipy.org/doc/numpy/reference/generated/numpy.linalg.eig.html
    eigenvalues, eigenvectors = np.linalg.eig(X_covariance)
    X_project = -(eigenvectors.T.dot(X_center.T).T)

    # calculate explainable ratio of each pricipal components
    explainable_ratio_list = []
    for i in range(eigenvalues.shape[0]):
        eigenvalues_totals = np.sum(eigenvalues)
        ratio = eigenvalues[i] / eigenvalues_totals
        explainable_ratio_list.append(ratio)

    # calculate how many principal components needed to explain 50, 60, 70, 80, 90% of total variance
    total_explainable_ratio = 0
    total_explainable_ratio_list = []
    culmulative_explainable_ratio_list = []
    components_needed_list = []
    for i, j in enumerate(explainable_ratio_list):
        total_explainable_ratio += j
        total_explainable_ratio_list.append(total_explainable_ratio)
        culmulative_explainable_ratio_list.append(np.sum(explainable_ratio_list[(i):]))

    total_explainable_ratio = 0
    for i, j in enumerate(explainable_ratio_list):
        total_explainable_ratio += j
        if total_explainable_ratio >= 0.9:
            if len(components_needed_list) < 5:
                components_needed_list.append(i + 1)
                print('90%:', str(i + 1), 'pricipal components needed.')
            break
        elif total_explainable_ratio >= 0.8:
            if len(components_needed_list) < 4:
                components_needed_list.append(i + 1)
                print('80%:', str(i + 1), 'pricipal components needed.')
        elif total_explainable_ratio >= 0.7:
            if len(components_needed_list) < 3:
                components_needed_list.append(i + 1)
                print('70%:', str(i + 1), 'pricipal components needed.')
        elif total_explainable_ratio >= 0.6:
            if len(components_needed_list) < 2:
                components_needed_list.append(i + 1)
                print('60%:', str(i + 1), 'pricipal components needed.')
        elif total_explainable_ratio >= 0.5:
            if len(components_needed_list) < 1:
                components_needed_list.append(i + 1)
                print('50%:', str(i + 1), 'pricipal components needed.')

    # Plot the scree plot
    if len(explainable_ratio_list) <= 15:
        x_list = ['PC' + str(i) for i in range(1, len(explainable_ratio_list) + 1)]
        plt.plot(total_explainable_ratio_list)
        plt.bar(x_list, culmulative_explainable_ratio_list, align='center', alpha=0.5)
    else:
```

```
        x_list = ['PC' + str(i) for i in range(1, 16)]
        plt.plot(total_explainable_ratio_list[:15])
        plt.bar(x_list, culmulative_explainable_ratio_list[:15], align='center', alpha=0.5)
    plt.xticks(rotation=90)
    plt.show()

    return eigenvalues, eigenvectors, X_project
```

In [170]:

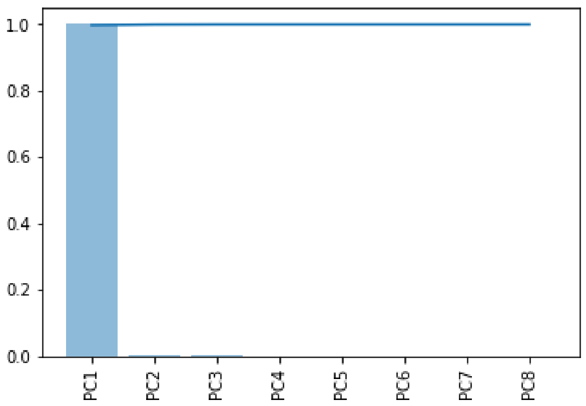
```
# implement PCA with AutoMPG dataset
AutoMPG = pd.read_csv('./data/AutoMPG.csv')

eigenvalues, eigenvectors, AutoMPG_project = PCA_decomposition(AutoMPG, True)
print('PCA with Correlation Matrix')
print('eigenvalues:', eigenvalues)
print()
print('eigenvectors:', eigenvectors)
print()
print('AutoMPG_project:',  AutoMPG_project)



eigenvalues, eigenvectors, AutoMPG_project = PCA_decomposition(AutoMPG, False)
print('PCA without Correlation Matrix')
print('eigenvalues:', eigenvalues)
print()
print('eigenvectors:', eigenvectors)
print()
print('AutoMPG_project:',  AutoMPG_project)
```

60%: 1 pricipal components needed.
70%: 2 pricipal components needed.
80%: 3 pricipal components needed.
90%: 4 pricipal components needed.




```
PCA with Correlation Matrix
eigenvalues: [5.37579947 0.94368068 0.81167701 0.48615382 0.18286556 0.11430911
 0.03196814 0.0535462 ]

eigenvectors: [[-0.38584973  0.0768251   0.29233712  0.09995932 -0.74033405  0.38734891
   0.11517032  0.19587534]
 [ 0.4023881   0.13847377  0.07222155 -0.21604069 -0.48258829 -0.53093056
   0.41770206 -0.27886477]
 [ 0.4164447   0.12637099  0.0742187  -0.13582082 -0.30329986 -0.00696426
  -0.82916136  0.08426507]
 [ 0.4018397  -0.11139899  0.23608904 -0.11972353  0.08431898  0.66672875
   0.13477375 -0.53501633]
 [ 0.40157692  0.21103513 -0.00094958 -0.32248018  0.1312946   0.23577482
   0.30996269  0.72201169]
 [-0.2647346   0.41672193 -0.63954802 -0.49281516 -0.09771744  0.20294159
  -0.03519452 -0.22890465]
 [-0.21387799  0.6905824   0.58696339 -0.10603199  0.30146799 -0.11007417
  -0.05431804 -0.12503449]
 [-0.2778673  -0.50144442  0.30745409 -0.74327015  0.04736686 -0.12085714
  -0.07951279  0.03453075]]

AutoMPG_project: [[-272.83046976 -117.33197165  -11.96660362 ... -138.12273605
```

-74.54082255 -376.28573339]
[-381.32241659 -158.44667041 -23.19016013 ... -205.12681707
-102.05824478 -497.50945919]
[-258.40565926 -101.72696191 -18.20890927 ... -135.14507474
-47.07325409 -317.6444326]
...
[309.15514682 146.39744394 1.76117171 ... 173.37431414
165.41119263 485.63717067]
[186.08782859 75.30797004 9.34206647 ... 98.03570443
51.80210414 247.8975295]
[147.85462174 55.28848696 8.93567461 ... 72.97178584
21.0029035 180.92813672]]
90%: 1 pricipal components needed.



PCA without Correlation Matrix
eigenvalues: [7.32159308e+05 1.51405465e+03 2.61318016e+02 1.23093098e+01
2.90140740e+00 1.88860286e+00 3.57475600e-01 2.57690315e-01]

eigenvectors: [[-3.22887929e-03 -7.39571693e-03 -1.68367301e-02 3.58097463e-01
1.37761080e-01 9.14674563e-01 -1.07540868e-01 6.51528870e-02]
[1.79261370e-03 1.33241105e-02 -7.29020365e-03 -2.55793640e-03
-1.87354959e-02 -1.58567937e-02 3.90300201e-01 9.20229787e-01]
[1.14340720e-01 9.45741882e-01 -3.03364700e-01 9.48215207e-03
1.03606815e-02 -1.56514533e-03 6.34075260e-04 -1.63781474e-02]
[3.89668646e-02 2.98262863e-01 9.48521534e-01 4.74430424e-02
8.52720884e-02 -1.29726089e-02 -8.28994320e-03 8.28033286e-03]
[9.92668234e-01 -1.20773235e-01 -2.48833009e-03 -5.94826172e-04
-2.71266838e-03 3.10360543e-03 -2.33657118e-04 -1.09049120e-04]
[-1.35283053e-03 -3.48265951e-02 -7.69835369e-02 -5.00497666e-02
9.86437038e-01 -1.30450550e-01 1.00203344e-02 1.33435475e-02]
[-1.33685177e-03 -2.38692393e-02 -4.30407932e-02 9.31043932e-01
-4.59157658e-03 -3.59349460e-01 3.51338495e-02 -1.85917655e-02]
[-5.51536058e-04 -3.24365681e-03 1.24486303e-02 8.41269228e-03
1.43429869e-02 1.29573354e-01 9.13617393e-01 -3.84800879e-01]]

AutoMPG_project: [[-5.36449619e+02 -5.08358444e+01 1.07053844e+01 ... -1.50243791e+00
-2.00383876e-01 -7.71177669e-01]
[-7.30349183e+02 -7.91426100e+01 -9.03776726e+00 ... -4.71265361e-01
-2.49066403e-02 -2.46702756e-01]
[-4.70986617e+02 -7.54516690e+01 -5.17422460e+00 ... -1.14517452e+00
-4.74281894e-02 -7.50696497e-01]
...
[6.85187239e+02 -2.00931891e+01 -2.93030701e-01 ... 1.38148524e-01
1.02831913e+00 1.82533974e-01]
[3.59520648e+02 3.56706345e+01 1.23051772e+00 ... 1.49371911e+00
8.20525580e-01 3.16047114e-02]
[2.65219821e+02 4.72323729e+01 -1.59888239e+00 ... 1.69806277e-01
9.97862955e-01 -9.33253015e-02]]

Q4

4. Transpose the ORL face dataset to let **X** be a 2576×400 data matrix. Apply PCA to **X**, using the PCA function you created in EX3.
   a. (10%) How many principal components are needed to explain 50%, 60%, 70%, 80%, and 90% of the total variance?
   b. (10%) Rescale the first principal component (PC) into the range of $[0, 255]$. Reshape the first PC (initially an 2576×1 vector) into a 46×56 matrix. Plot an image from the 46×56 matrix using the rescaled PC scores as the grayscale values.

Answer

(a)

50%: 2 pricipal components needed.

60%: 3 pricipal components needed.

70%: 6 pricipal components needed.

80%: 15 pricipal components needed.

90%: 47 pricipal components needed.

(b)

如下圖程式碼output所示

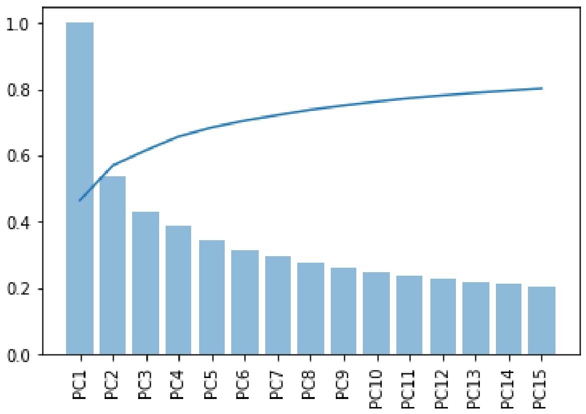
In [100]:

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import cv2
```

In [107]:

```
ORL_data = pd.read_csv('./data/ORL_data.csv').iloc[:,:-1]

ORL_data_array = np.transpose(ORL_data.to_numpy())
# print(ORL_data_array.shape)
eigenvalues, eigenvectors, X_project = PCA_decomposition(ORL_data_array, False)
# print(eigenvalues.shape)
```

```
50%: 2 pricipal components needed.
60%: 3 pricipal components needed.
70%: 6 pricipal components needed.
80%: 15 pricipal components needed.
90%: 47 pricipal components needed.
```

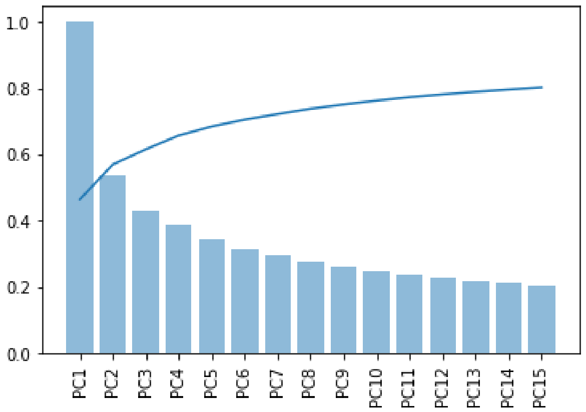


In [108]:

```
ORL_data = pd.read_csv('./data/ORL_data.csv').iloc[:,:-1]

ORL_data_array = np.transpose(ORL_data.to_numpy())
# print(ORL_data_array.shape)
eigenvalues, eigenvectors, X_project = PCA_decomposition(ORL_data_array, False)
# print(eigenvalues.shape)
# print(X_project.shape)
```

```
pc1_X_project = X_project[:, 0]
print(pc1_X_project.shape)
# print(pc1_X_project[0])
min_pc1 = np.min(pc1_X_project)
range_pc1 = np.max(pc1_X_project) - np.min(pc1_X_project)

for i, j in enumerate(pc1_X_project):
    pc1_X_project[i] = 255 * ((j - min_pc1) / range_pc1)

# print(pc1_X_project)
# print(pc1_X_project.shape)
```

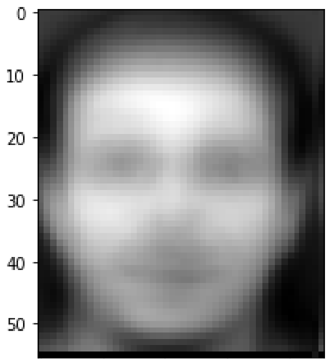
50%: 2 pricipal components needed.
60%: 3 pricipal components needed.
70%: 6 pricipal components needed.
80%: 15 pricipal components needed.
90%: 47 pricipal components needed.

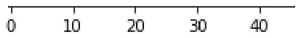


(2576,)

In [109]:

```
pixel_list_number = []
pixel_list_x = []
pixel_list_y = []

for i, j in enumerate(pc1_X_project.tolist()):
    value = (i+1)
    row = value / 46
    column = value % 46 - 1
    pixel_list_number.append(value)
    pixel_list_x.append(row)
    pixel_list_y.append(column)


sample = np.zeros(shape=(56, 46))
for i, j in enumerate(pixel_list_x):
    x_value = int(j - 1)
    y_value = int(pixel_list_y[i] - 1)
    sample[x_value, y_value] = pc1_X_project[i]

imgplot = plt.imshow(sample, cmap='gray', vmin=0, vmax=255)
plt.show()
```




0
10
20
30
40